##

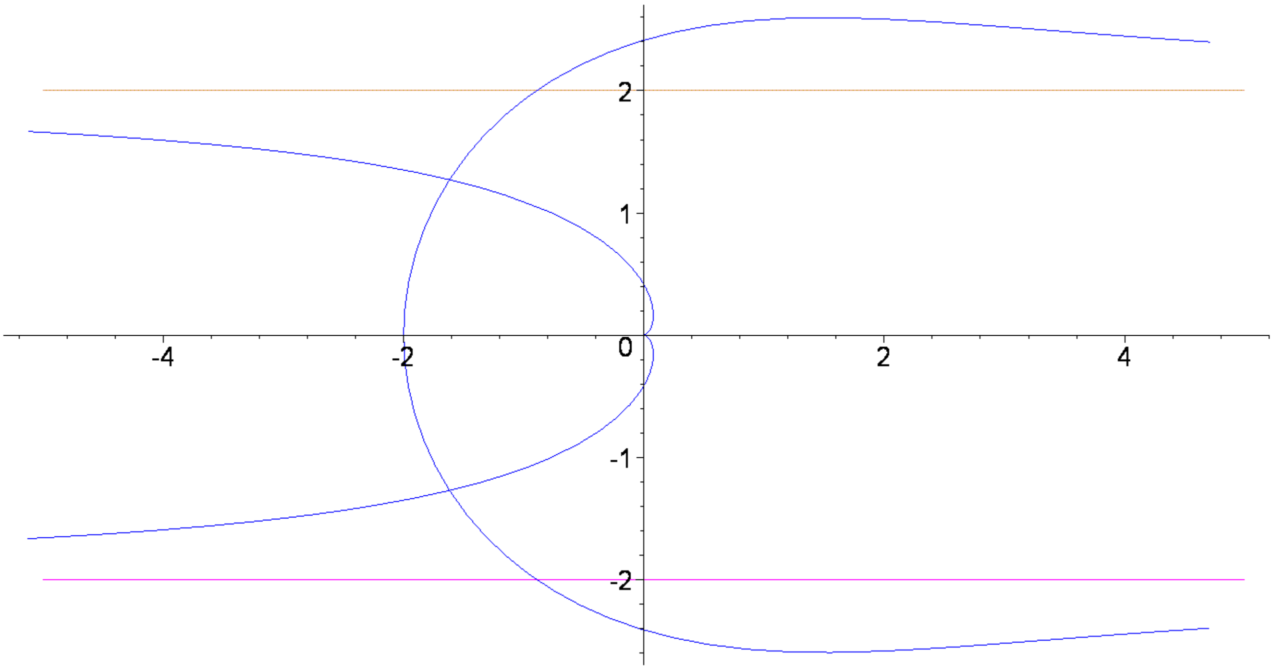
```
> ################### Exercice I
```

On considère la courbe définie en coordonnées polaires par
rho(theta)=1+sin(theta/2)^(-1)
1. Déterminer le domaine de définition et la période de l'arc paramétrée.

```
> restart:with(plots):P1:=polarplot([1+sin(t/2)^(-1),t,t=0.15*Pi..
  1.85*Pi],color=blue):
  P11:=polarplot([1+sin(t/2)^(-1),t,t=2.1*Pi..3.9*Pi],color=blue):

  P2:=plot([t, 2, t=-5..5],color=gold):
  P3:=plot([t, -2, t=-5..5],color=magenta):
  display(P1, P11, P2, P3, scaling=CONSTRAINED);
```

```
Warning, the name changecoords has been redefined
```




```
> restart:r:=(1+sin(t/2)^(-1));rp:=diff((1+sin(t/2)^(-1)),t);
>
```

$$r := 1 + \frac{1}{\sin\left(\frac{t}{2}\right)}$$

$$rp := -\frac{1}{2}\frac{\cos\left(\frac{t}{2}\right)}{\sin\left(\frac{t}{2}\right)^2}$$

```
> v:=rp*[cos(t),sin(t)]+r*[-sin(t),cos(t)];
```

$$v := -\frac{1}{2}\frac{\cos\left(\frac{t}{2}\right)[\cos(t), \sin(t)]}{\sin\left(\frac{t}{2}\right)^2} + \left(1 + \frac{1}{\sin\left(\frac{t}{2}\right)}\right)[-\sin(t), \cos(t)]$$

