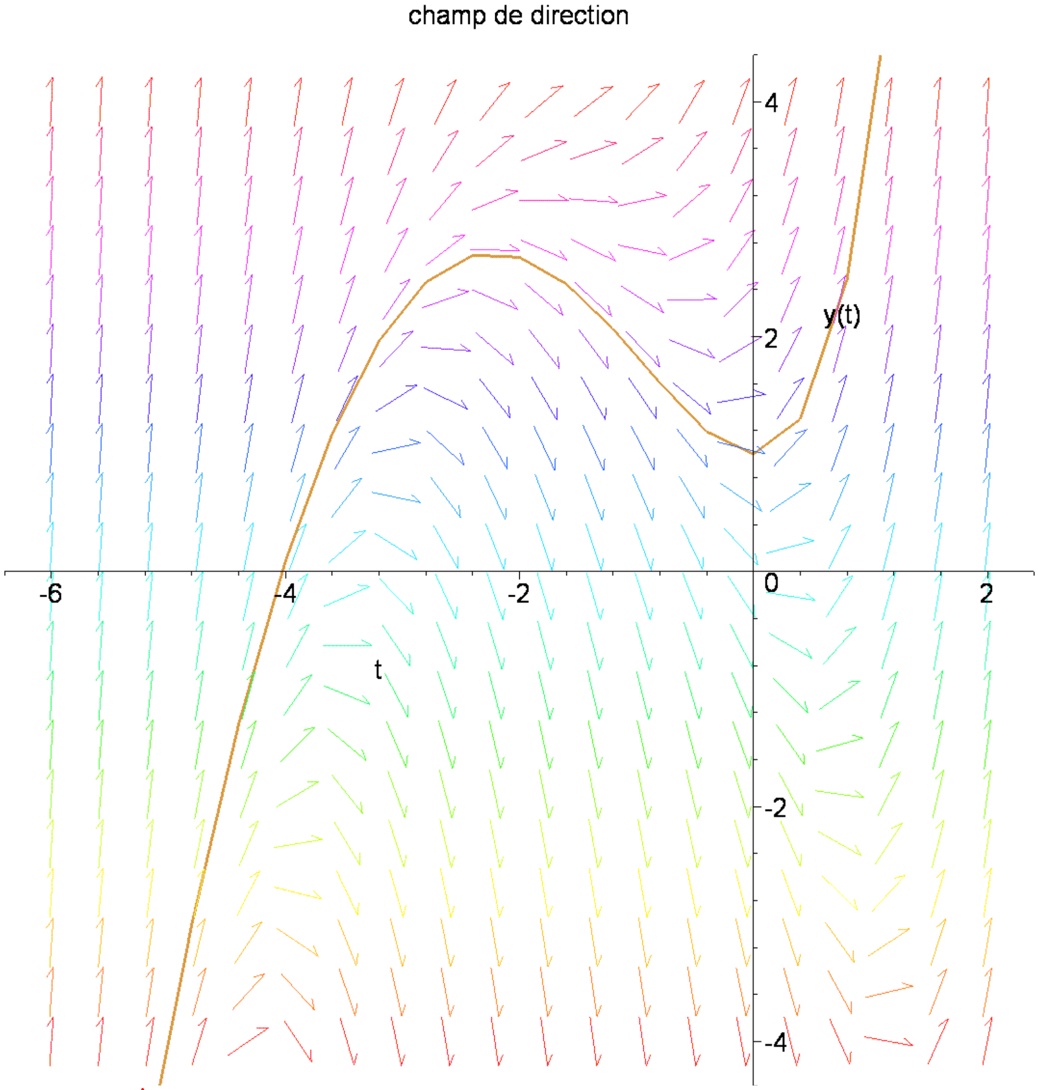
```
> ##############################   TD 12/11/07   Mat237
> ############################### Ex1(2)
   restart;with(DEtools):
  ode := diff(y(t),t)=y(t)+t^2+3*t-1;
  dsolve({ode, y(0)=1});
> DEplot(ode,y(t),t=-6..2,
  y=-4..4,[[y(0)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,\mathrm{y}(t) = \mathrm{y}(t) + t^2 + 3\,t - 1$$

$$\mathrm{y}(t) = -4 - 5\,t - t^2 + 5\,\mathbf{e}^t$$




```
> y:=-4-5*t-t^2+5*exp(t);
```

$$y := -4 - 5\,t - t^2 + 5\,\mathbf{e}^t$$

```
> fsolve(y=0, t=-5..1);
```

$$-4.029355723$$

```
> yp:=diff(-4-5*t-t^2+5*exp(t),t);
```

$$yp := -5 - 2\,t + 5\,\mathbf{e}^t$$

```
> evalf(solve(yp=0));
```

-2.231611884, 0.

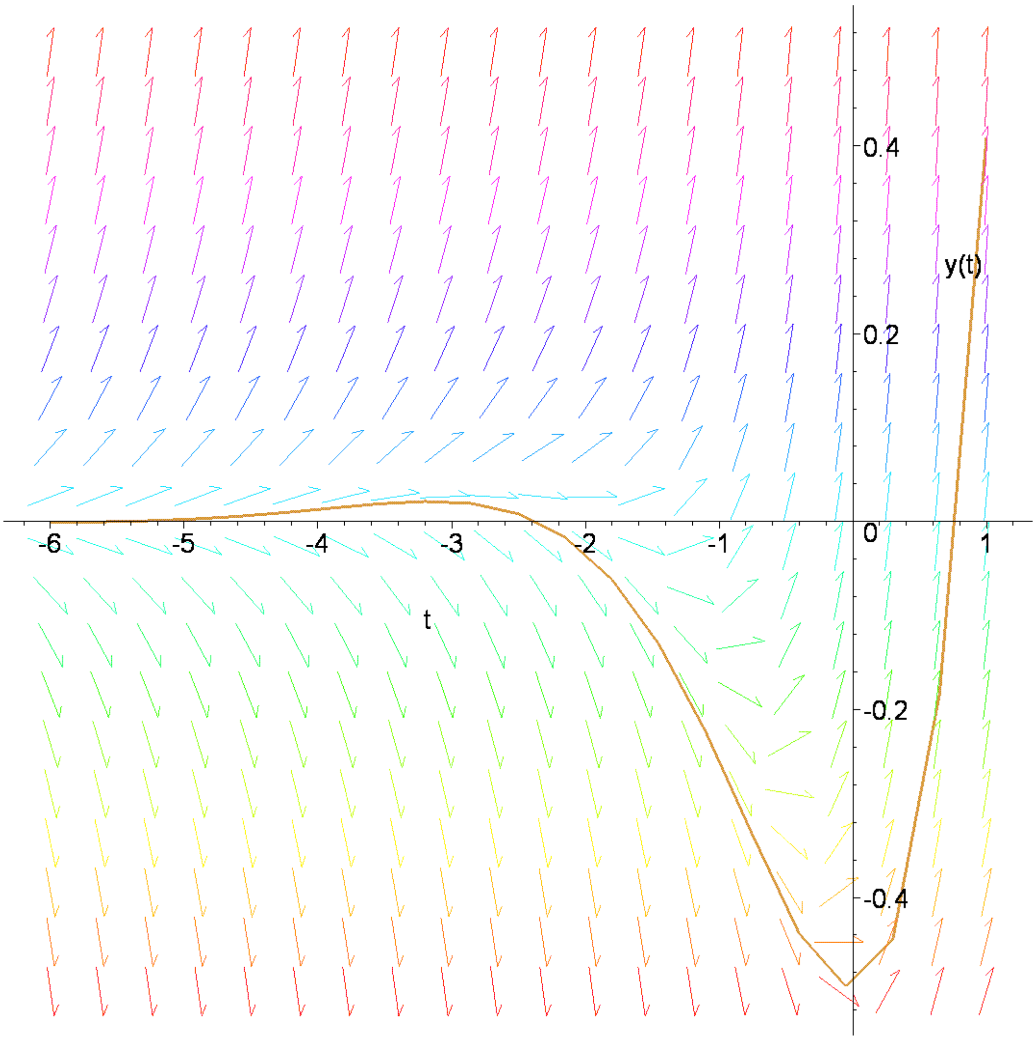
```
> ################################ Ex1(3)

