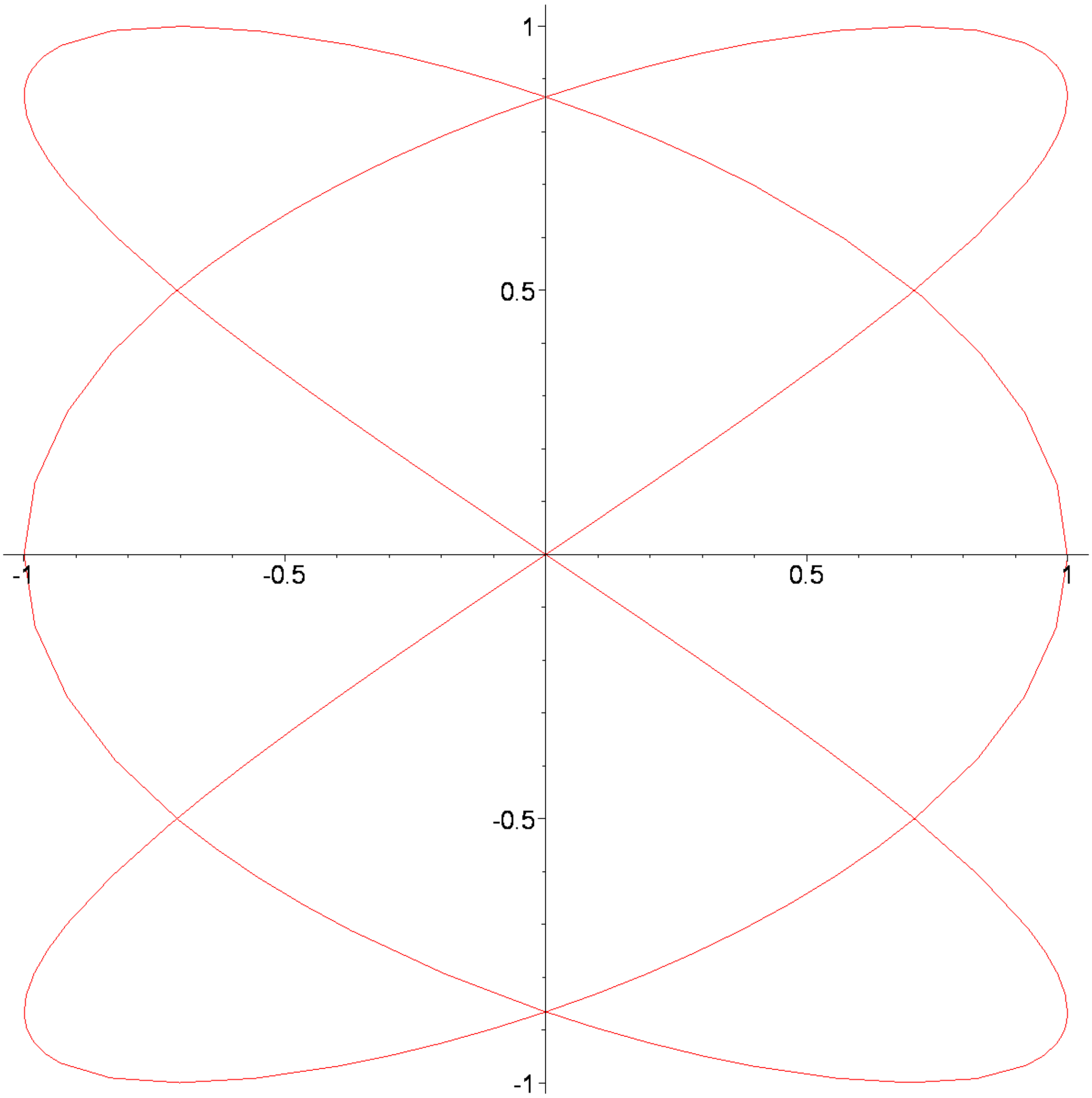
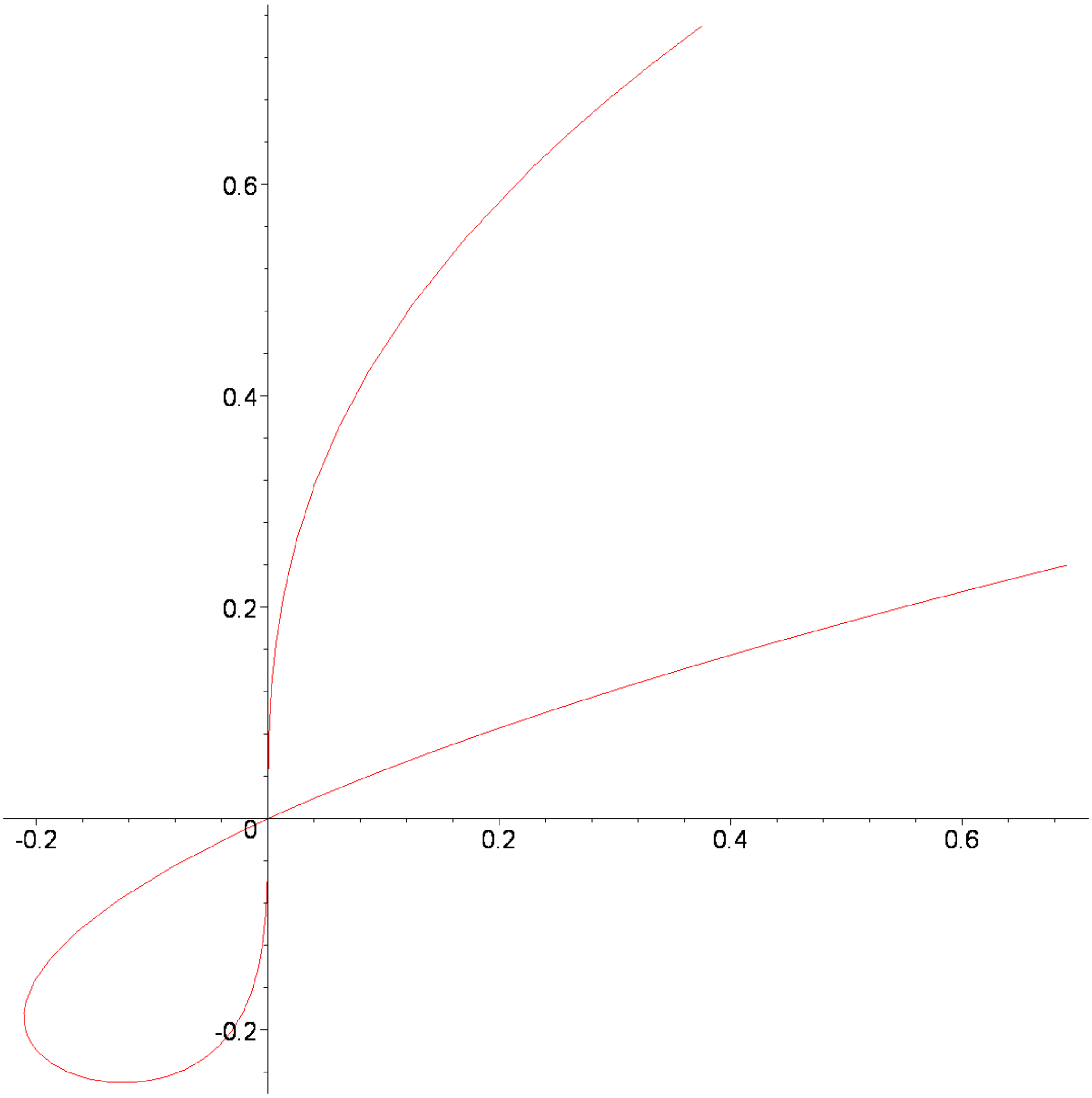
```
> #####################     FEUILLE N3 237
  #####################################    Exercice 1.
  plot([cos(3*t), sin(2*t), t=-Pi..Pi]);
```

