A semianalytic approach to the modelling and control of the biped gait for the design of interactive orthesis in Rehabilitation tasks

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Abstract

The configurations space of humanoid robots is usually modelled in terms of rotational joints. Passive devices allow to maintain the upright standing posture with a minimal energy expenditure. Usual optimization and Lyapunov stability criteria are based on quadratic scalar functionals defined onto the configurations space and their prolongations. To include a changing interaction with itself and the terrain, we introduce bivector functionals defined onto oriented elements of working space and their prolongations linked to the muscular effort. Extremals w.r.t. vector fields of both scalar and bivector functionals allow us a feedforward geometric control, which is based onto a min-max principle. Critical points for the scalar and bivector functionals are characterized onto a stratified mechanics as events (singularities of mappings) which separate control states between them. So, a geometric representation between adjacent attractors given by relative extrema organize the dynamics around alternating typical postures. Switching modes between adjacent states for the control are associated to transitions between typical postures. The functional coupling between anticipatory and compensatory human locomotion behaviour modelled as an alternance between forward and inverse mechanics around periodically changing optimal values. This alternance is translated to a coupling between active and passive mechanical robotic devices. So, our approach provides a theoretical foundations to integrate active and passive orthesis based on reciprocal mechanical devices for rehabilitation tasks.

1 Introduction

A good understanding and modelling of human gait is crucial to improve passive and active orthesis devices in the assistance to disabled persons. The usual analysis of passive reciprocal mechanical devices is restricted to the sagital plane. Hence, it ignores implicitely the small internal rotational movements holding for longitudinal components (tibia, femur) and the transversal rotation which holds at pelvis, also. The combination of rotational effects holding at frontal and horizontal planes improves the mobility and charge transmission. Furthermore, the swing of arms across the body in a direction parallel to the motion contribute to reinforce the compensatory effects in pelvic rotation. Such effects are very important to increase the marginal stability around optimal trajectories of the body components along the human locomotion ([8]).

A multibody is given as a collection of open kinematic chains which are connected to a common body. Usual biomechanical modelling of a multibody follows an increasing complexity which involves to the geometry of the musculoskeletal system, the kinematics related to the task to be performed, and the dynamics including the optimal integration of all subsystems to improve a control based onto propioceptive sensors. So, we have a changing dynamics arising from an evolving interaction with the environment and with another components of the body itself. The usual motion's equations are given as local sections of $J^2\mathcal{C}$ where $\mathcal{C}$ is the configurations space. Their direct image via $j^2\pi$ gives the motion's equations onto $J^2\mathcal{W}$, where $\mathcal{W}$ is the working space. A stratified framework for kinematics have been recently introduced ([5]. Our first contribution extends this approach to a mechanical hierarchy (going from geometry to dynamics through the kinematics) by means of a big diagram (EMAD) of stratified maps between semianalytic spaces ([3]). Stratifications induced by the rank have good incidence properties and support usual optimization and control models.

Non-linear dynamics for multibodies requires to evaluate changes at the position-orientation of the end-effector for each component and the global effect for the center of gravity. Hence, we must coordinate an unsupervised decentralized analysis for each component and a supervised centralized analysis for coordination. Superposition principles are easy to manage in the linear case, but the accumulative affect of non-linear models produce generically unstable effects. Thus, it is important to reformulate dynamics in linear terms to avoid the initial non-linear character.

There are different proposals which we shall label as 0-dimensional (by introducing a reformulation in terms of barycentric coordinates), 1-dimensional or 2-dimensional. Our second contribution is linked to a two-dimensional representation of dynamic effects associated to a changing architecture. The 2D representation is in some sense dual to the more usual geometric representation based on points.

Any control law in regular strata can be represented in terms of local sections of a stratification associated to the equations describing the kinematics or the dynamics. In this framework, motion's equations are interpreted as generalized vector fields in terms of generalized dynamic coordinates $(\mathbf{q}, \mathbf{p}, \mathbf{r})$. Ordinary vector fields onto strata are patched together to obtain stratified vector fields ([7]) onto $J^k\mathcal{C}$. The transference map $\pi : \mathcal{C} \to \mathcal{W}$ is a stratified map, and their k-th order prolongations $j^k\pi$ are also stratified maps between semianalytic spaces (Theorem 1). The forward images via $j^k\pi$ of sections defined onto $J^k\mathcal{C}$ are also sections defined $J^k\mathcal{W}$. Stratified vector fields allow us to make compatibilize behaviors at adjacent strata which are changing at a singularity ([3]), and to compute scape trajectories by using simple variational principles ([4]). Our third contribution displays an application of traditional variational calculus applied in this case to optimization problems in the space of generalized coordinates.