  restart;with(DEtools):
  ode := diff(y(t),t)=2*y(t)+exp(t)*cos(t);
  dsolve({ode, y(Pi/4)=0});
> DEplot(ode,y(t),t=-6..1,
  y=-0.5..0.5,[[y(Pi/4)=0]],
  linecolor=[gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,\mathrm{y}(t) = 2\,\mathrm{y}(t) + \mathbf{e}^{t}\cos(t)$$

$$\mathrm{y}(t) = -\frac{1}{2}\,\mathbf{e}^{t}\cos(t) + \frac{1}{2}\sin(t)\,\mathbf{e}^{t}$$

champ de direction

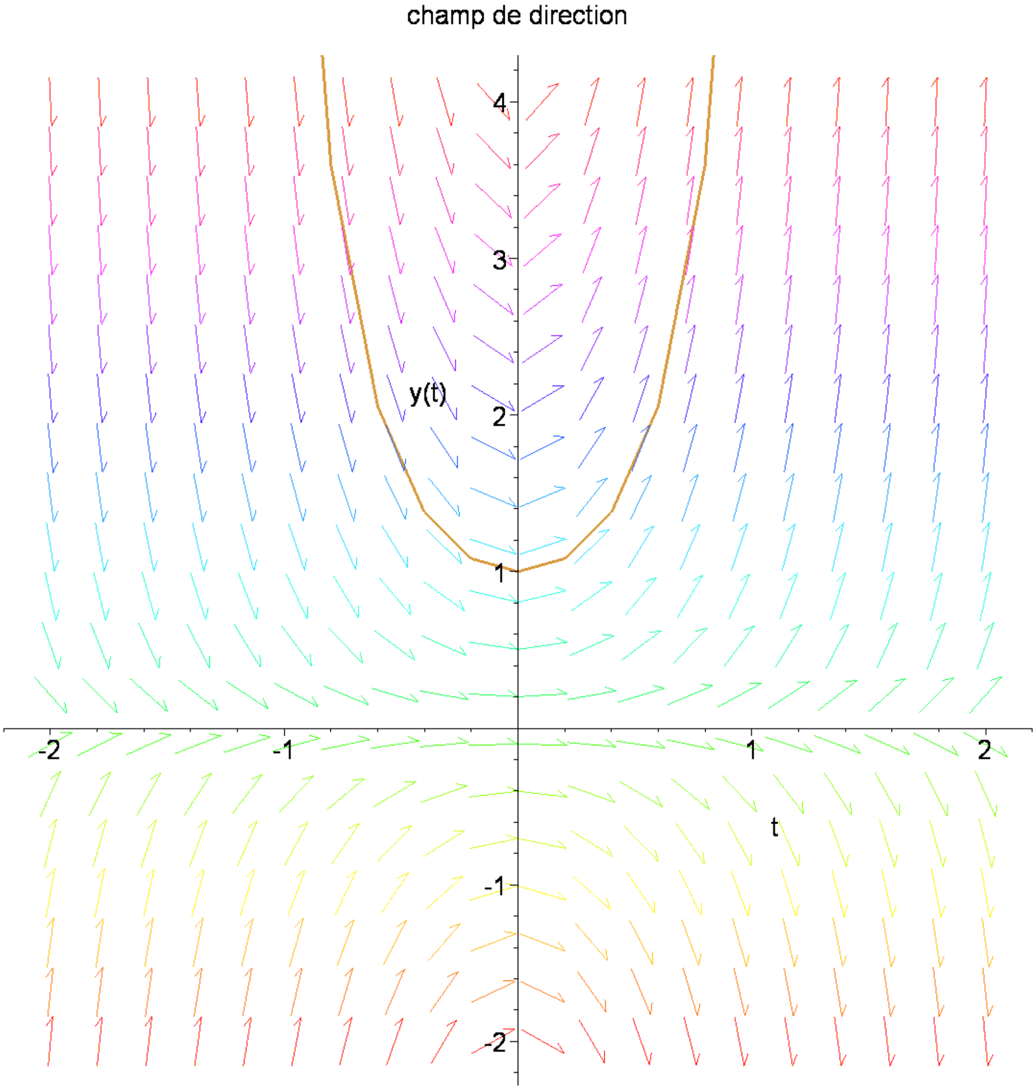



```
> ################################ Ex1(4)
  restart;with(DEtools):
  ode := diff(y(t),t)=4*t*y(t);
  dsolve({ode, y(0)=1});
```

```
> DEplot(ode,y(t),t=-2..2,
  y=-2..4,[[y(0)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,y(t) = 4\,t\,y(t)$$

$$y(t) = \mathbf{e}^{(2\,t^2)}$$

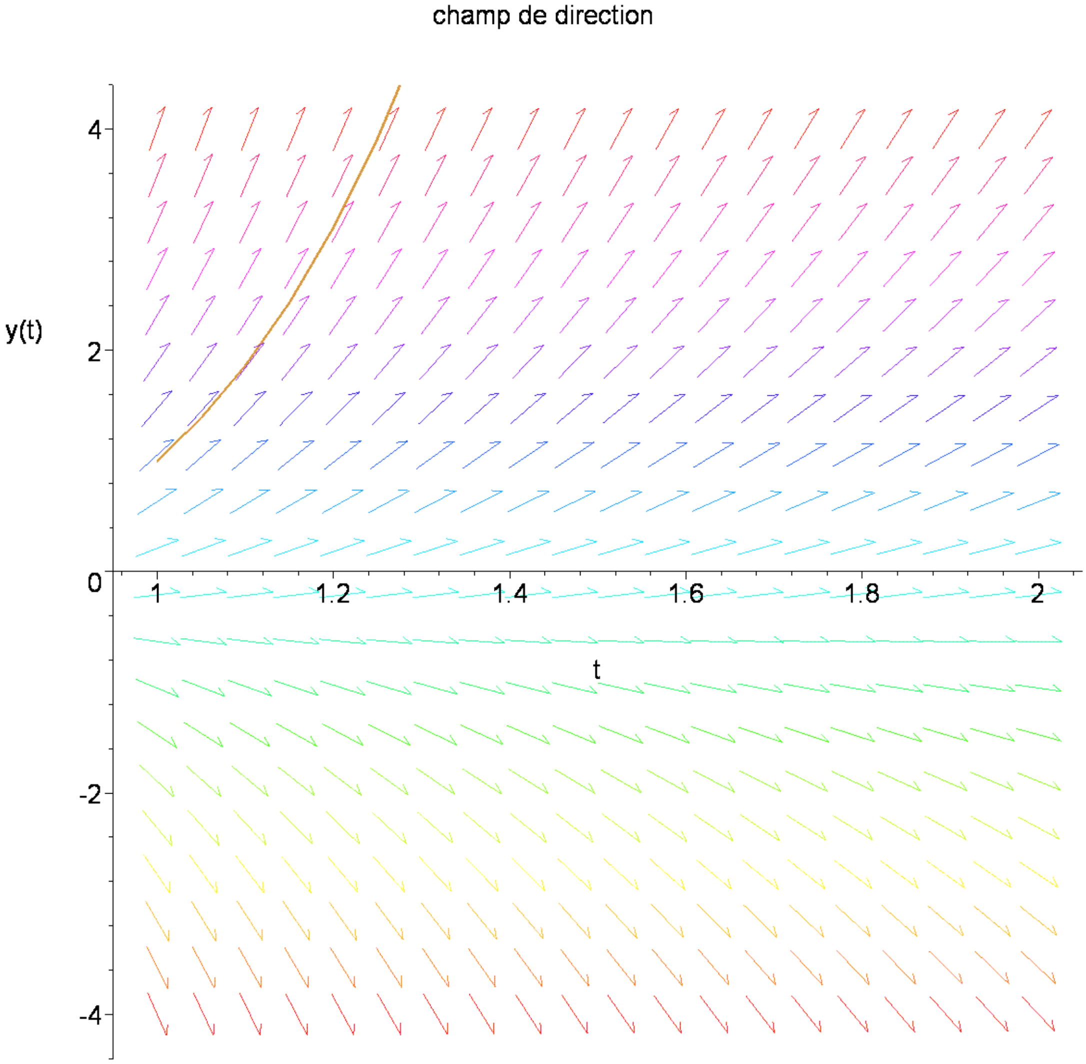