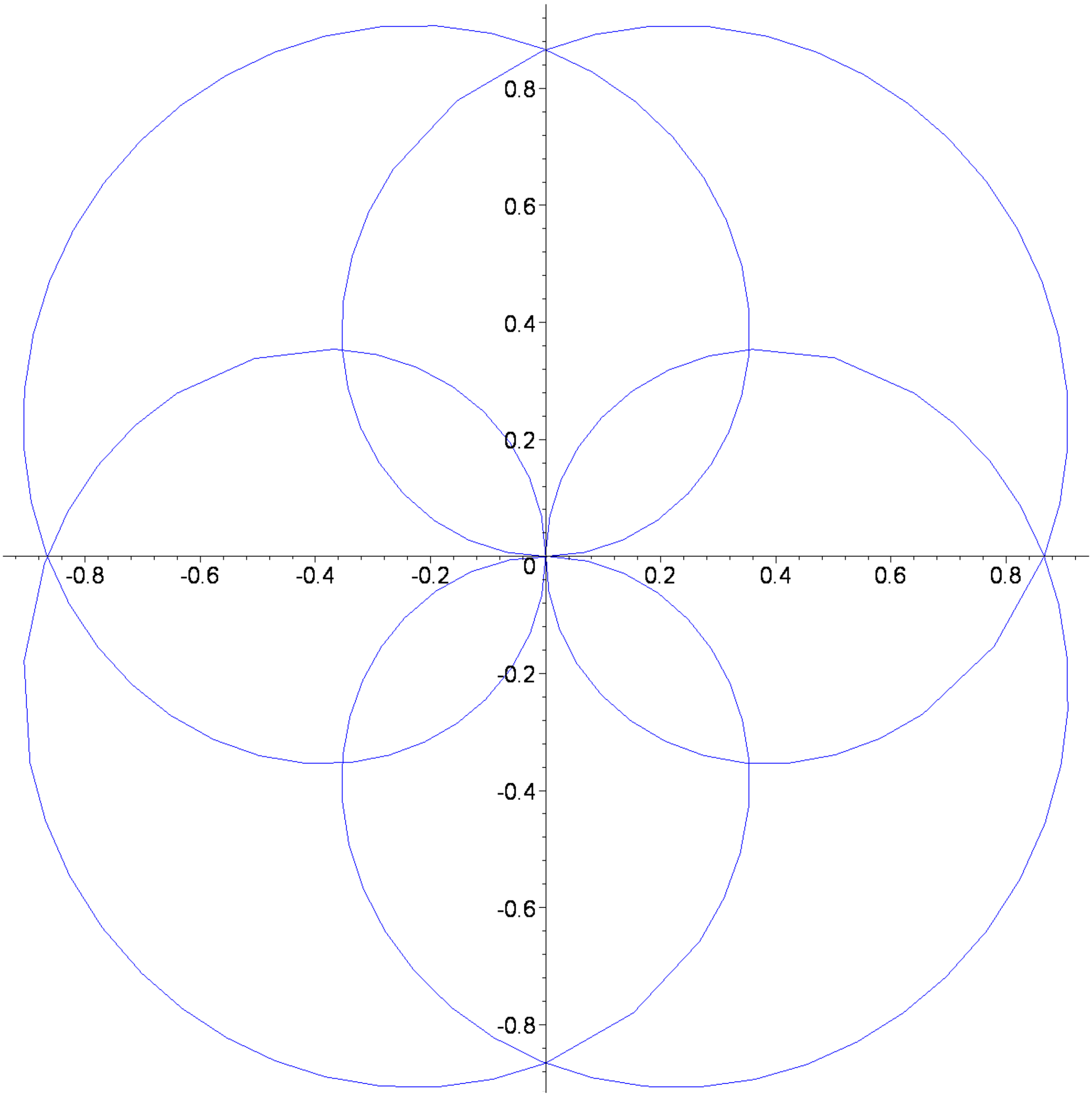



```
> #####################################    Exercice 2.
  restart:plot([2*t^4-2*t^3,t^2-t, t=-0.5...1.2]);
```




```
>
> ########################################     Exercice 3.
  with(plots):
  polarplot([sin(2*t/3),t,t=0..6*Pi],color=blue);
```

```
Warning, the name changecoords has been redefined
```




```
> ######################################     Exercice 4
> with(VectorCalculus): assume(a>0):
> SetCoordinates( 'polar' );
  A:=ArcLength( <a*cos(2*t), t>, t=0..Pi/2) ;
```

polar

$$A := 2\ a\sim\ \text{EllipticE}\left(\frac{\sqrt{3}}{2}\right)$$

```
> r:=a*cos(2*t):simplify(int(sqrt(r^2+diff(r,t)^2),t)):
> with(VectorCalculus):assume(a>0):
> SetCoordinates( 'cartesian' );
  B:=ArcLength( <2*a*cos(t),a*sin(t)>, t=0..Pi/2);
```