The above hierarchised representation allow us to include modal changes for the control design. Modal changes are associated to the singularities linked to 1) the task, 2) the boundary of original strata relative to $j^k\pi$ (or its extended version associated to additional constraints), 3) the existence of multiple solutions (hyperredundant mechanisms), or singularities at the boundary of the semianalytic support associated to constraints. To avoid the lack of uniqueness, we must to incorporate additional optimization criteria. The motion dynamics can be extended at such singularities if we introduce different representations involving to 1) a duality between low-codimension singularities associated to the tasks, 2) the specification of scape trajectories to connect adjacent strata, 3) an extension of the original space to guarantee unique optimal solutions.

Our goal is to show how the application of advanced mathematical tools can be useful to explain some dynamic effects linked to the alternance between anticipatory and compensatory effects in simplified models of the human gait. Thus, we have avoided to give proofs of our results, and we have limited ourselves to sketch the main ideas. Technical details will appear elsewhere.

2 A hierarchised approach for Mechanics

A general framework for a hierarchised model for Mechanics has been developed in terms of the Extended Main Analytic Diagram, EMAD ([3]). Hierarchies of the mechanical model are formulated in terms of the k-th order prolongations of the composition $g \circ \pi \circ \Gamma : \Pi_{i=1}^{N}[t_{0i}, t_{1i}] \to \mathcal{C} \to \mathcal{W} \to \mathbf{R}^p$ where 1) $\pi : \mathcal{C} \to \mathcal{W}$ is the *transference map* from the configurations $\mathcal{C}$ onto the working space $\mathcal{W}$, 2) the task to be performed is represented by a multipath $\Gamma : \Pi_{i=1}^{N}[t_{0i}, t_{1i}] \to \mathcal{X}$, 3) with the corresponding constraints $g : \mathcal{X} \to \mathbf{R}^p$ for the optimal behavior, being $\mathcal{X}$ the configurations $\mathcal{C}$ or the working space $\mathcal{W}$. The multipath Γ represents the set of trajectories described by the control points (including the end-effectors linked to the upper or lower extremities and/or the center of gravity $\mathbf{G}$). A decoupled analysis is based on the description of a constrained path for control nodes (at the hip, ankle and knee for each leg, e.g.) of each kinematic chain modelled as a free-floating manipulator (a floating base and a serial articulated arm). The movement of the c.o.g. $\mathbf{G}$ requires to specify a model for the dynamic coupling between kinematic chains (see below).

The transference between small movements at joints and motions in the ambient working space is represented by means of a general commutative Extended Main Analytic Diagram (EMAD in the successive):

$$\begin{array}{ccccccc}
J^2\mathbf{R}^n & \to & J^2\mathcal{C} & \to & J^2\mathcal{W} & \to & J^2\mathbf{R}^p \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
J^1\mathbf{R}^n & \to & J^1\mathcal{C} & \to & J^1\mathcal{W} & \to & J^1\mathbf{R}^p \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
J^0\mathbf{R}^n & \to & J^0\mathcal{C} & \to & J^0\mathcal{W} & \to & J^0\mathbf{R}^p
\end{array}$$

where horizontal floors correspond to the geometric, kinematic and dynamic aspects of the mechanics in terms of the configuration $\mathcal{C}$ and working $\mathcal{W}$ spaces. Tasks to be developed are represented as multipaths by the left column, whereas mechanical constraints are represented by the right column of the EMAD. There exists a natural stratification by the rank of the differential map which decomposes each semianalytic space in strata where the rank of the differential map is constant. More formally, an adaptation of the Ehresman's fibration theorem gives us:

Theorem *With the above notation, 1) The canonical projection $\pi : \mathcal{C} \to \mathcal{W}$ is a stratified map between stratified semianalytic spaces with respect to its differential locally given by the ideal generated by fixed size minors of the Jacobian map. Another said, the restriction to each stratum is a locally trivial topological fibration. 2) The k-th order prolongation $j^k\pi$ of*

π is also a stratified map for any $k \geq 0$..