```
> ############################### Ex1(5)

  restart;with(DEtools):
  ode := diff(y(t),t)=5*y(t)/t+(t+1)/t;
  dsolve({ode,y(1)=1});
> DEplot(ode,y(t),t=1..2,
  y=-4..4,[[y(1)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,y(t) = \frac{5\,y(t)}{t} + \frac{t+1}{t}$$

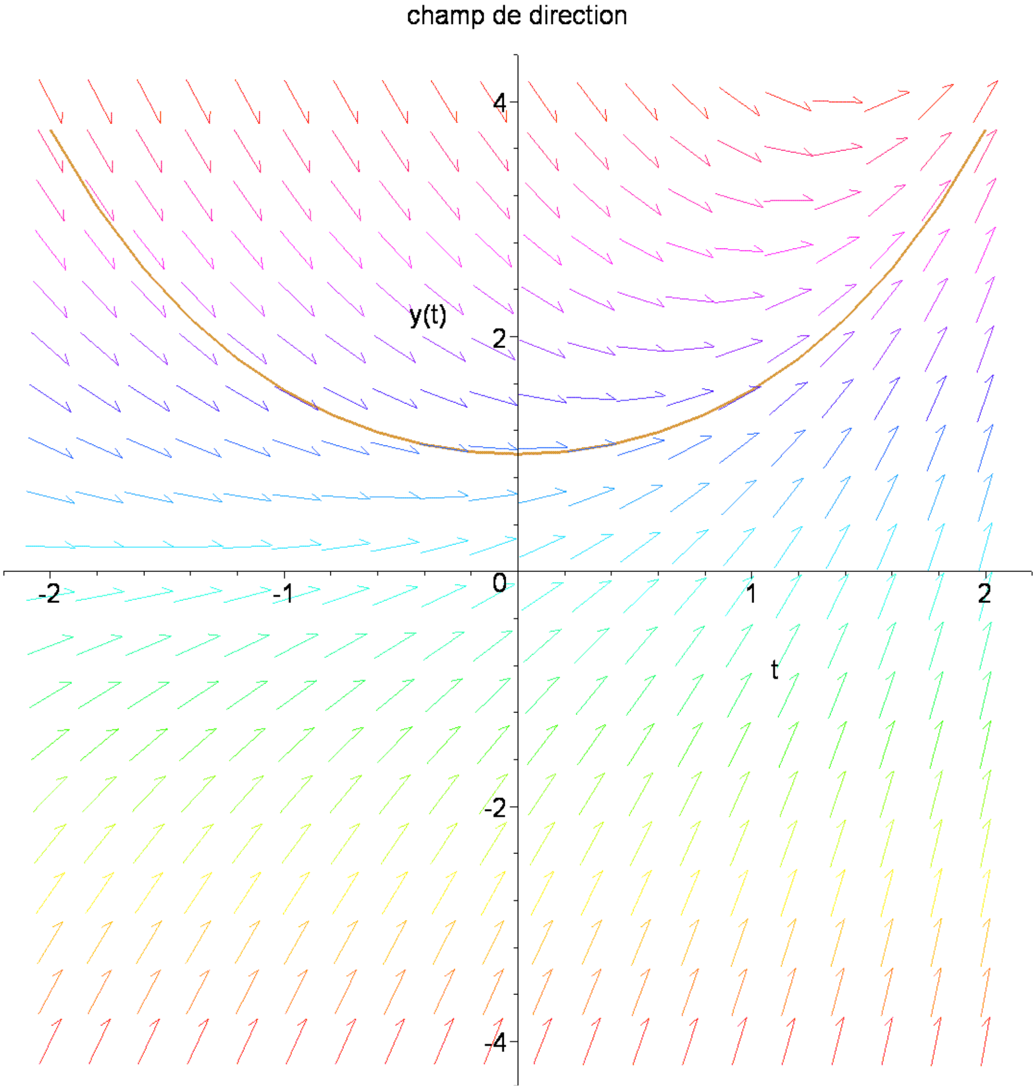
$$y(t) = -\frac{1}{4}t - \frac{1}{5} + \frac{29}{20}t^5$$




```
> ################################ Ex2
  restart;with(DEtools):
  ode := diff(y(t),t)=-y(t)+exp(t);
  dsolve({ode});
> DEplot(ode,y(t),t=-2..2,
  y=-4..4,[[y(0)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}y(t) = -y(t) + e^t$$

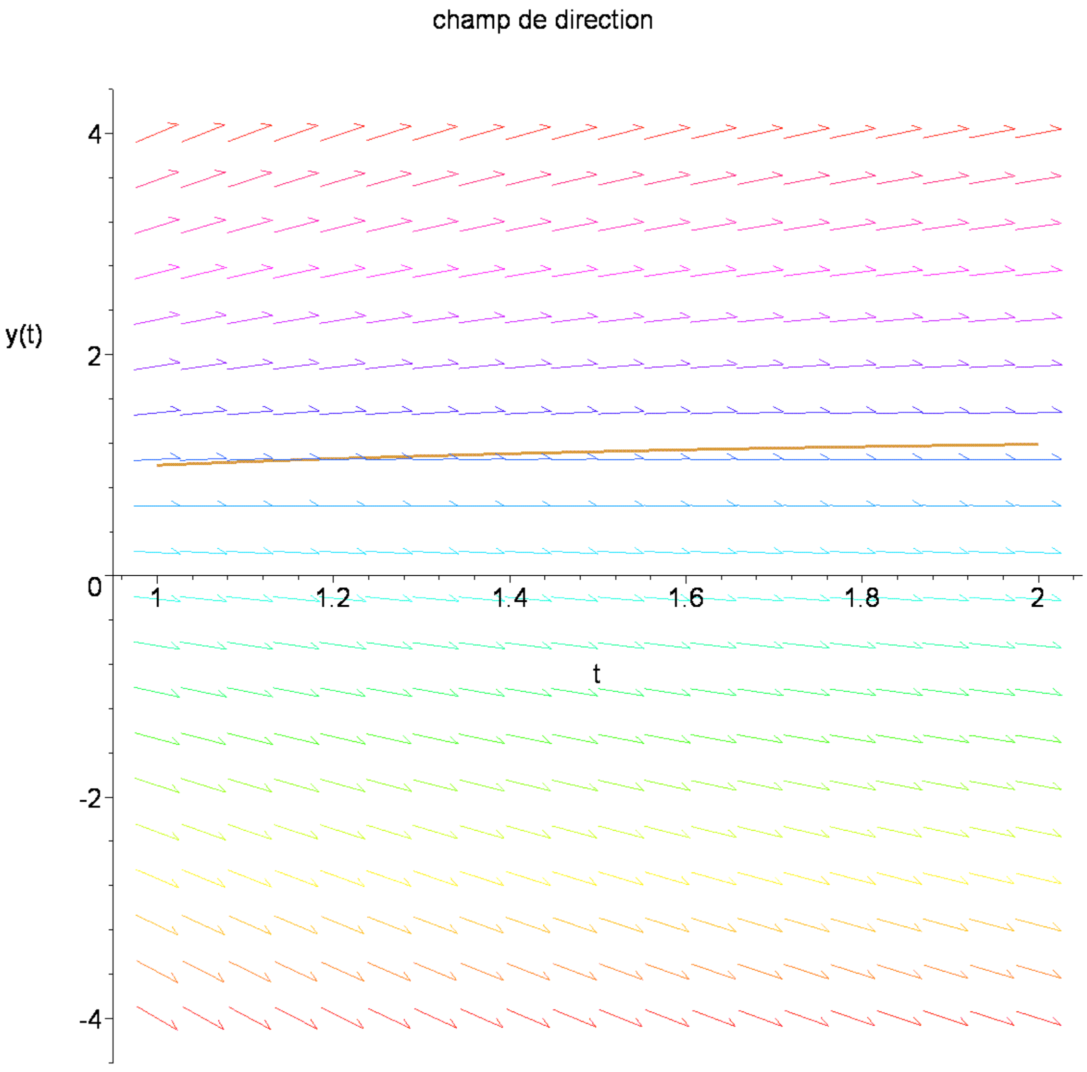
$$\{y(t) = \frac{1}{2}e^t + e^{(-t)}\_C1\}$$