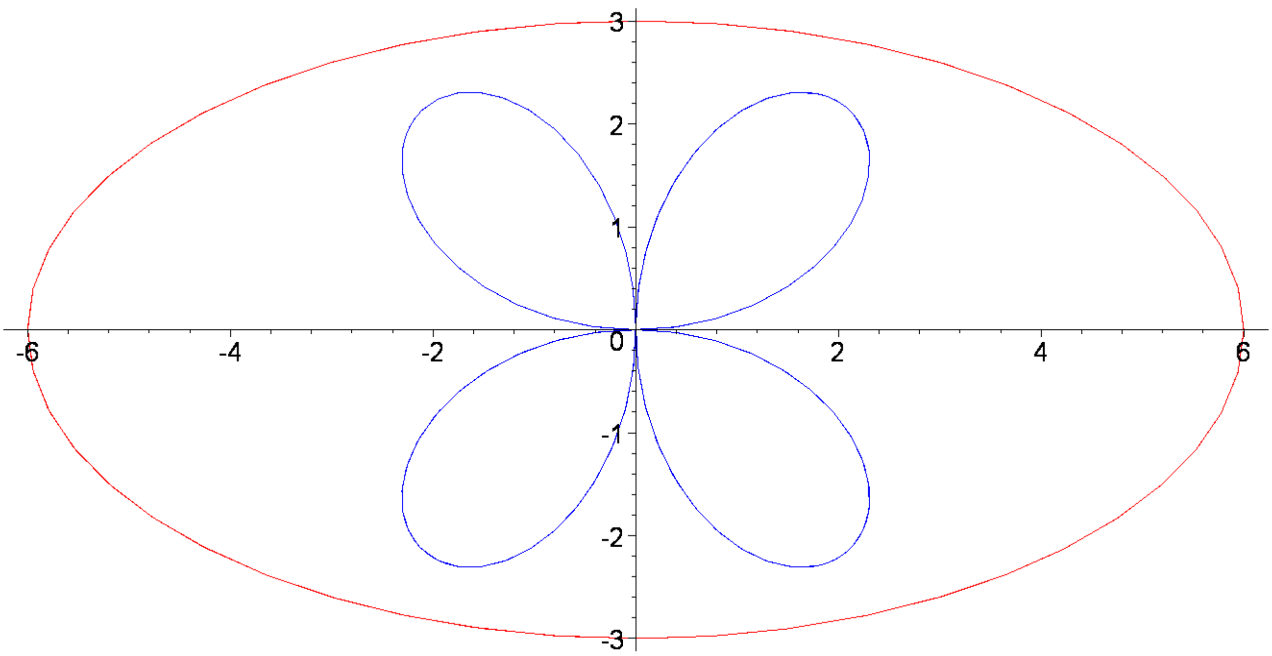
cartesian

$$B := 2\ a\sim\ \text{EllipticE}\left(\frac{\sqrt{3}}{2}\right)$$

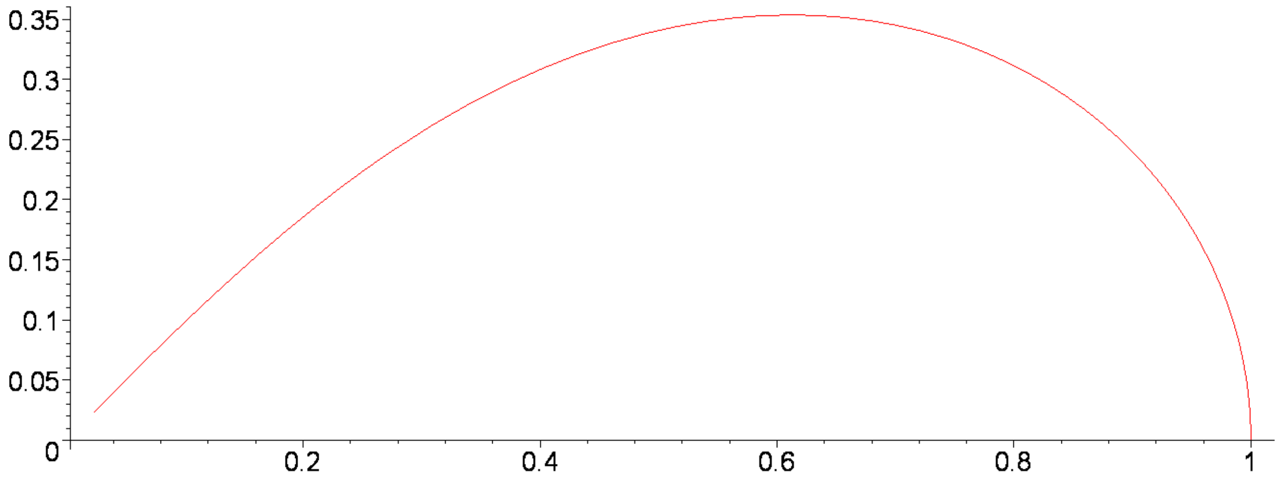
```
> simplify(A-B);
```

$$2\,a\sim \mathrm{EllipticE}\left(\frac{\sqrt{3}}{2}\right) - 2\,a\sim \mathrm{EllipticE}\left(\frac{\sqrt{3}}{2}\right)$$

```
> with(plots):P1:=polarplot([3*sin(2*t),t,t=0..4*Pi],color=blue):
  P2:=plot([6*cos(t),3*sin(t),t=0..4*Pi],color=red):
  display(P1,P2, scaling=CONSTRAINED);
Warning, the name changecoords has been redefined
```




```
> ######################################     Exercice 5.
  restart:with(plots):
  P:=polarplot([(cos(2*t))^(1/2), t, t=0..Pi/4]):

> display(P, scaling=CONSTRAINED);
Warning, the name changecoords has been redefined
```




```
> r:=(cos(2*t))^(1/2);rp:=diff(r,t);
  ds:=simplify((sqrt(r^2+rp^2)))*dt;
```

$$r := \sqrt{\cos(2\,t)}$$

$$rp := -\frac{\sin(2\,t)}{\sqrt{\cos(2\,t)}}$$

$$ds := \sqrt{\frac{1}{\cos(2\,t)}}\,dt$$

```
>
> ############################################
  #######################################    Exercice 6.
  SetCoordinates( 'polar' );
  ArcLength( <1, t>, t=0..2*Pi) ;
```

$$polar$$

$$2\,\pi$$

```
> restart;F:=x^2+y^2-1;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=0.3; x0:=sqrt(-y0^2+1);
```

$$F := x^2 + y^2 - 1$$

$$Fx := 2\,x$$

$$Fy := 2\,y$$

$$y0 := 0.3$$

$$x0 := 0.9539392014$$

```
> with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
  numpoints=1000):
  y0:=0.4; x0:=sqrt(-y0^2+1);
  Fx0:=eval( Fx,[x=x0, y=y0]);
  Fy0:=eval( Fy,[x=x0, y=y0]);
  l0:=(Fx0^2+Fy0^2)^(1/2);
  b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, length=[0.5],
  width=[0.02, relative], head_length=[0.1, relative],
  color=red):
  b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,
  length=[0.5],width=[0.02,relative], color=blue):

  display(b0, b1, b2, scaling=CONSTRAINED);
```

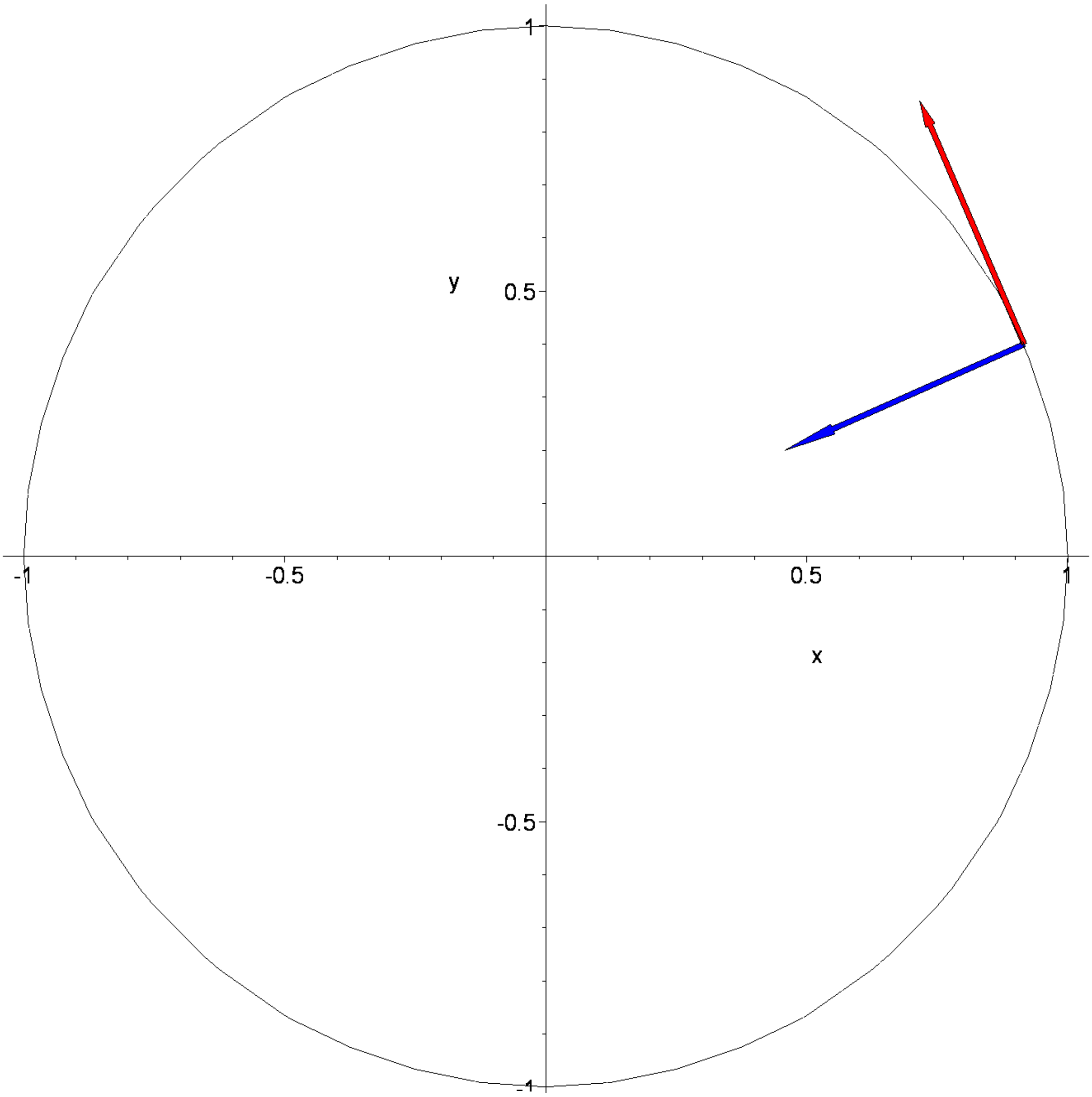
$$y0 := 0.4$$

$$x0 := 0.9165151390$$

$$Fx0 := 1.833030278$$

$$Fy0 := 0.8$$

$$l0 := 2.000000000$$




```
> ######################################    Exercice 7.
  with(VectorCalculus):
  SetCoordinates( 'cartesian' ):
> ArcLength( <sin(t),sin(t)>, t=0..2*Pi ) ;

> SetCoordinates( 'cartesian' ):simplify(Curvature(
  <sin(t),sin(t)> ))assuming t::real ;
```

$$4\sqrt{2}$$

$$0$$

```
>
> ######################################    Exercice 8.
  #Hyperbole

> restart;F:=x^2-y^2-1;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=1; x0:=sqrt(y0^2+1);
```