Horizontal floors are given by the k-th order prolongations $j^k\pi : J^k\mathcal{C} \to J^k\mathcal{W}$ of the natural transference map $\pi : \mathcal{C} \to \mathcal{W}$. Jumps in the rank located at the closures of strata correspond to changes in the dimensionality of the meaningful parameter space for dynamics, and they can be interpreted as abrupt changes in the interaction with the environment.

The motion's equations can be interpreted as local sections of vertical fibrations $J^{k+1}\mathcal{X} \to J^{\ell}\mathcal{X}$ where $\ell \geq 1$. Each stratum is a union of integrability locuses, where the distribution associated to motion's equation is involutive. Vertical fiber bundle support natural *contact structures* given by the 1-forms $\underline{p}dt - d\underline{q}$ and $\underline{r}dt - d\underline{p}$. In terms of the contact constraints, linear momentum is integrable, but angular contact is not. Contact structural constraints are associated to the usual Legendre's interpretation of Mechanics in terms of generalized coordinates $(\underline{q}, \underline{p}, \underline{r})$.

In particular, the two lower horizontal lines concern to relations between the geometry and kinematics $\dot{\mathbf{x}} = J_{\mathbf{q}_a}\dot{\mathbf{q}}_a + J_{\mathbf{q}_p}\dot{\mathbf{q}}_p$ (where the subindex a, p is relative to the active, passive character of joints corresponding to such generalized coordinates). Similarly, the two upper rows concern to the relations between the kinematics and dynamics for $j^2\pi$ relative to a Lagrangian $\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = T(\mathbf{q}, \dot{\mathbf{q}}) - U(\mathbf{q})$, giving the well-known Euler-Lagrange dynamic model at the configurations or joints space:

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) \;=\; \Gamma - \mathbf{f} \;,$$

where $\mathbf{A}(\mathbf{q})$ is the symmetric positive definite inertia matrix $\mathbf{A}(\mathbf{q})$ and $\mathbf{C}(q, \dot{q})\dot{q}$ is the vector of Coriolis and centrifugal generalized forces. From these data, one constructs a *skew-symmetric* matrix ([6])

$$\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \;:=\; \dot{\mathbf{A}}(\mathbf{q}, \dot{\mathbf{q}}) - 2C(\mathbf{q}, \dot{\mathbf{q}}) \;,$$

which becomes degenerated at phase transitions (take-off and landing for the biped gait). The analysis of degenerate cases for $\mathbf{A}$ and $\mathbf{N}$ can be performed in terms of complete symmetric and skewsymmetric matrices ([**?**]). One obtains the forward or the inverse interpretation by taking sections (local inverses) of stratified maps. Furthermore lifted constraints arising from taking k-th order prolongations of geometric constraints g_k, there are specific kinematic or dynamic constraints; a typical example does correspond to the active and passive joints, with a changing character depending on the motion phase. A splitting of the lagrangian dynamics in terms of active and passive generalized coordinates gives ([9]):

$$\begin{pmatrix} B_a & B_{ap} \\ B_{ap}^T & B_p \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_p \end{pmatrix} + \begin{pmatrix} C(\mathbf{q}_a, \dot{\mathbf{q}}_a) \\ C(\mathbf{q}_p, \dot{\mathbf{q}}_p) \end{pmatrix} + \begin{pmatrix} g(\mathbf{q}_a) \\ g(\mathbf{q}_p) \end{pmatrix} = \begin{pmatrix} \mathcal{F}_a \\ \mathcal{F}_p \end{pmatrix}$$

where $\mathcal{F}_{ex}$ represents an external wrench which is applied on $\mathbf{x} \in \mathcal{W}$ corresponding to the current configurations $\mathbf{q}$.