```
> ################################ Ex3
  restart;with(DEtools):
  ode := -t*diff(y(t),t)+y(t)=2*t/(t+2);
  dsolve({ode});
> DEplot(ode,y(t),t=1..2,
  y=-4..4,[[y(1)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := -t\left(\frac{d}{dt}\,y(t)\right) + y(t) = \frac{2\,t}{t+2}$$

$$\{y(t) = (-\ln(t) + \ln(t+2) + \_C1)\,t\}$$




```
> ##########################################      Ex4
> restart;with(DEtools):
  assume(a::real,b::real, c::real):
   ode  := diff(y(t),t)-y(t)=a*t^2+b*t+c;
  dsolve({ode});
```

$$ode := \left(\frac{d}{dt}\,\mathrm{y}(t)\right) - \mathrm{y}(t) = a\sim t^2 + b\sim t + c\sim$$

$$\{\mathrm{y}(t) = -c\sim - b\sim - 2\,a\sim - b\sim t - 2\,a\sim t - a\sim t^2 + \mathbf{e}^t\,\_C1\}$$

```
> ###########################################
  int(exp(-t)*(t^2+t+2),t);
```

$$-(5 + 3\,t + t^2)\,\mathbf{e}^{(-t)}$$

Exercice 5. (a) Résoudre l'équation différentielle suivante
y'= y + t + 2 ; y(0) = 1.
(b) Tracer la solution et étudier le comportement de la solution en -1 et en +1.
(c) Trouver le point où la solution atteint son minimum sur R, et donner la valeur de ce minimum.

```
> ##################################################      Ex 5
```

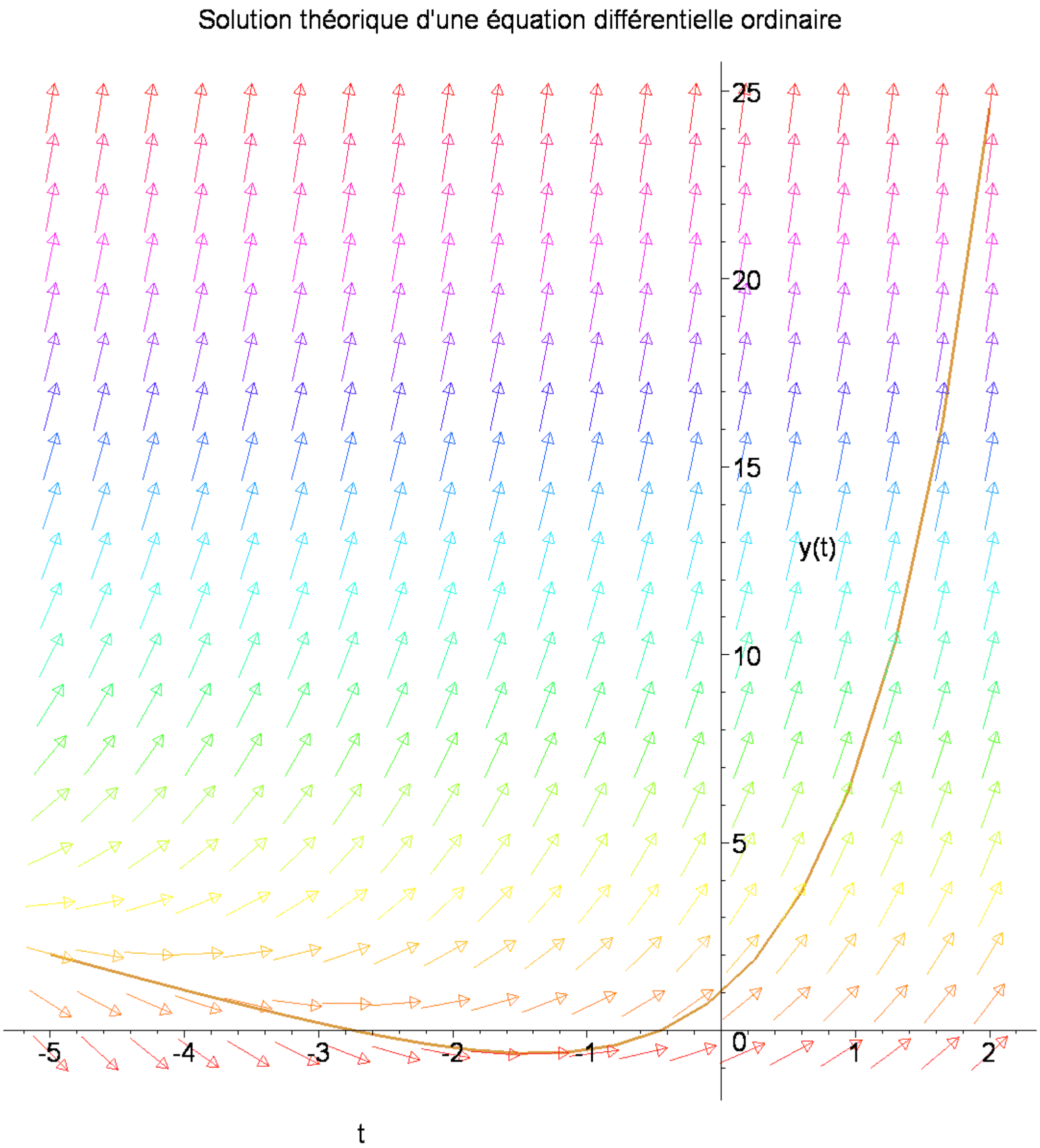
```
#####################################################
restart;ode:=diff(y(t),t)=y(t)+t+2;
########################################
> ##########################
sol:=dsolve({ode, y(0)=1}, y(t));
f(t):=solve(sol,y(t));
tmin:=solve(diff(f(t),t)=0);
######################################
with(DEtools):DEplot(ode,y(t),t=-5..2,
[[y(0)=1]],
title=`Solution théorique d'une équation différentielle
ordinaire`,
colour=y,
#labels=[t, y],
arrows=MEDIUM,
linecolor=[gold]
);
```

$$ode := \frac{d}{dt}\mathrm{y}(t) = \mathrm{y}(t) + t + 2$$

$$sol := \mathrm{y}(t) = -3 - t + 4\,\mathbf{e}^{t}$$

$$\mathrm{f}(t) := -3 - t + 4\,\mathbf{e}^{t}$$

$$tmin := -2\ln(2)$$




```
> ymin:=eval(f(t),t=tmin);
>
```

$$ymin := -2 + 2\ln(2)$$

Exercice 6. Équations différentielles à variables séparées : résoudre l'équation différentielle suivante :