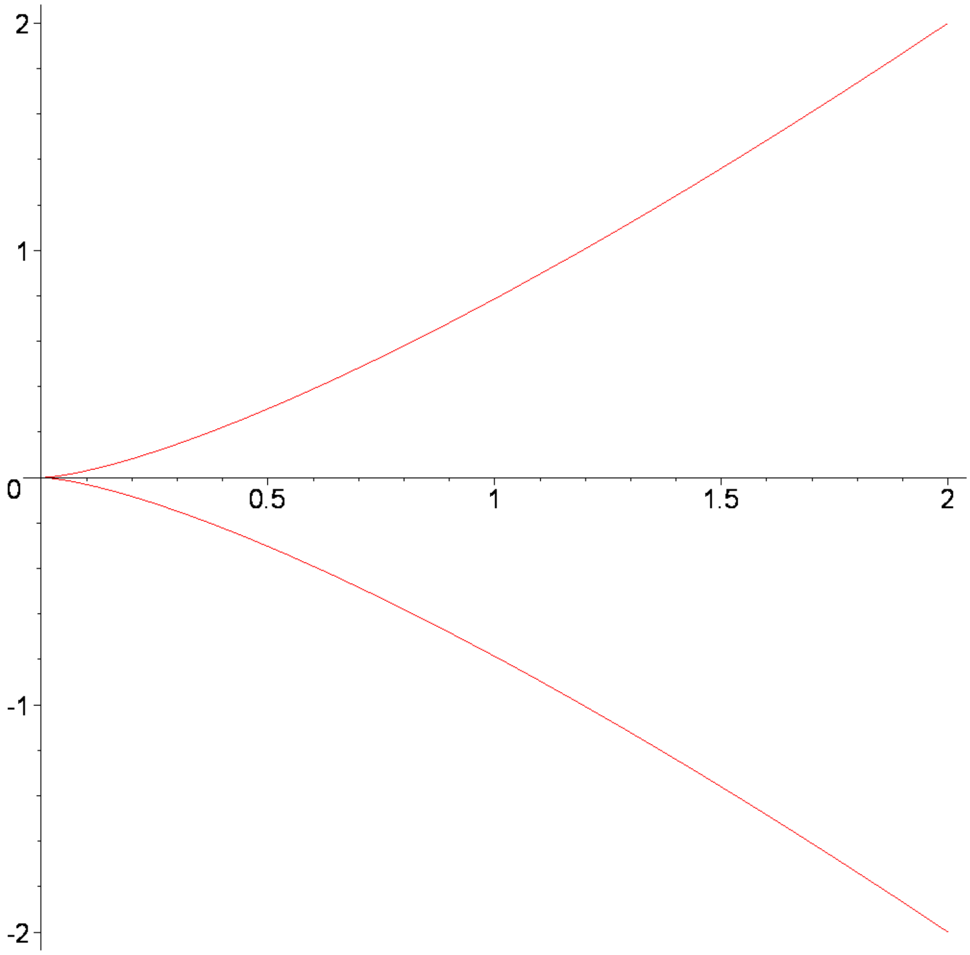
```
> restart:limit((1+sin(t/2)^(-1))*sin(t), t=0);
  limit((1+sin(t/2)^(-1))*sin(t), t=2*Pi);
```