>

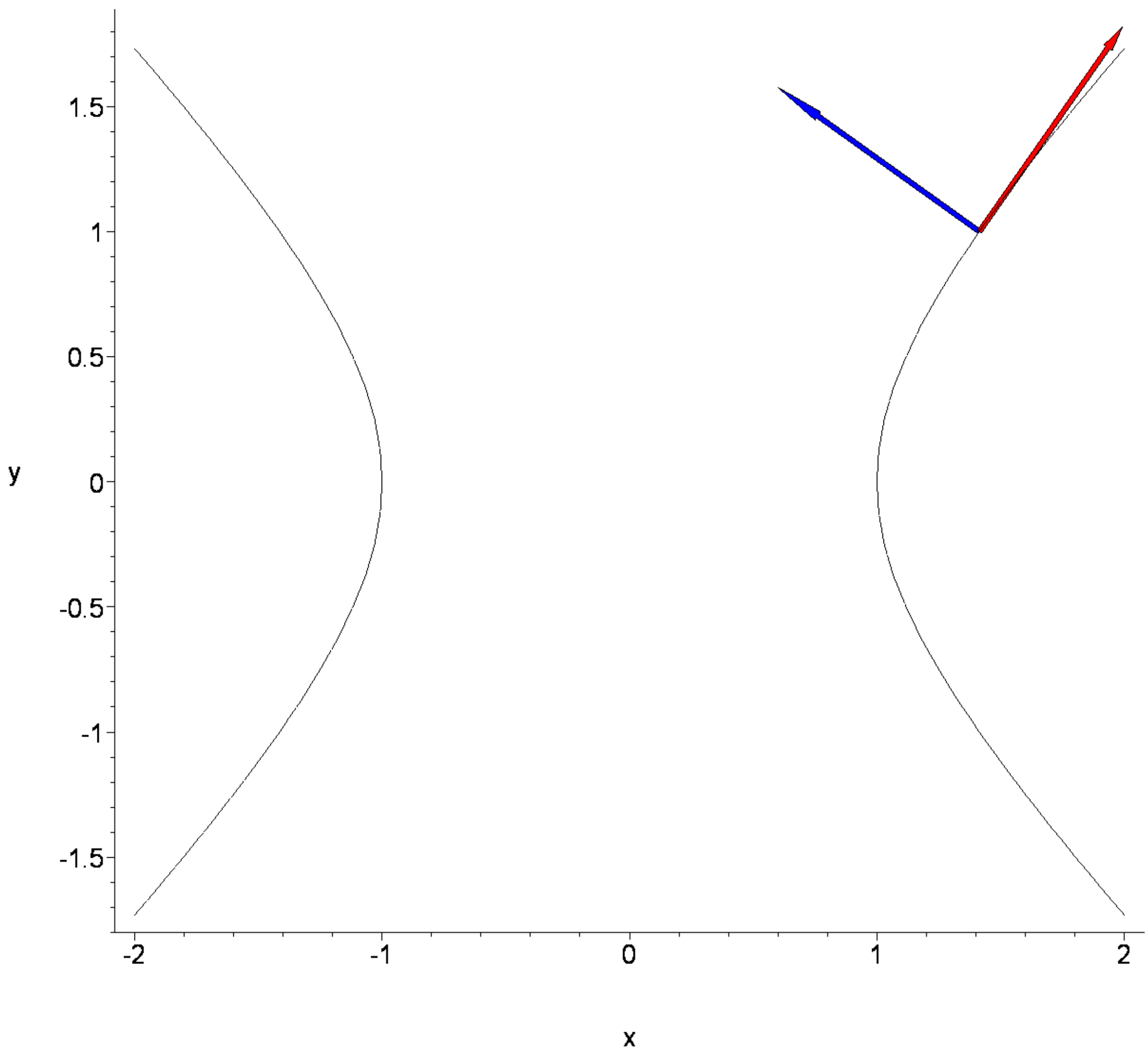
$$F := x^2 - y^2 - 1$$
$$Fx := 2\,x$$
$$Fy := -2\,y$$
$$y0 := 1$$
$$x0 := \sqrt{2}$$

>

```
 with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
numpoints=1000):
#y0:=1; x0:=sqrt(y0^2+1);
Fx0:=eval( Fx,[x=x0, y=y0]);
Fy0:=eval( Fy,[x=x0, y=y0]);
l0:=(Fx0^2+Fy0^2)^(1/2);
b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, width=[0.02, relative],
head_length=[0.1, relative],  color=red):
b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,width=[0.02,relative],
color=blue):

display(b0, b1, b2, scaling=CONSTRAINED, axes=FRAMED);
```

```
Warning, the name changecoords has been redefined
```

$$Fx0 := 2\,\sqrt{2}$$
$$Fy0 := -2$$
$$l0 := \sqrt{12}$$




```
> ######################################     Exercice 9
  #Ellipse
> restart;F:=2*x^2+3*y^2-1;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=.1; solve(F,x)=0;x0:=max(eval(solve(F,x), y=y0));
>
```