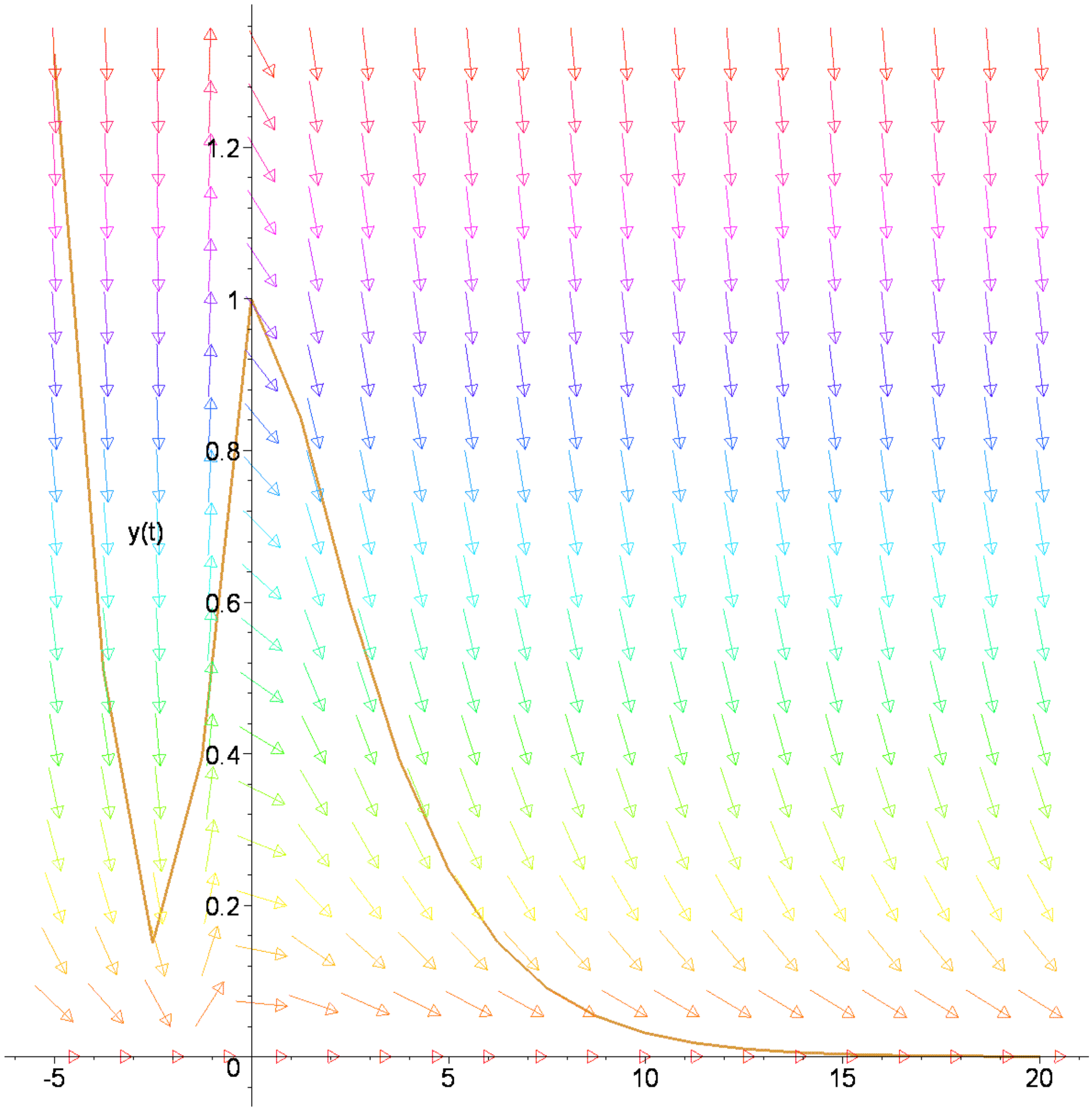
$(2t + 3)y'(t) + ty(t) = 0$; $y(0) = 1$.

```
> restart;ode:=(2*t+3)*diff(y(t),t)+t*y(t)=0;
  #######################################   Ex6
> ##########################
  #sol:=dsolve({ode, y(0)=1}, y(t));
  #f(t):=solve(sol,y(t));
  #tmin:=solve(diff(f(t),t)=0);
  #####################################
  with(DEtools):DEplot(ode,y(t),t=-5..20,
  [[y(0)=1]],
  title=`Solution théorique d'une équation différentielle
  ordinaire`,
```

```
colour=y,
#labels=[t, y],
arrows=MEDIUM,
# stepsize=0.01,
linecolor=[gold]
);
```

$$ode := (2\,t+3)\left(\frac{d}{dt}\,\mathrm{y}(t)\right)+t\,\mathrm{y}(t)=0$$

Solution théorique d'une équation différentielle ordinaire




```
> ###############################################################
restart;ode:=((2*t+3)*diff(y(t),t)+t*y(t)=0);
dsolve({ode}, y(t));
dsolve({ode,y(0)=1}, y(t));
#######################################
with(DEtools):DEplot(ode,y(t),t=0..20,
```

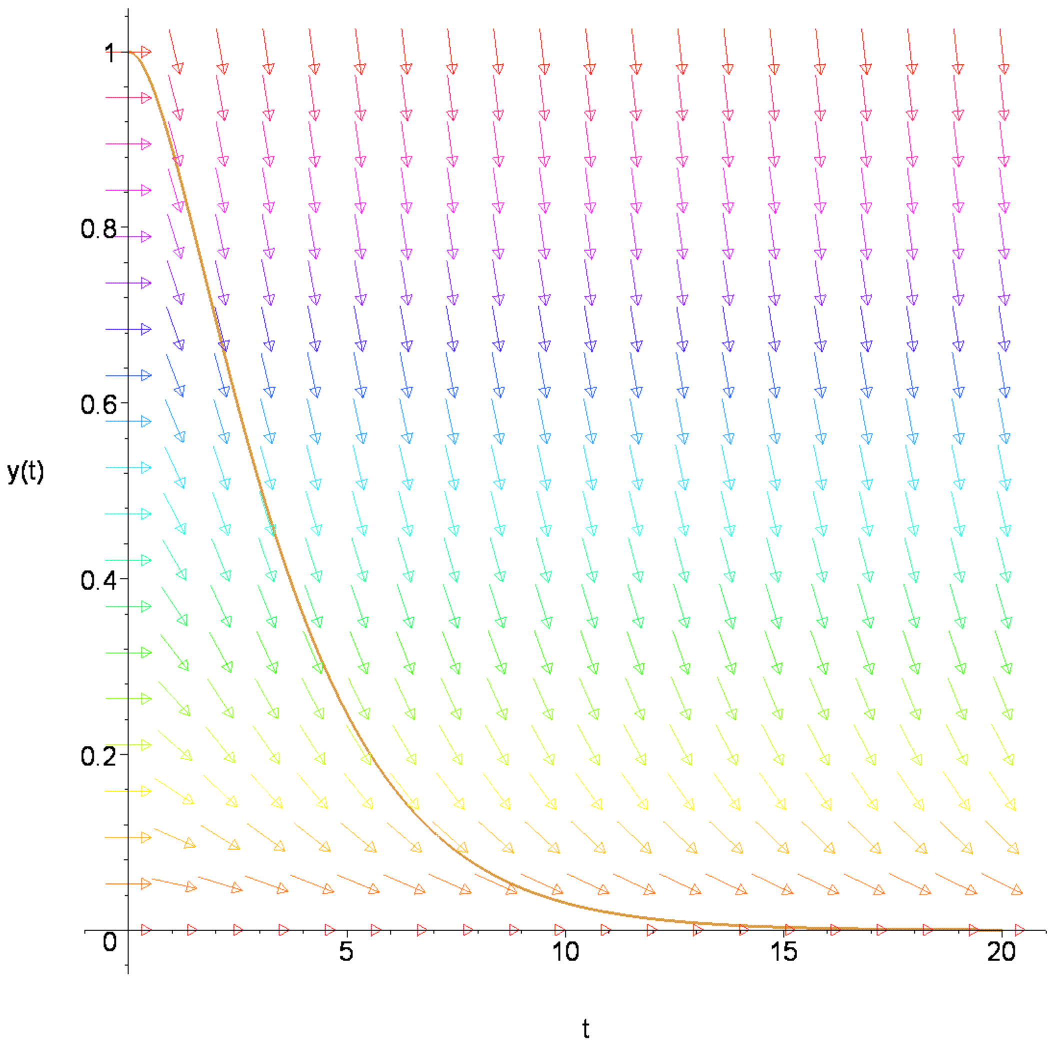
```
[[y(0)=1]],
title=`Solution théorique d'une équation différentielle
ordinaire`,
colour=y,
#labels=[t, y],
arrows=MEDIUM,
linecolor=[gold], stepsize=0.001
);
```

$$ode := (2\,t+3)\left(\frac{d}{dt}\,\mathrm{y}(t)\right)+t\,\mathrm{y}(t)=0$$

$$\{\mathrm{y}(t)=\_C1\,\mathbf{e}^{\left(-\frac{t}{2}\right)}\,(2\,t+3)^{(3/4)}\}$$

$$\mathrm{y}(t)=\frac{1}{3}\,3^{(1/4)}\,\mathbf{e}^{\left(-\frac{t}{2}\right)}\,(2\,t+3)^{(3/4)}$$

Solution théorique d'une équation différentielle ordinaire



Exercice 7.

