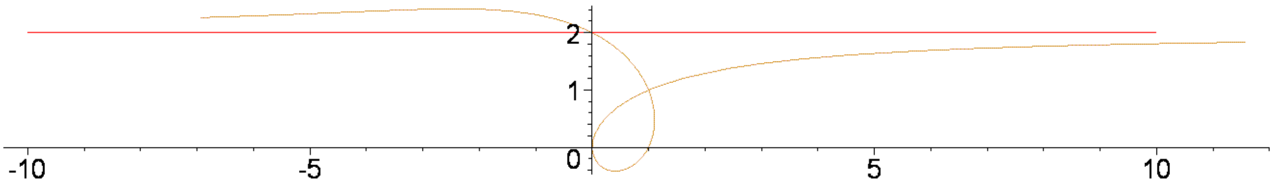
On considére la courbe définie en coordonnées polaires par
rho(theta)=1+tan(theta/2)
1. Déterminer le domaine de définition et la periode de l'arc paramétrée.

```
> restart:with(plots):P1:=polarplot([1+tan(t/2),t,t=-19*Pi/20..9*Pi/10],color=gold):
P2:=plot([t, 2, t=-10..10]):
display(P1, P2);
```

```
Warning, the name changecoords has been redefined
```



```
> restart:r:=(1+tan(t/2));rp:=diff((1+tan(t/2)),t);
>
```

$$r := 1 + \tan\left(\frac{t}{2}\right)$$

$$rp := \frac{1}{2} + \frac{1}{2}\tan\left(\frac{t}{2}\right)^2$$

```
> v:=rp*[cos(t),sin(t)]+r*[-sin(t),cos(t)];
```

$$v := \left(\frac{1}{2} + \frac{1}{2}\tan\left(\frac{t}{2}\right)^2\right)[\cos(t), \sin(t)] + \left(1 + \tan\left(\frac{t}{2}\right)\right)[-\sin(t), \cos(t)]$$

```
> t:=0;v;t:=-Pi/2;v;
>
```

$$t := 0$$

$$\left[\frac{1}{2}, 1\right]$$

$$t := -\frac{\pi}{2}$$

$$[0, -1]$$

```
> restart:limit((1+tan(t/2))*sin(t), t=Pi);
  limit((1+tan(t/2))*sin(t), t=-Pi);
```

$$2$$

$$2$$

Exercie 2

Etudier les branches infinies de la courbe
x(t)=(t^2+2*t-2)/(t-1), y(t)=(t^2+3*t-2)/(t-1).

```
> limit((t^2+3*t-2)/(t^2+2*t-2), t=1);
```