$$F := 2\,x^2 + 3\,y^2 - 1$$

$$Fx := 4\,x$$

$$Fy := 6\,y$$

$$y0 := 0.1$$

$$\left(\frac{\sqrt{-6\,y^2+2}}{2}, -\frac{\sqrt{-6\,y^2+2}}{2}\right) = 0$$

$$x0 := 0.6964194140$$

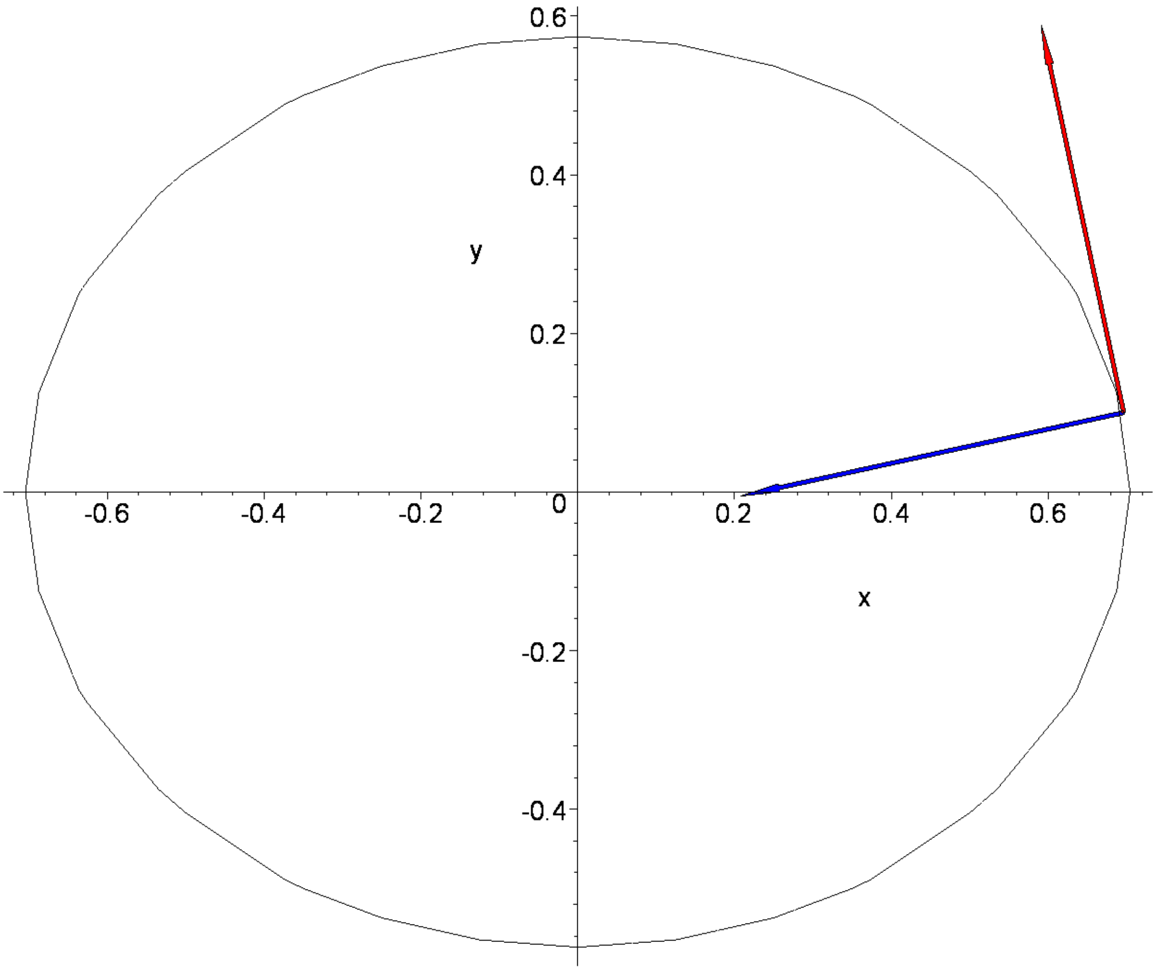
```
>
   with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
  numpoints=1000):
```

```
#y0:=1; x0:=sqrt(y0^2+1);
Fx0:=eval( Fx,[x=x0, y=y0]);
Fy0:=eval( Fy,[x=x0, y=y0]);
l0:=(Fx0^2+Fy0^2)^(1/2);
b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, length=[0.5],
width=[0.01, relative], head_length=[0.1, relative],
color=red):
b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,
length=[0.5],width=[0.01, relative], head_length=[0.1,
relative], color=blue):
display(b0, b1, b2, scaling=CONSTRAINED, axes=NORMAL);
```

$$Fx0 := 2.785677656$$

$$Fy0 := 0.6$$

$$l0 := 2.849561370$$




```
>
> ######################################     Exercice 10
>
#Parabole
```

```
> restart;F:=y^2-6*x;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=1; solve(F,x)=0;x0:=max(eval(solve(F,x), y=y0));

>
```

$$F := y^2 - 6\,x$$

$$Fx := -6$$

$$Fy := 2\,y$$

$$y0 := 1$$

$$\frac{y^2}{6} = 0$$

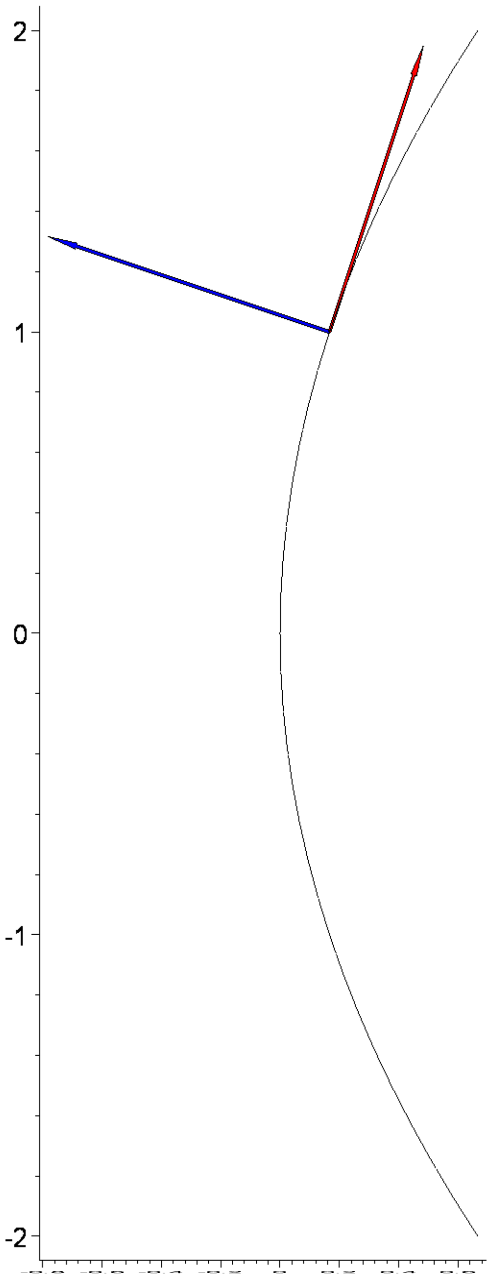
$$x0 := \frac{1}{6}$$

```
>
   with(plots):
  b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
  numpoints=1000,coords=cartesian):
  #y0:=1; x0:=sqrt(y0^2+1);
  Fx0:=eval( Fx,[x=x0, y=y0]);
  Fy0:=eval( Fy,[x=x0, y=y0]);
  l0:=(Fx0^2+Fy0^2)^(1/2);
  b1 := arrow(<x0,y0>, <Fy0/l0,-Fx0/l0>, width=[0.01, relative],
  head_length=[0.1, relative], color=red):
  b2 := arrow( <x0,y0>, <Fx0/l0,Fy0/l0>,width=[0.01, relative],
  head_length=[0.1, relative], color=blue):

  display(b0, b1, b2, scaling=CONSTRAINED, axes=FRAMED);
>
Warning, the name changecoords has been redefined
```

$$Fx0 := -6$$

$$Fy0 := 2$$

$$l0 := \sqrt{40}$$




```
>
########################################
#Une courbe elliptique (cet exemple n'est pas inclus dans la
feuille 3)
```

```
> restart;F:=y^2-x^3+x;Fx:=diff(F, x);Fy:=diff(F, y);
x0:=-.1; solve(F,y)=0;y0:=max(eval(solve(F,y), x=x0));

>
```