1) La désintégration de 50 % de matière radioactive s'est produite dans 30 jours. Dans combien de temps restera-t-il 1 % de toute la quantité initiale ?

```
> ################################################    Ex 7
  evalf(solve(x(0)*2^(-t/30)=
  x(0)/100));

  evalf(log(2));
```

$$199.3156857$$

$$0.6931471806$$

```
> ################################
  evalf(30*log(100)/log(2));

>
```

$$199.3156857$$

```
> ############
> evalf(log(.5)/log(1-.00044));
```

$$1574.987902$$

```
> evalf(log(2)/.00044);
>
```

$$1575.334502$$

Exercice 7.

2) Selon les expériences, la désintégration annuelle du radium est de l'ordre 0,44 mg par gramme. En combien d'années la moitié de toute la réserve de radium se désintégrera-t-elle ?

```
> #########################################      Ex8a
> restart;wi*th(DEtools):
   ode := diff(y(t),t)=y(t)*(5-y(t));
  dsolve({ode});
```

$$ode := \frac{d}{dt}\,y(t) = y(t)\,(5 - y(t))$$

$$\{y(t) = \frac{5}{1 + 5\,e^{(-5\,t)}\,\_C1}\}$$

```
> #########################################      Ex8b
> restart;with(DEtools):
   ode := diff(y(t),t)=y(t)^2+10;
  dsolve({ode});
```

$$ode := \frac{d}{dt}\,y(t) = y(t)^2 + 10$$

$$\{y(t) = \sqrt{10}\,\tan(\sqrt{10}\,t + \sqrt{10}\,\_C1)\}$$

```
> #########################################      Ex8c
> restart;with(DEtools):
   ode := diff(y(t),t)=y(t)^2-10*y(t)+9;
  dsolve({ode});
```

$$ode := \frac{d}{dt}\,y(t) = y(t)^2 - 10\,y(t) + 9$$

$$\{y(t) = \frac{-9 + e^{(8\,t)}\,\_C1}{e^{(8\,t)}\,\_C1 - 1}\}$$

```
>
>

  ##############################
```

Exercice 9. On considère les deux équations différentielles dépendant de deux paramètres réels a et b.

(E1) u'= au + b

(E2) u' = au + bu^2.