$$2$$

$$-2$$

Exercice II
Etudier les branches infinies et les points singuliers de la courbe
x(t)=t^2+t^4, y(t)=t^3+t^5.Tracer la courbe

```
> plot([t^2+t^4, t^3+t^5, t=-1..1]);
```




```
> #Exercice III. Repère de Frenet de la néphroide
  restart:assume((t>0),(t<Pi)):x:=3*cos(t) - cos(3*t);y:=3*sin(t)
  - sin(3*t);
```

$$x := 3\cos(t\sim) - \cos(3\,t\sim)$$

$$y := 3\sin(t\sim) - \sin(3\,t\sim)$$

```
> u:=diff(3*cos(t) - cos(3*t),t):v:=diff(3*sin(t) -
  sin(3*t),t):[u,v];
  l:=simplify(sqrt(u^2+v^2));
```

$$[-3\sin(t\sim)+3\sin(3\,t\sim),\,3\cos(t\sim)-3\cos(3\,t\sim)]$$

$$l := 6\sin(t\sim)$$

```
> tau:=[simplify(u/l),simplify(v/l)];
```

$$\tau := [2\cos(t\sim)^2-1,\,2\sin(t\sim)\cos(t\sim)]$$

```
> eta:=[-simplify(v/l),simplify(u/l)];
```

$$\eta := [-2\sin(t\sim)\cos(t\sim),\,2\cos(t\sim)^2-1]$$

```
> t0:=Pi/4; x0:=subs(t=t0,x);
  y0:=subs(t=t0,y);
  u0:=subs(t=t0,u);
  v0:=subs(t=t0,v);
  with(plots):
  b0:=
  plot([x,y,t=0..4*Pi]):
```

$$t0 := \frac{\pi}{4}$$

$$x0 := 3\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{3\,\pi}{4}\right)$$