$$F := y^2 - x^3 + x$$

$$Fx := -3\,x^2 + 1$$

$$Fy := 2\,y$$

$$x0 := -0.1$$

$$(\sqrt{x^3 - x}, -\sqrt{x^3 - x}) = 0$$

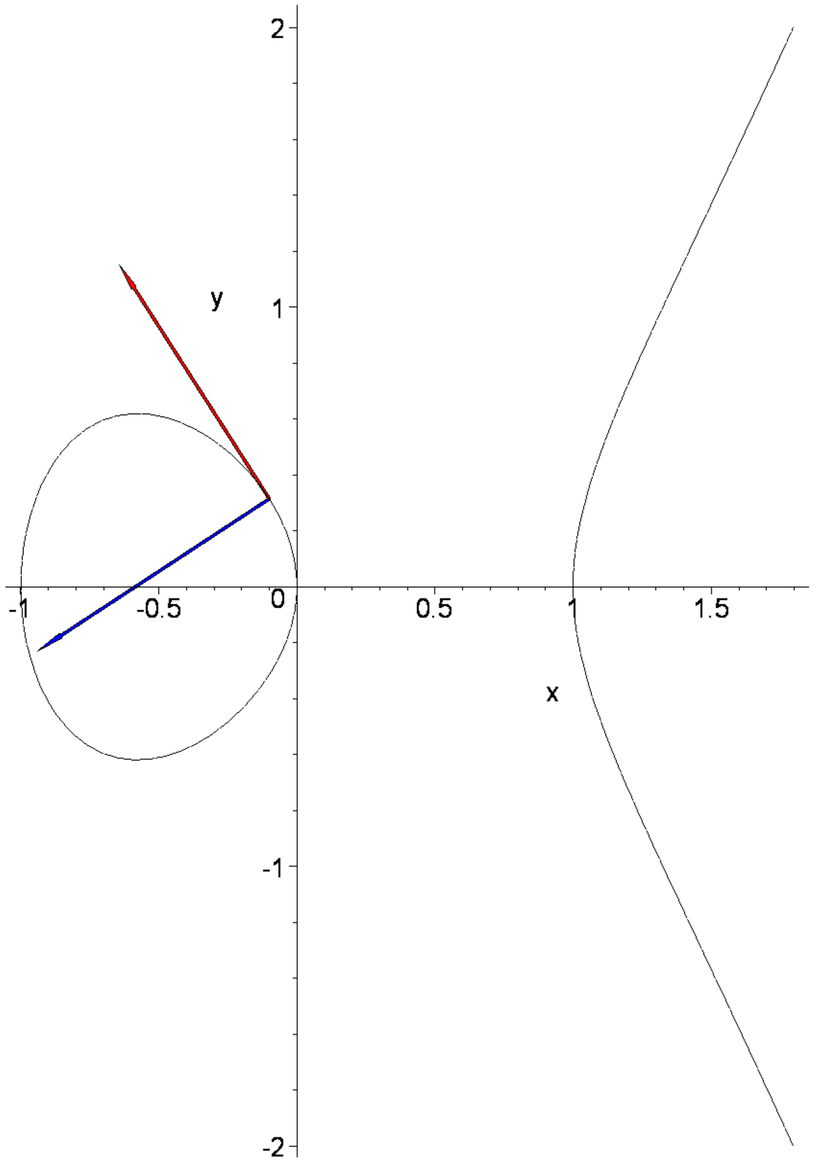
$$y0 := 0.3146426545$$

```
>
 with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
numpoints=5000):
#y0:=1; x0:=sqrt(y0^2+1);
Fx0:=eval( Fx,[x=x0, y=y0]);
Fy0:=eval( Fy,[x=x0, y=y0]);
l0:=(Fx0^2+Fy0^2)^(1/2);
b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, width=[0.01, relative],
head_length=[0.1, relative], color=red):
b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,width=[0.01, relative],
head_length=[0.1, relative], color=blue):

display(b0, b1, b2, scaling=CONSTRAINED);
Warning, the name changecoords has been redefined
```

$$Fx0 := 0.97$$

$$Fy0 := 0.6292853090$$

$$l0 := 1.156243919$$



```
> ######################################     Exercice 11
  with(VectorCalculus):
  SetCoordinates( 'cartesian' );
  K:=simplify(Curvature( <(1+cos(t)^2)*sin(t),sin(t)^2*cos(t)> ))
  assuming t::real ;
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, ., D,
Vector, diff, int, limit, series
```

$$cartesian$$

$$K := \frac{1}{\left|3 \cos(t)^2 - 1\right|}$$

```
> R:=1/K;
```

$$R := \left|3 \cos(t)^2 - 1\right|$$

```
> x:=(1+cos(t)^2)*sin(t);y:=sin(t)^2*cos(t);
  xp:=diff(x,t);yp:=diff(y,t); xs:=diff(xp,t);ys:=diff(yp,t);
```

$$x := (\cos(t)^2 + 1) \sin(t)$$

$$y := \sin(t)^2 \cos(t)$$

$$xp := -2 \sin(t)^2 \cos(t) + (\cos(t)^2 + 1) \cos(t)$$

$$yp := 2 \sin(t) \cos(t)^2 - \sin(t)^3$$

$$xs := -6 \sin(t) \cos(t)^2 + 2 \sin(t)^3 - (\cos(t)^2 + 1) \sin(t)$$

$$ys := 2 \cos(t)^3 - 7 \sin(t)^2 \cos(t)$$

```
> K:=simplify(((xp*ys-yp*xs)/(xp^2+yp^2)^(3/2))) assuming t::real;
```

$$K := \frac{1}{\left|3 \cos(t)^2 - 1\right|}$$

```
> X:=simplify(x-(yp*(xp^2+yp^2))/(xp*ys-yp*xs));Y:=simplify(y+xp*(
  xp^2+yp^2)/(xp*ys-yp*xs));
>
```

$$X := 2 \sin(t)^3$$

$$Y := 2 \cos(t)^3$$

```
> ##################################################### Exercice
  12
> with(VectorCalculus):
  SetCoordinates( 'cartesian' );
  assume(t>0):K:=simplify(Curvature( <t-sinh(t)*cosh(t),2*cosh(t)>
  ))  assuming t::real ;
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, ., D,
Vector, diff, int, limit, series
```

$$cartesian$$

$$K := \frac{1}{2} \frac{1}{\cosh(t\sim)^2 \sinh(t\sim)}$$

```
> R:=1/K;
```

$$R := 2\cosh(t\sim)^2 \sinh(t\sim)$$

```
> x:=t-sinh(t)*cosh(t);y:=2*cosh(t);
  xp:=diff(x,t);yp:=diff(y,t); xs:=diff(xp,t);ys:=diff(yp,t);
```

$$x := t\sim - \sinh(t\sim)\cosh(t\sim)$$

$$y := 2\cosh(t\sim)$$

$$xp := 1 - \cosh(t\sim)^2 - \sinh(t\sim)^2$$

$$yp := 2\sinh(t\sim)$$

$$xs := -4\sinh(t\sim)\cosh(t\sim)$$

$$ys := 2\cosh(t\sim)$$

```
> K:=simplify(((xp*ys-yp*xs)/(xp^2+yp^2)^(3/2))) assuming t::real;
```

$$K := \frac{1}{2}\frac{1}{\cosh(t\sim)^2 \sinh(t\sim)}$$

```
> X:=simplify(x-(yp*(xp^2+yp^2))/(xp*ys-yp*xs));Y:=simplify(y+xp*(
  xp^2+yp^2)/(xp*ys-yp*xs));
```

$$X := -3\sinh(t\sim)\cosh(t\sim) + t\sim$$

$$Y := -2\cosh(t\sim)\left(\cosh(t\sim)^2 - 2\right)$$

```
>
> ##################################################################
  ######################
> #######################################     Exercice 13

  r:=tan(t/2); x:=r*cos(t);y:=r*sin(t);r := tan(1/2*t);
```

$$r := \tan\left(\frac{t\sim}{2}\right)$$

$$x := \tan\left(\frac{t\sim}{2}\right)\cos(t\sim)$$

$$y := \tan\left(\frac{t\sim}{2}\right)\sin(t\sim)$$

$$r := \tan\left(\frac{t\sim}{2}\right)$$

```
> with(VectorCalculus):
  SetCoordinates( 'polar' );
  K:=simplify(Curvature( <tan(t/2),t> ))  assuming t::real ;
```

polar

$$K := -\frac{\left(\cos(t\sim)^2 - \cos(t\sim) - 2\right)\left(\cos(t\sim) + 1\right)}{\left(-\cos(t\sim)^2 + 2\right)^{(3/2)}}$$

```
> rp:=diff(r,t);rs:=diff(rp,t);
```

$$rp := \frac{1}{2} + \frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2$$

$$rs := \tan\left(\frac{t\sim}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)$$

```
> xp:=diff(x,t);yp:=diff(y,t); xs:=diff(xp,t);ys:=diff(yp,t);
```

$$xp := \left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\cos(t\sim)-\tan\left(\frac{t\sim}{2}\right)\sin(t\sim)$$

$$yp := \left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\sin(t\sim)+\tan\left(\frac{t\sim}{2}\right)\cos(t\sim)$$