The description of trajectories performed by control points is impractical due to some troubles to identify a) initial or boundary conditions to solve ODE or PDE associated to temporal or spatial propagation models; b) an imperfect knowledge of the manipulator response, specially when it interacts with the environment. In view of these difficulties, it suffices often to have 1) a qualitative *robust* information about the dynamic coupling to compensate motions in a stable way, 1) a measure of internal response to *adapt* modal changes when the multibody interacts with itself and the environment. To solve both problems we add a term for the inverse dynamics controller to achieve robustness, and another geometric optimization to adapt response parameters. A more refined analysis having in account singularities will need more control nodes. Hyperredundant mechanisms require an extension of usual optimization techniques due to the apparition of additional ordinary singularities (technical details relative to the singular case will appear elsewhere). Some optimization criteria for scape trajectories linked to singularities of biped locomotion have been proposed in [4]. Our above results can be extended to the reaction null-space.

3 Dynamical Modelling

A multibody with an intermittent contact with the ground has an evolving interaction with itself in the configurations space $\mathcal{C}$ and with the environment in the working space $\mathcal{W}$. The first one is modelled as a *coupling map* giving small displacement of the flexible beam induced by the manipulation reaction under a quasi-static conditions.

The existence of local sections defined onto incidence varieties allow us to construct a local splitting of fibrations which is translated to a decoupled mechanics. The transference between horizontal properties of the base space and vertical properties lying onto the fiber is performed in terms of connections which are compatible with the stratified structure of the EMAD.

3.1 Equivariant aspects

The action of $N = N_1 + N_2$ copies of $SO(n)$ corresponding to N_1 rotational ($n = 2$) and N_2 spherical ($n = 3$) joints induces a locally symmetric structure for strata. Some *meaningful results* relative to the local action of $SO(n)^N$ are the following: 1) This action induces an $SO(n)$-equivariant stratification for the EMAD. The locally symmetric structure of incidence varieties along equivariant strata gives structural symmetries for the dynamic coupling which can be propagated to adjacent strata by using elementary reflections or their infinitesimal versions given by stratified vector fields. 2) Additional kinematic constraints can be deduced from the conservation of the angular momentum along low codimension strata (in particular, we shall have a Momentum Map as it is usual in the Lagrangian or the Legendrian formulation of Mechanics). 3) Intrinsic differentiation given by a connection can be interpreted as a parallel displacement along an orbit for the momentum map. 4) Trajectories through the closure of orbits don't conserve the angular momentum. At singular points where the rank of the differential map jumps up, we have a capture-breaking symmetries phenomenon associated to the action of the product group. 5) The changing dynamic coupling can be interpreted as an exchange between the active and passive character of joints along the motion. The resulting capture-breaking of symmetries gives changes in the dimensionality of the maximal integral subspace of the distribution associated to the motion's equations.

3.2 Linearization for a splitting onto strata

If we take a suitable set of dynamic parameters, then the dynamic model of a manipulator can be linearized ([6], Chap.4.2.2). Dynamic parameters for the kinetic energy of an augmented link are given by the mass, the three components of the first moment of inertia, the six components of the inertia tensor and the moment of inertia of the rotor (the situation for the potential energy is easier). This linearization has been extended to the multibody case ([**?**]). It can be reformulated in a graded algebra which extends the space of Lagrange's generalized coordinates. The 2-dimensional components are given as changing bivectors which are supported onto mobile oriented parallelograms associated to pairs of incident vectors with variable angle and different contact point with the architecture (tendons). An optimal control for this linearized dynamics requires to implement vector fields acting as Lie derivatives onto such bivectors to find extremal values according to multiobjective optimization criteria.