$$y0 := 3\sin\left(\frac{\pi}{4}\right)-\sin\left(\frac{3\,\pi}{4}\right)$$

$$u0 := -3\sin\left(\frac{\pi}{4}\right)+3\sin\left(\frac{3\,\pi}{4}\right)$$

$$v0 := 3\cos\left(\frac{\pi}{4}\right)-3\cos\left(\frac{3\,\pi}{4}\right)$$

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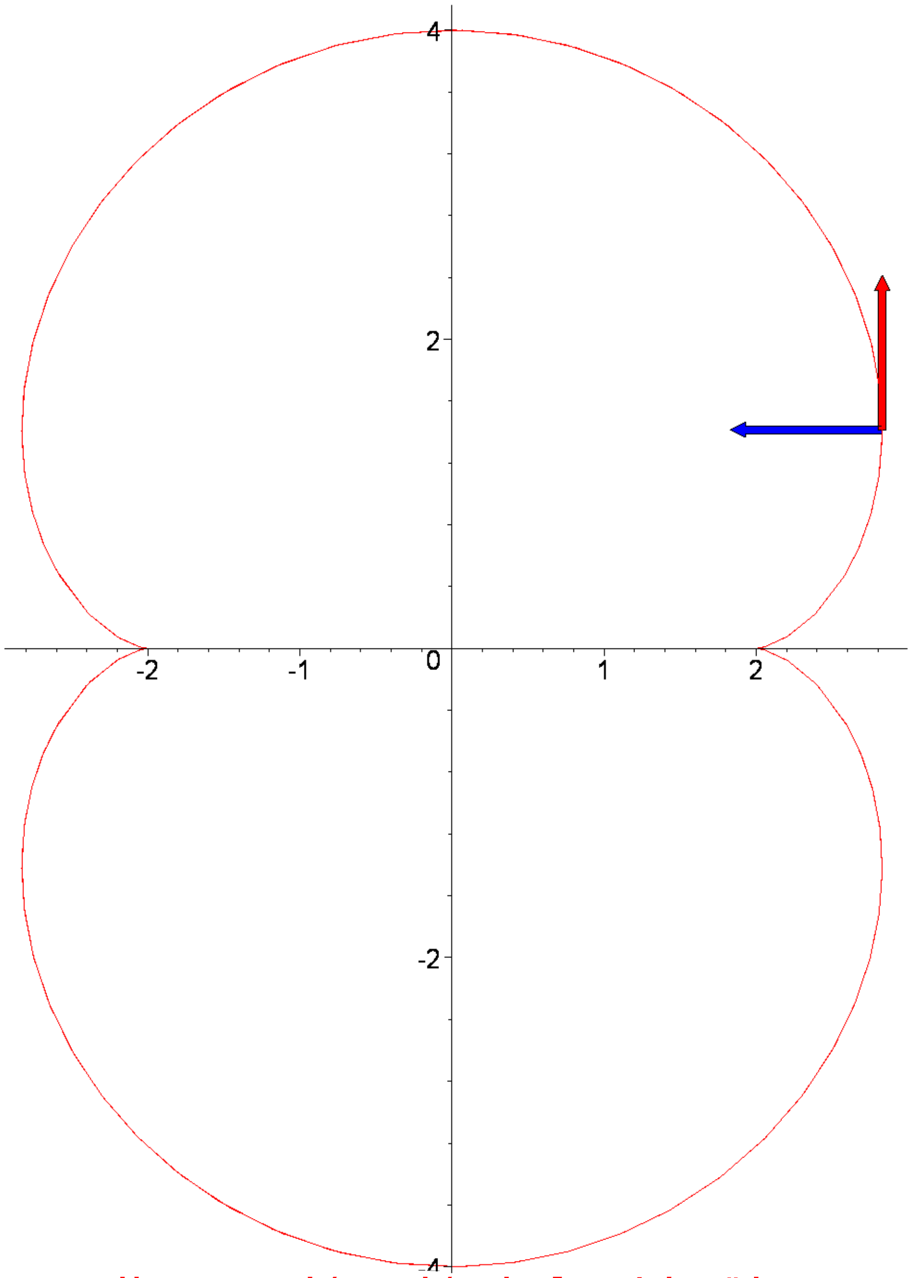
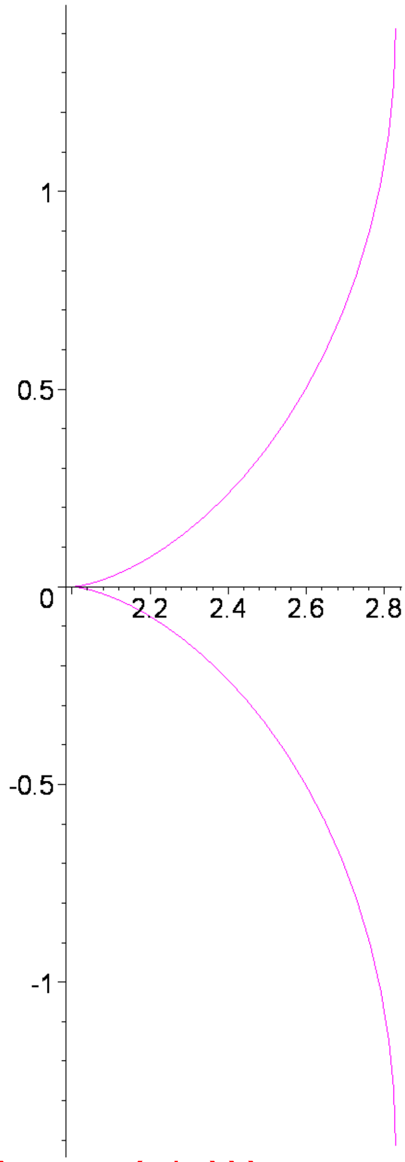
```
> b1 := arrow(<x0,y0>, <u0,v0>, length=[1], width=[0.05,
  relative], head_length=[0.1, relative], color=red):
  b2 := arrow( <x0,y0>, <-v0,u0>,length=[1],width=[0.05,
  relative], head_length=[0.1, relative], color=blue):

  display(b0, b1, b2, scaling=CONSTRAINED);
```




```
> #la longueur d'arc t=-Pi/4..Pi/4 de la néphroïde
  restart:
  with(plots):plot([3*cos(t) - cos(3*t),3*sin(t) - sin(3*t),
  t=-Pi/4..Pi/4],color=magenta, scaling=CONSTRAINED);
```

```
Warning, the name changecoords has been redefined
```




```
> x:=simplify((3*cos(t) - cos(3*t)));
  y:=simplify((3*sin(t) - sin(3*t)));
  simplify((3*cos(t) - cos(3*t))^2+(3*sin(t) - sin(3*t))^2);
```