$$xs := \tan\left(\frac{t\sim}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\cos(t\sim)-2\left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\sin(t\sim)-\tan\left(\frac{t\sim}{2}\right)\cos(t\sim)$$

$$ys := \tan\left(\frac{t\sim}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\sin(t\sim)+2\left(\frac{1}{2}+\frac{1}{2}\tan\left(\frac{t\sim}{2}\right)^2\right)\cos(t\sim)-\tan\left(\frac{t\sim}{2}\right)\sin(t\sim)$$

```
> K:=simplify(((xp*ys-yp*xs)/(xp^2+yp^2)^(3/2))) assuming t::real;
```

$$K := \frac{(\cos(t\sim)^2+2-3\cos(t\sim))\,(\cos(t\sim)+1)^3}{\sin(t\sim)^2\,(-\cos(t\sim)^2+2)^{(3/2)}}$$

```
> X:=simplify(x-(yp*(xp^2+yp^2))/(xp*ys-yp*xs));Y:=simplify(y+xp*(
xp^2+yp^2)/(xp*ys-yp*xs));
```

$$X := -\frac{2\sin(t\sim)\,(-1+\cos(t\sim))}{\cos(t\sim)^2-\cos(t\sim)-2}$$

$$Y := \frac{(2\cos(t\sim)^2-3)\cos(t\sim)}{\cos(t\sim)^3-3\cos(t\sim)-2}$$

```
>
#######################################     Exercice 14


restart:print('(d/dt)*arcsin(sqrt(t))'=diff(arcsin(sqrt(t)),
t));
```

$$\frac{d\arcsin(\sqrt{t})}{dt}=\frac{1}{2\sqrt{t}\sqrt{1-t}}$$

```
> print('(d/dt)*arcsin(sqrt(t))/(sqrt(t*(1-t)))'=diff((arcsin(sqrt
(t)))/(sqrt(t*(1-t))), t));
```

$$\frac{d\arcsin(\sqrt{t})}{dt\sqrt{t\,(1-t)}}=\frac{1}{2\sqrt{t}\sqrt{1-t}\sqrt{t\,(1-t)}}-\frac{1}{2}\frac{\arcsin(\sqrt{t})\,(1-2t)}{(t\,(1-t))^{(3/2)}}$$

```
> simplify(diff((arcsin(sqrt(t)))/(sqrt(t*(1-t))), t));
```

$$\frac{1}{2}\frac{-t^2+t-\arcsin(\sqrt{t})\sqrt{t}\sqrt{1-t}+2\arcsin(\sqrt{t})\,t^{(3/2)}\sqrt{1-t}}{t^{(3/2)}\,(1-t)^{(3/2)}\sqrt{-t\,(-1+t)}}$$

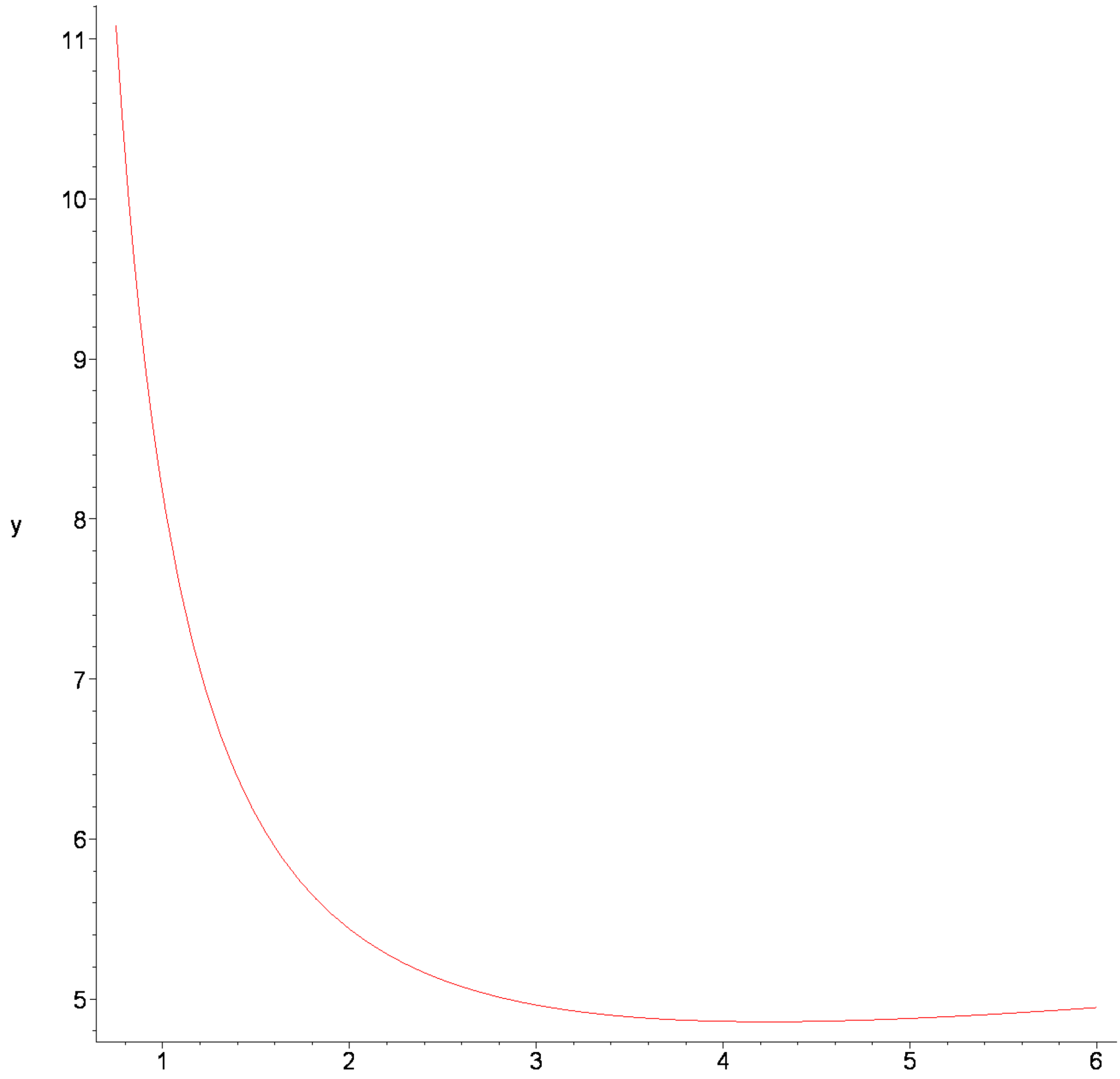
```
> with(VectorCalculus):
SetCoordinates( 'cartesian' );
print('arcsin'(sqrt(t))/(sqrt(t*(1-t))));
K:=simplify(Curvature( <t,(arcsin(sqrt(t)))/(sqrt(t*(1-t)))> ))
```

```
assuming t::positive :
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, ., D,
Vector, diff, int, limit, series
```

$$cartesian$$

$$\frac{\arcsin(\sqrt{t})}{\sqrt{t(1-t)}}$$

```
> R:=1/K:
> ######################################    Exercice 15


restart;with(plots): plot([t^2+t,(t+1)*exp(1/t), t=0.5..2],
labels=[x, y]);
Warning, the name changecoords has been redefined
```




```
> 
  with(VectorCalculus):
  SetCoordinates( 'cartesian' );
  K:=simplify(Curvature( <t^2+t,(t+1)*exp(1/t)> ))  assuming
  t::positive :
```

Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, ., D, Vector, diff, int, limit, series

cartesian

```
> ######################################    Exercice 16
  restart;P:=(x-y)/(x^2+y^2);Q:=(x+y)/(x^2+y^2);Py:=diff(P,y);
  Qx:=diff(Q,x);'Py'-'Qx'=simplify(Py-Qx);F:=int(P,x);'Q'=simplify
  (diff(int(P,x),y));
```

$$P := \frac{x-y}{x^2+y^2}$$

$$Q := \frac{x+y}{x^2+y^2}$$

$$Py := -\frac{1}{x^2+y^2} - \frac{2\,(x-y)\,y}{(x^2+y^2)^2}$$

$$Qx := \frac{1}{x^2+y^2} - \frac{2\,(x+y)\,x}{(x^2+y^2)^2}$$

$$Py - Qx = 0$$

$$F := \frac{1}{2}\ln(x^2+y^2) - \arctan\left(\frac{x}{y}\right)$$

$$Q = \frac{x+y}{x^2+y^2}$$

```
> assume( a > 0 );
  FP1:=evala(subs({x=a, y=-a},F));
  FP2moins:=evala(limit((subs({x=a},F), y=0, left)));
  FP2plus:=evala(limit((subs({x=a},F), y=0, right)));
  FP3moins:=evala(limit((subs({x=a},F), y=0, right)));
  FP3plus:=evala(limit((subs({x=a},F), y=0, left)));
```

$$FP1 := \frac{1}{2}\ln(2\,a{\sim}^2) + \frac{\pi}{4}$$

$$FP2moins := \ln(a{\sim}) + \frac{\pi}{2}$$

$$FP2plus := \ln(a{\sim}) - \frac{\pi}{2}$$

$$FP3moins := \ln(a{\sim}) - \frac{\pi}{2}$$

$$FP3plus := \ln(a{\sim}) + \frac{\pi}{2}$$

```
> print('(F(P[2,moins])-F(P[1]))+F(P[3,plus])-F(P[2,plus])+(F(P[1]
  )-F(P[3,moins]))'
  =FP2moins-FP1+FP3plus-FP2plus+FP1-FP3moins);
```

$$\mathrm{F}(P_{2,\,moins}) - \mathrm{F}(P_1) + \mathrm{F}(P_{3,\,plus}) - \mathrm{F}(P_{2,\,plus}) + \mathrm{F}(P_1) - \mathrm{F}(P_{3,\,moins}) = 2\,\pi$$

```
> x:=a*cos(t);y:=a*sin(t);
  print('P*diff(x,t)+Q*diff(y,t)'=P*diff(x,t)+Q*diff(y,t));
  simplify(P*diff(x,t)+Q*diff(y,t));
```

$$x := a\cos(t)$$

$$y := a\sin(t)$$

$$P\left(\frac{\partial}{\partial t}x\right)+Q\left(\frac{\partial}{\partial t}y\right)=-\frac{(a\cos(t)-a\sin(t))\,a\sin(t)}{a^2\cos(t)^2+a^2\sin(t)^2}+\frac{(a\cos(t)+a\sin(t))\,a\cos(t)}{a^2\cos(t)^2+a^2\sin(t)^2}$$

$$1$$

```
> print('int(1,t=0..2*Pi)'=int(1,t=0..2*Pi));
```

$$\int_0^{2\pi} 1\,dt=2\pi$$

```
> #####################################     Exercice 17
> restart;P:=exp(x)*cos(y)+x*y^2;Q:=(-exp(x)*sin(y)+x^2*y);
```

$$P:=\mathbf{e}^x\cos(y)+x\,y^2$$

$$Q:=-\mathbf{e}^x\sin(y)+x^2\,y$$

```
> diff(P,y);
  diff(Q,x);
```

$$-\mathbf{e}^x\sin(y)+2\,x\,y$$

$$-\mathbf{e}^x\sin(y)+2\,x\,y$$

```
> F:=exp(x)*cos(y)+x^2*y^2/2;'P'=diff(F,x);'Q'=diff(F,y);
```

$$F:=\mathbf{e}^x\cos(y)+\frac{x^2\,y^2}{2}$$

$$P=\mathbf{e}^x\cos(y)+x\,y^2$$

$$Q=-\mathbf{e}^x\sin(y)+x^2\,y$$

```
> int(P,x);
```

$$\mathbf{e}^x\cos(y)+\frac{x^2\,y^2}{2}$$

```
> #Soit a;b > 0. Calculer int( x^2*dy + y^2*dx, o u  a pour
   equation cart esienne l'une des
  # equations suivantes: x^2 +y^2-a*x=0; (x/a)^2 +(y/b)^2=1;
  #  (x/a)^2 +(y/b)^2-2*(x/a)-2*(y/b)=0;
> assume(a>0);assume(b>0);
> x:=(a/2)*(cos(t)+1);y:=(a/2)*(sin(t));
```

$$x:=\frac{1}{2}\,a\sim(\cos(t)+1)$$

$$y:=\frac{1}{2}\,a\sim\sin(t)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp:=-\frac{1}{2}\,a\sim\sin(t)$$

$$yp:=\frac{1}{2}\,a\sim\cos(t)$$

```
> x^2*yp+y^2*xp;
```

$$a^2\left(\sqrt{2}\cos(t)+1\right)^2 b\sqrt{2}\cos(t)-b^2\left(\sqrt{2}\sin(t)+1\right)^2 a\sqrt{2}\sin(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{24} a\sim^3 (\cos(t)^2 \sin(t) + 3 \sin(t) \cos(t) + 5 \sin(t) + 3 t - \cos(t)^3 + 3 \cos(t))$$

```
> x^2*yp+y^2*xp;
```

$$\frac{1}{8} a\sim^3 (\cos(t) + 1)^2 \cos(t) - \frac{1}{8} a\sim^3 \sin(t)^3$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$\frac{a\sim^3 \pi}{4}$$

```
> #################################################
  x:=a*(cos(t));y:=(b)*(sin(t));
```

$$x := a \cos(t)$$

$$y := b \sin(t)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp := -a \sin(t)$$

$$yp := b \cos(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{3} a\, b\, (a \cos(t)^2 \sin(t) + 2 a \sin(t) + 3 b \cos(t) - b \cos(t)^3)$$

```
> x^2*yp+y^2*xp;
```

$$a^2 \cos(t)^3 b - b^2 \sin(t)^3 a$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$0$$

```
> #########################################
  restart;P:=y^2;Q:=x^2;
```

$$P := y^2$$

$$Q := x^2$$

```
> diff(P,y);
  diff(Q,x);
```

$$2 y$$

$$2 x$$

```
> ##############################################################
> restart;((x/a)-1)^2+((y/b)-1)^2=2;
```

$$\left(\frac{x}{a} - 1\right)^2 + \left(\frac{y}{b} - 1\right)^2 = 2$$

```
> x:=a*(sqrt(2)*cos(t)+1);y:=(b)*(sqrt(2)*sin(t)+1);
```

$$x := a (\sqrt{2} \cos(t) + 1)$$

$$y := b (\sqrt{2} \sin(t) + 1)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp := -a\sqrt{2}\,\sin(t)$$

$$yp := b\sqrt{2}\,\cos(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{3}\,a\,b\,(2\,a\cos(t)^2\sin(t)\sqrt{2}+6\,a\cos(t)\sin(t)+7\,a\sqrt{2}\,\sin(t)+6\,b\sin(t)\cos(t)-6\,b\,t+9\,b\sqrt{2}\,\cos(t)+6\,a\,t-2\,b\sqrt{2}\,\cos(t)^3)$$

```
> x^2*yp+y^2*xp;
```

$$a^2\,(\sqrt{2}\,\cos(t)+1)^2\,b\sqrt{2}\,\cos(t)-b^2\,(\sqrt{2}\,\sin(t)+1)^2\,a\sqrt{2}\,\sin(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
```

$$\frac{1}{3}\,a\,b\,(2\,a\cos(t)^2\sin(t)\sqrt{2}+6\,a\cos(t)\sin(t)+7\,a\sqrt{2}\,\sin(t)+6\,b\sin(t)\cos(t)-6\,b\,t+9\,b\sqrt{2}\,\cos(t)+6\,a\,t-2\,b\sqrt{2}\,\cos(t)^3)$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$4\,a^2\,b\,\pi-4\,a\,b^2\,\pi$$

```
>
```