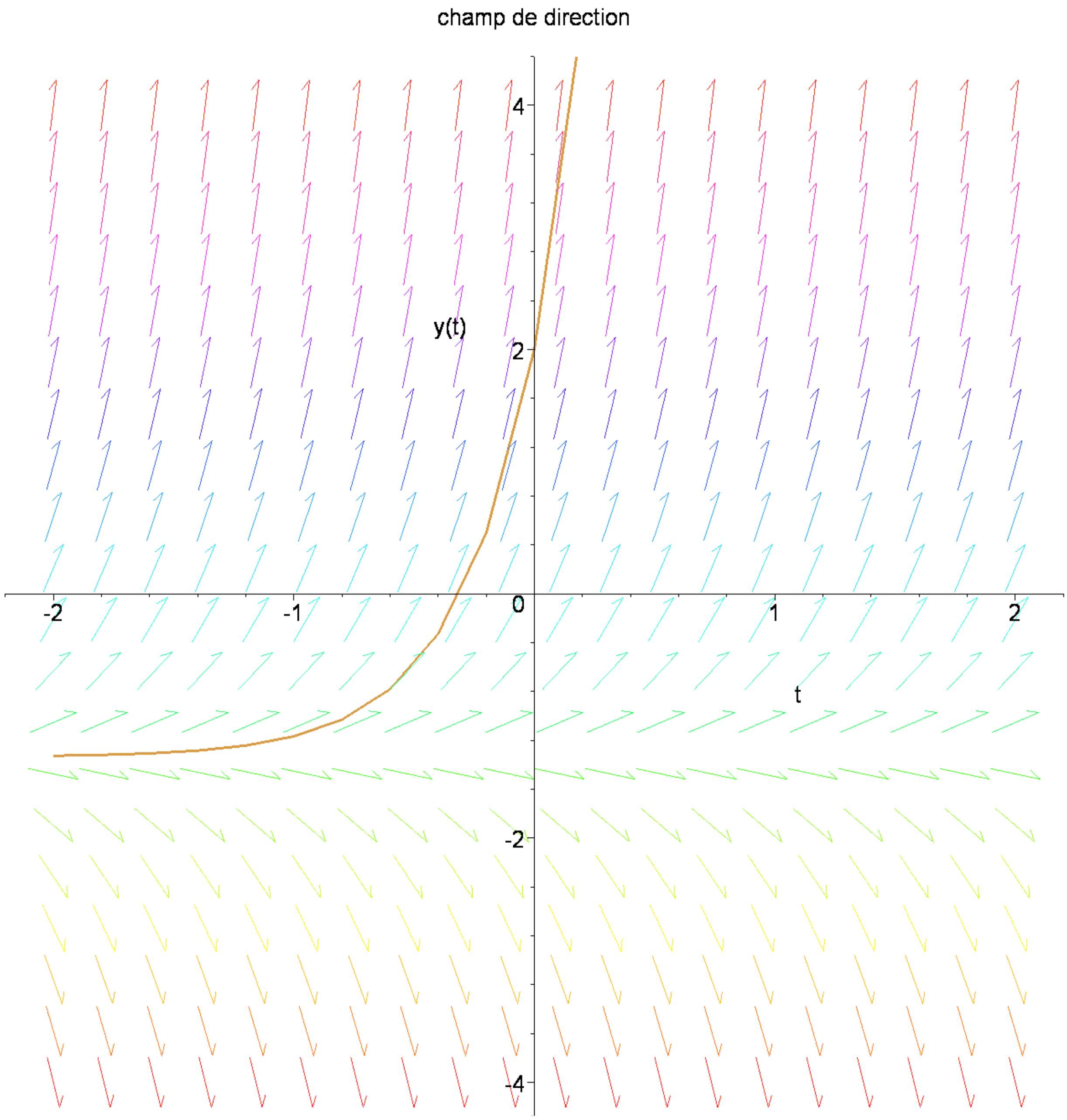
Pour chacune d'elles

1. Montrer que la donnée d'une condition initiale u(0) détermine u de façon unique.
2. Déterminer les solutions stationnaires.
3. Résoudre analytiquement, et donner l'allure des solutions.

```
> ##############################################    Ex 9
> restart;with(DEtools):
  ode := diff(y(t),t)=3*y(t)+4;
  dsolve({ode,y(0)=2});
> DEplot(ode,y(t),t=-2..2,
  y=-4..4,[[y(0)=2]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,y(t) = 3\,y(t) + 4$$

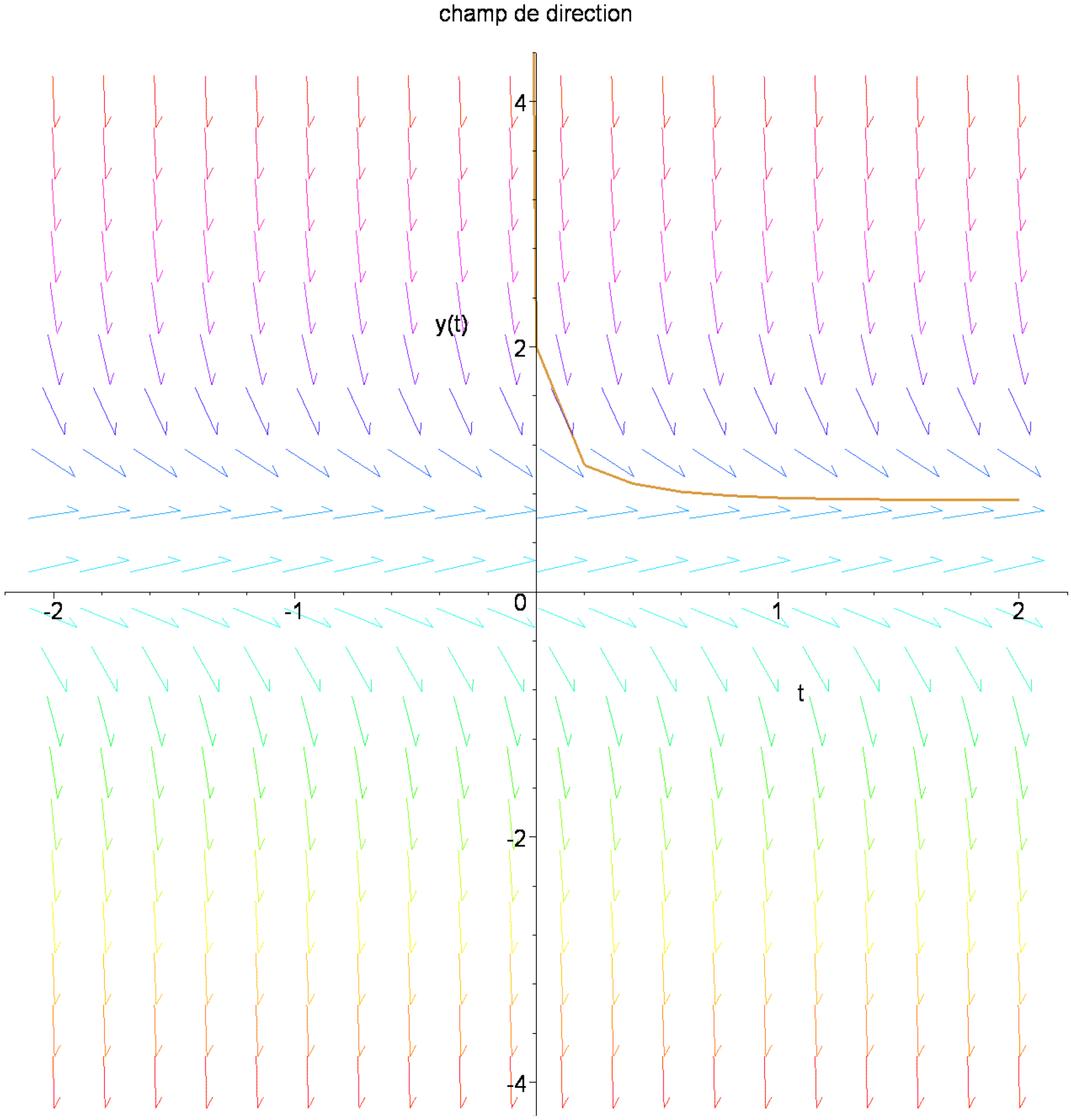
$$y(t) = -\frac{4}{3} + \frac{10}{3}\,e^{(3\,t)}$$




```
> ############################################
> restart;with(DEtools):
  ode := diff(y(t),t)=3*y(t)-4*y(t)^2;
  dsolve({ode,y(0)=2});
> DEplot(ode,y(t),t=-2..2,
  y=-4..4,[[y(0)=2]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,y(t) = 3\,y(t) - 4\,y(t)^2$$

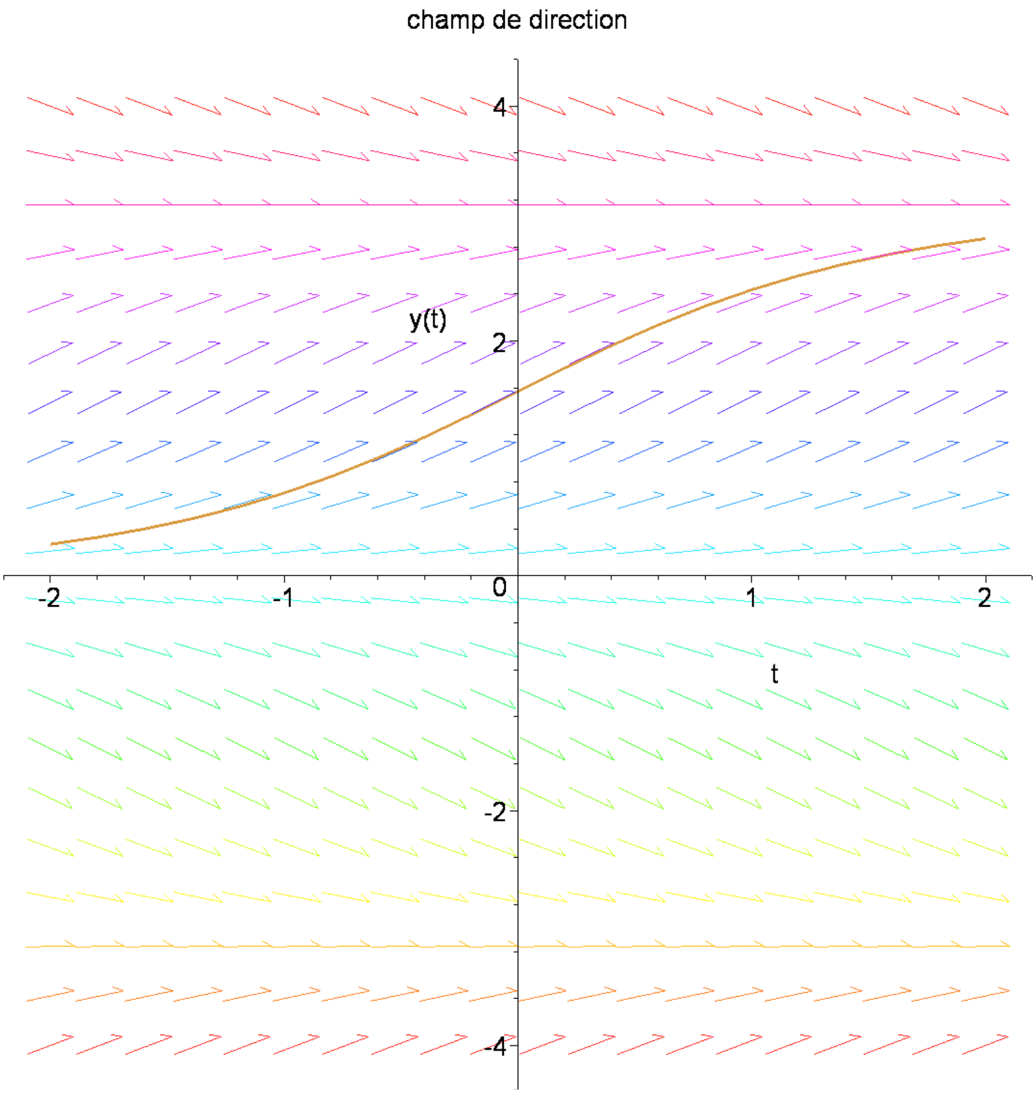
$$y(t) = -\frac{6}{-8 + 5\,e^{(-3\,t)}}$$




```
> ##########################################
> ##########################################    Ex 9(supplem)
  restart;with(DEtools):
  ode := diff(y(t),t)=sin(y(t));
  dsolve({ode});
> DEplot(ode,y(t),t=-2..2,
  y=-4..4,[[y(0)=Pi/2]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\, y(t) = \sin(y(t))$$

$$\{ y(t) = \arctan\left(\frac{2\, e^{t}\, \_C1}{1 + e^{(2\,t)}\, \_C1^2}, \frac{-e^{(2\,t)}\, \_C1^2 + 1}{1 + e^{(2\,t)}\, \_C1^2} \right) \}$$

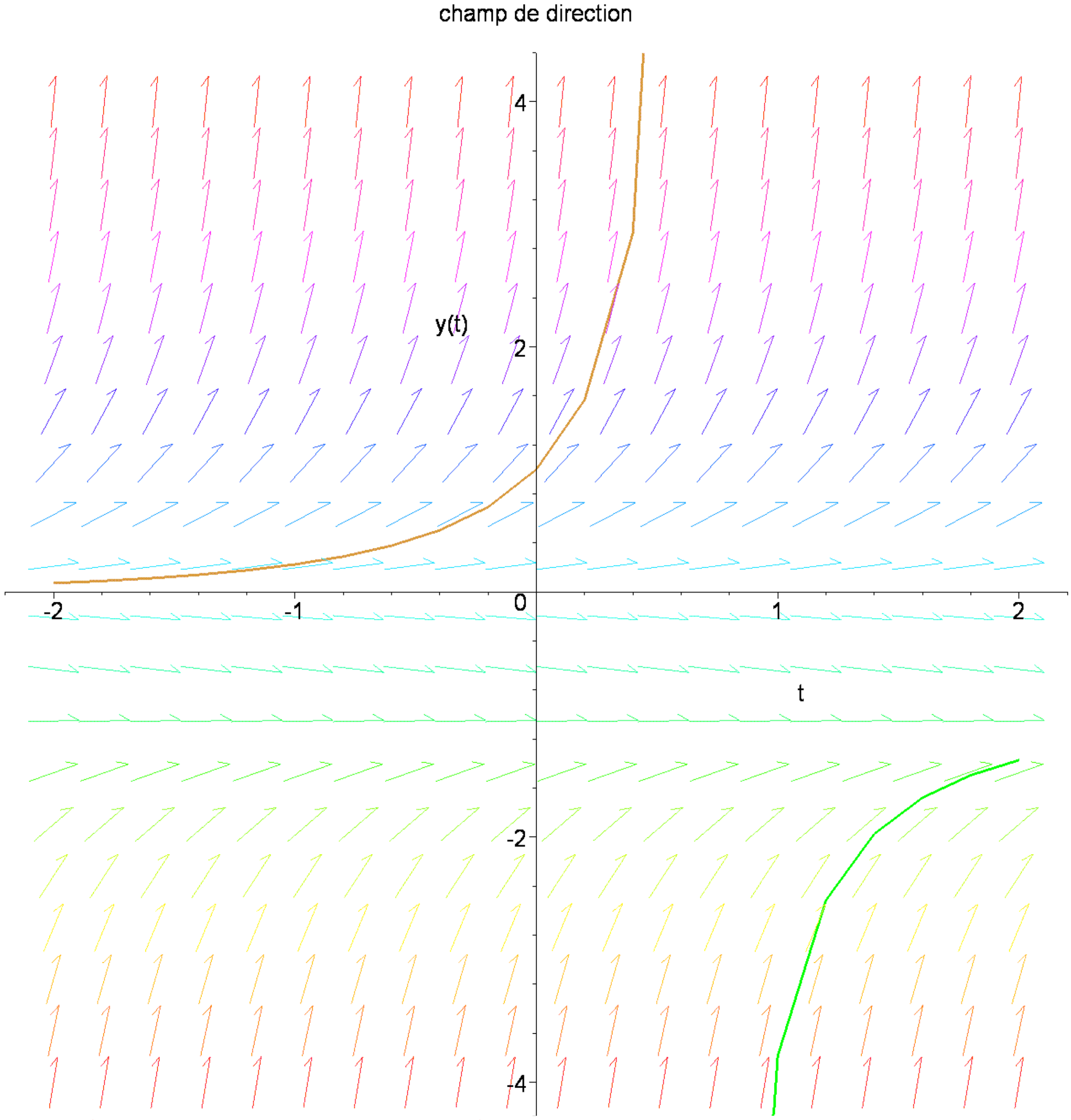