$$x := -2\cos(t)\,(-3 + 2\cos(t)^2)$$

$$y := 4\sin(t)^3$$

$$-12\cos(t)^2 + 16$$

```
> xp:=diff(3*cos(t) - cos(3*t),t);yp:=diff(3*sin(t) - sin(3*t),t);
```

$$xp := -3\sin(t) + 3\sin(3\,t)$$

$$yp := 3\cos(t) - 3\cos(3\,t)$$

```
> xs:=diff(xp,t);ys:=diff(yp,t);
```

$$xs := -3\cos(t) + 9\cos(3\,t)$$

$$ys := -3\sin(t) + 9\sin(3\,t)$$

```
> simplify(xp);simplify(yp);simplify(xs);simplify(ys);simplify(xp^
  2+yp^2);
  simplify(u*ys-v*xs);s:=simplify(sqrt(xp^2+yp^2));
```

$$6\,(2\cos(t)^2 - 1)\sin(t)$$

$$12\sin(t)^2\cos(t)$$
$$6\cos(t)(-5+6\cos(t)^2)$$
$$12(-1+3\cos(t)^2)\sin(t)$$
$$36\sin(t)^2$$
$$36\sin(t)\,u\cos(t)^2-12\,u\sin(t)-36\,v\cos(t)^3+30\,v\cos(t)$$
$$s := 6\,\mathrm{csgn}(\sin(t))\sin(t)$$

```
> int(s, t=-Pi/4..Pi/4);
```

$$-6\sqrt{2}+12$$

Exercie IV

On considère un cercle de rayon 1, roulant sans glisser sur une axe faisant un angle avec l'horizontale. On note H l'intersection du cercle et de l'axe, et M un point fixe sur le cercle.
On suppose qu'en t = 0 on a M = H = (0, 0) dans le repère (i,j).
Donner dans ce repère un paramétrage de la courbe décrite par M lorsque le cercle roule sur l'axe.

```
> restart:
  with(plots):p1:=plot([t, -t/sqrt(3), t=-4..4], color=magenta,
  scaling=CONSTRAINED):
  p2:=plot([t, 0, t=-4..4], scaling=CONSTRAINED):
  t0:=0:
  p3:=plot([t0+1/2+cos(-th+Pi/6), -t0/3+(sqrt(3)/2)+sin(-th+Pi/6),
  th=-4..4], color=red, scaling=CONSTRAINED):
  t0:=1:
  p4:=plot([t0+1/2+cos(-th), -t0/sqrt(3)+(sqrt(3)/2)+sin(-th),
  th=-4..4], color=blue, scaling=CONSTRAINED):
```