$$2$$

```
> limit(((t^2+3*t-2)/(t-1))-2*(t^2+2*t-2)/(t-1), t=1);
```

$$-3$$

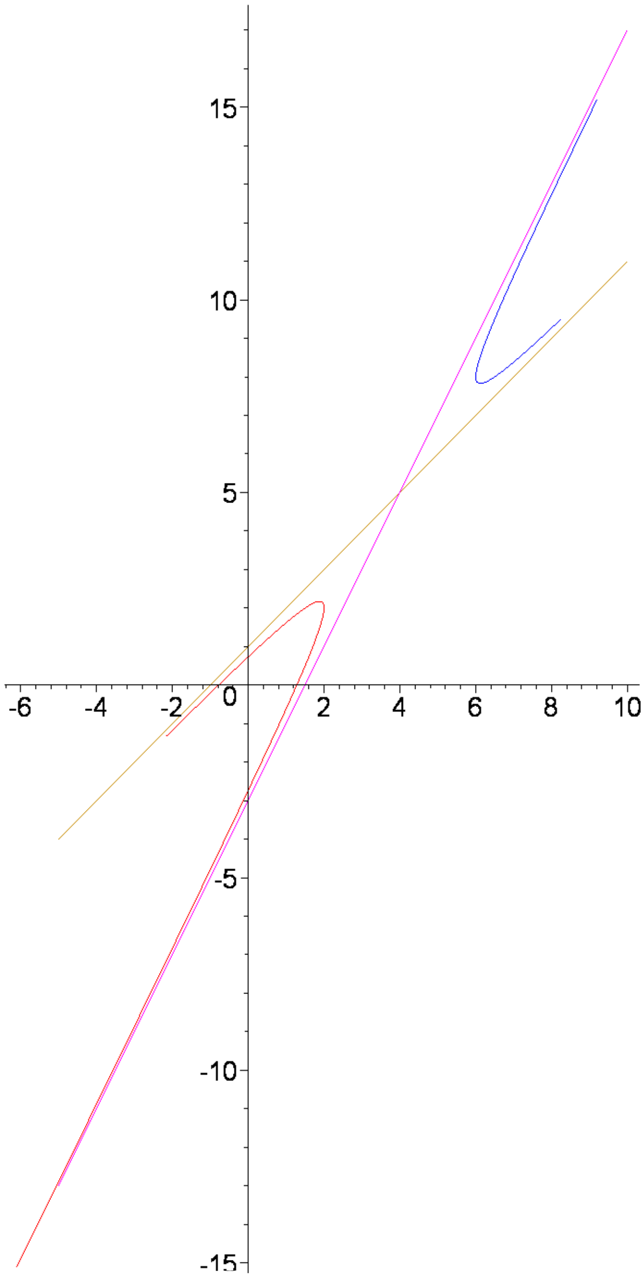
```
> limit(((t^2+3*t-2)/(t-1))-(t^2+2*t-2)/(t-1), t=infinity);
```

$$1$$

```
> #étudier les branches infinies
  with(plots):
  B1:=plot([(t^2+2*t-2)/(t-1), (t^2+3*t-2)/(t-1),
  t=-5..0.9],color=red):
  B2:=plot([(t^2+2*t-2)/(t-1), (t^2+3*t-2)/(t-1),
  t=1.2..5],color=blue):
  B3:=plot([t,2*t-3,t=-5..10],color=magenta): # asymptote oblique
  B4:=plot([t,t+1,t=-5..10],color=gold): # asymptote oblique

  display(B1,B2,B3, B4);
```

```
Warning, the name changecoords has been redefined
```



Exercie 3

Montrer que la courbe n'a pas de point singulier. Donner le repère de Frenet en tout point.

```
> #Repère de Frenet
```

```
> x:=simplify(exp(t/2)*cos(t));y:=simplify(exp(t/2)*sin(t));
```

$$x := \mathbf{e}^{\left(\frac{t}{2}\right)} \cos(t)$$

$$y := \mathbf{e}^{\left(\frac{t}{2}\right)} \sin(t)$$

```
> u:=diff(x,t);v:=diff(y,t);[u,v]:
  l:=simplify(sqrt(u^2+v^2)):
```

$$u := \frac{1}{2} e^{\left(\frac{t}{2}\right)} \cos(t) - e^{\left(\frac{t}{2}\right)} \sin(t)$$

$$v := \frac{1}{2} e^{\left(\frac{t}{2}\right)} \sin(t) + e^{\left(\frac{t}{2}\right)} \cos(t)$$

```
> tau:=[simplify(u/l),simplify(v/l)];
```

$$\tau := \left[\frac{1}{5} \frac{e^{\left(\frac{t}{2}\right)} (\cos(t) - 2\sin(t))\sqrt{5}}{\sqrt{e^t}}, \frac{1}{5} \frac{e^{\left(\frac{t}{2}\right)} (\sin(t) + 2\cos(t))\sqrt{5}}{\sqrt{e^t}} \right]$$

```
> eta:=[-simplify(v/l),simplify(u/l)];
```

$$\eta := \left[-\frac{1}{5} \frac{e^{\left(\frac{t}{2}\right)} (\sin(t) + 2\cos(t))\sqrt{5}}{\sqrt{e^t}}, \frac{1}{5} \frac{e^{\left(\frac{t}{2}\right)} (\cos(t) - 2\sin(t))\sqrt{5}}{\sqrt{e^t}} \right]$$