```
> ##########################################################   Supplem

restart;
with(DEtools):
ode1 := diff(y(t),t)=(y(t))^2+y(t);
dsolve({ode1, y(0)=1});
> DEplot(ode1,y(t),t=-2..2,
y=-4..4,[[y(0)=1],[y(1)=1/(2*exp(-1)-1)]],
linecolor=[ gold, green],title=`champ de direction`,
color=y-1);
```

$$ode1 := \frac{d}{dt}\,y(t) = y(t)^2 + y(t)$$

$$y(t) = \frac{1}{-1 + 2\,\mathbf{e}^{(-t)}}$$

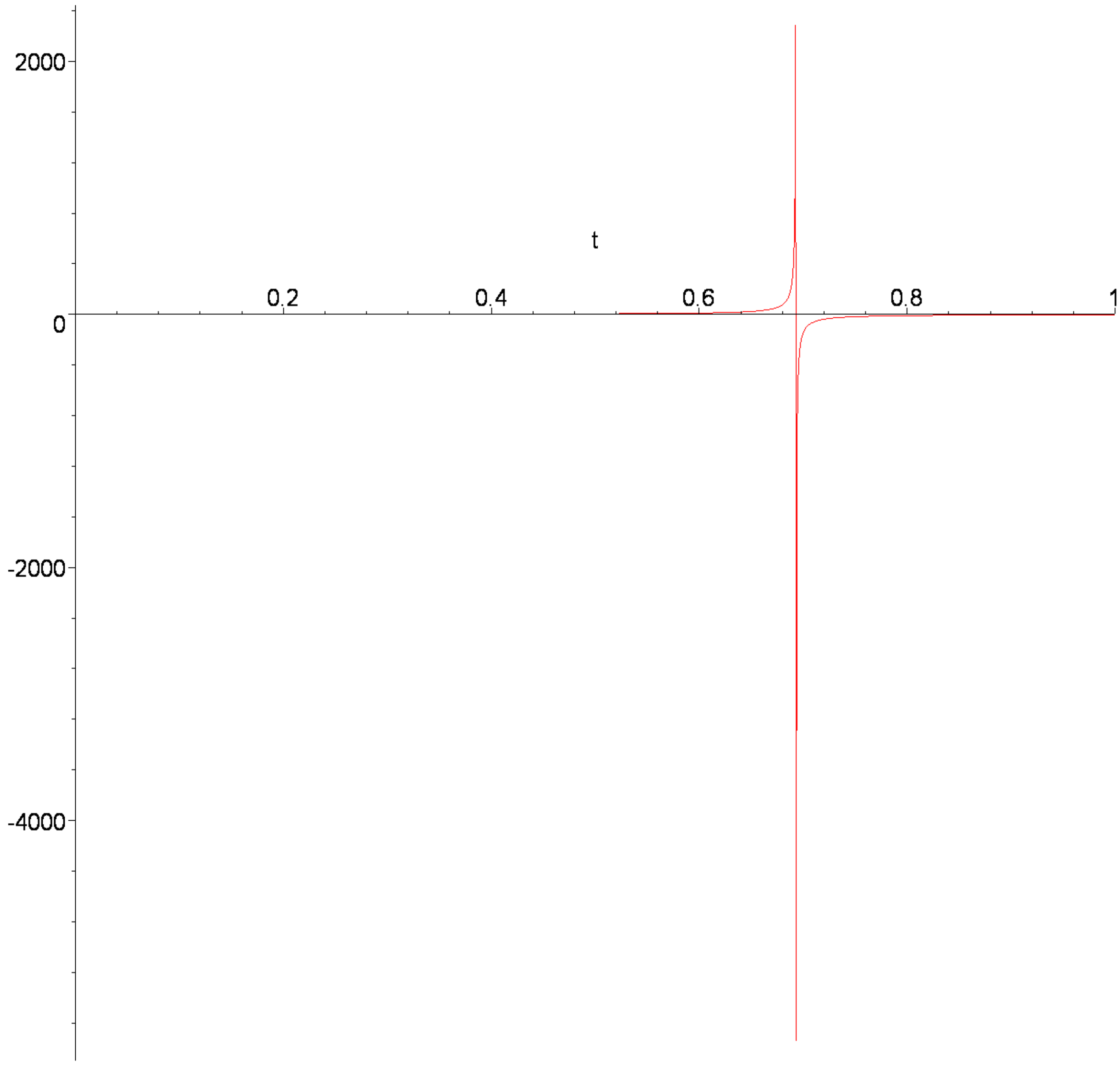



```
> y:=1/(-1+2*exp(-t));evalf(1/(2*exp(-1)-1));
```

$$y := \frac{1}{-1 + 2\,\mathbf{e}^{(-t)}}$$

-3.784422383

```
> plot(y, t=0..1);
```



2000
t
0.2
0.4
0.6
0.8
1
0
-2000
-4000

>

>

>