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Warning, the name changecoords has been redefined
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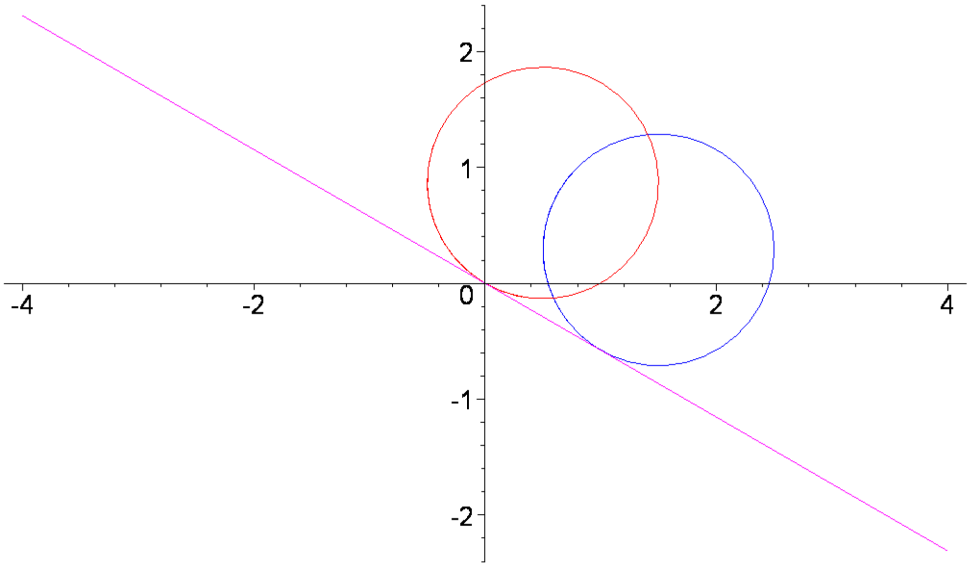
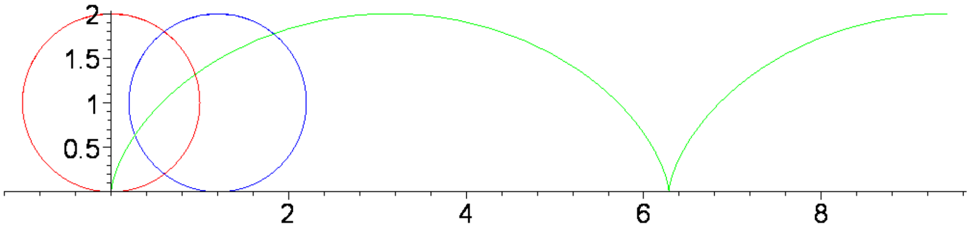
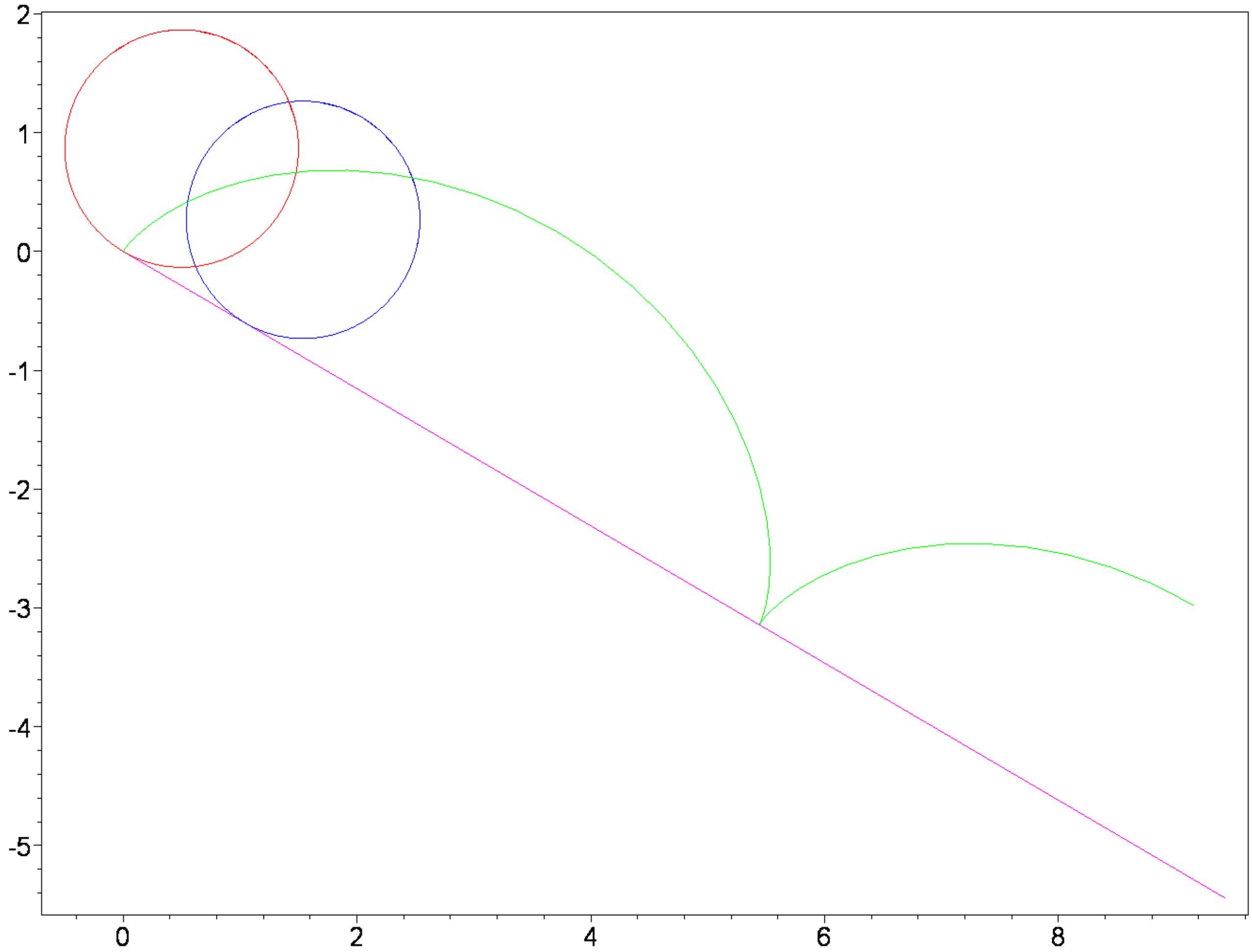
```
> display(p1, p2, p3, p4
  );
```




```
> restart:with(plots):R:=1:theta:=1.2:
  P0:=plot([ R*(t-sin(t)), R*(1-cos(t)),t=0..3*Pi], color=green):
  C0:=plot([ R*cos(t), R*(sin(t)+1),t=0..3*Pi], color=red):
  C1:=plot([ R*(cos(t)+theta), R*(sin(t)+1),t=0..3*Pi],
  color=blue):
  display(P0, C0,C1,scaling=CONSTRAINED);
Warning, the name changecoords has been redefined
```




```
> restart:with(plots):R:=1:theta:=1.2:alpha:=Pi/6:
  P0:=plot([ (cos(alpha)*R*(t-sin(t)))+sin(alpha)*R*(1-cos(t)),
  -sin(alpha)*R*(t-sin(t))+cos(alpha)*R*(1-cos(t)),t=0..3*Pi],
  color=green,scaling=CONSTRAINED):
  C0:=plot([ (sqrt(3)*R*cos(t)/2)+R*(sin(t)+1)/2,
  -R*cos(t)/2+sqrt(3)*R*(sin(t)+1)/2,t=0..3*Pi], color=red):
  C1:=plot([ (sqrt(3)*R*(cos(t)+theta)/2)+R*(sin(t)+1)/2,
  -(cos(t)+theta)/2+sqrt(3)*R*(sin(t)+1)/2,t=0..3*Pi],
  color=blue):
  C2:=
  plot([ x, -x/sqrt(3),x=0..3*Pi], color=magenta):display(P0,
  C0,C1,C2,scaling=CONSTRAINED);
Warning, the name changecoords has been redefined
```


2
1
0
-1
-2
-3
-4
-5
0
2
4
6
8