4 Optimization Principles

The dynamic constraints involve to the architecture and the function: a) *Architectural constraints* are associated to the $3D$ geometry associated to the geometric configurations of control nodes. Geometric configurations are analyzed in terms of incident lines or planes. Their spatio-temporal evolution generates angular momenta whose total contribution must be controlled. The active or passive role of joints changes along a dynamic coupling; the selection of passive constraints allows to solve linearized dynamics following triangular elimination and back-propagation methods. b) *Functional constraints* involve to symmetric operators (like-distance, energy or a Lyapunov function). The allowable range of values is defined by a set of cost (in)equalities $g : J^1\mathcal{X} \to \mathbf{R}^p$ with components $g_1, \dots, g_p$. From constraints g_i we define an augmented Hamiltonian $\mathcal{H}(q, u, \lambda, t) = I(q(t), u(t), t) + \sum_{i=1}^{p} \lambda_i(t) g_i(q(t), u(t), t)$. The optimal control vector $u(t)$ is solved for the Euler-Lagrangian dynamics by using a variant of the gradient and Hessian criteria. Pontryagine's maximum principle gives necessary conditions for an extremal (local maximum, Hamilton-Jacobi equations and transversality conditions). Inversely, if the optimal values $u^* = u^*(q, \lambda, t)$, and the augmented hamiltonian $\mathcal{H}(q, u, \lambda, t)$ is concave in $q(t)$ for any $t \in [t_0, T]$, then the above necessary conditions of the maximum principle are also sufficient.

The *multiobjective optimization* for each leg is performed w.r.t. the internal and external constraints which are expressed in terms of functionals defined onto $J^k\mathcal{C}$ and $J^k\mathcal{W}$, respectively. Both types of constraints involve to *like-distance* and *like-volume functionals* which are expressed in terms of symmetric and skew-symmetric operators. The alternance between anticipatory and compensatory behaviors is characteristic of the biped gait. In the same way, we have a wandering alternance between extremals of like-energy symmetric at $J^k\mathcal{C}$ and like-volume skew-symmetric operators $J^k\mathcal{W}$ along the gait cycle: stance, take-off, flight and landing ([4]).

Following this approach, the Geometric Optimization is performed onto the graph $graph(j^k\pi)$ in terms of Lie derivatives w.r.t. fields associated to allowable transformations at joints associated to a scape trajectory as an integral curve of a stratified vector field. The vanishing of the Lie derivative along a field ξ of some of above scalar or bivector functional gives an extremal solution linked to a critical transition between typical postures (representing orbits for the induced action of $SO(n)^N$). In more advanced dissipative models (see below), extremals of integrals associated to variational problems are given by harmonic maps satisfying Euler-Lagrange equations.

A measure of dexterity and/or manipulability for articulated mechanisms is given by the product ${}^TJJ$, where ${}^TJ$ denotes the transposed of the jacobian matrix J representing the forward kinematics. If the jacobian matrix J is regular, the analogous to the normalized gradient $grad(f(x))/grad^2(f(x)$ is given by $J(^TJJ)^{-1}$. Let us remark that ^TJJ represents some kind of the square of the volume kinematic form in the source space $\mathcal{C}$ (in the singular case, take pseudo-inverse). For theoretical optimization questions, we introduce a kind of conjugation for the product ^TJJ relative to the metrics G and H defined onto the source $J^1\mathcal{C}$ and the target $J^1\mathcal{W}$ space. So, instead of taking ^TJJ, we must work with a functional defined by $^TJHJG^{-1}$ as a measure of smoothness associated to a kind of normalized dexterity/manipulability matrix in terms of the source coordinates. The extremal values associated to its trace are given by harmonic maps, verifying a system of Euler-Lagrange equations as structural constraints ([2]). In fact, the functions with a extremal opposite behavior for like-energy symmetric operator and like-volume skew-symmetric operator can be geometrically interpreted as solutions of an extended isoperimetric problem (similar to the Plateau's problem). Away from several classical well-known examples, it is very difficult to have an explicit analytic description of such solutions. This fact justifies our reduction to the topological approach.

The good incidence properties between tangent spaces along trajectories contained at adjacent strata of $J^1\mathcal{C}$, are ideally translated to a reflex behavior relative to constraints between cost functions associated to the optimization procedures for each leg. Indeed, if we take local coordinates $(\underline{q}, \mathbf{x}) \in graph(\pi) \subset \mathcal{C}x\mathcal{W}$ for the integral functional

$$F = [G(\underline{q}, \mathbf{x})]_i^f + \int_{\underline{q}_i}^{\underline{q}_f} H(x, y_k, \dot{y}_k)dx \ ,$$

the Erdmann-Weierstrass for the augmented Lagrangian $L = H + \sum \lambda_j g_j$, are given as

$$(\frac{\partial F}{\partial \dot{y}_k})_- = (\frac{\partial F}{\partial \dot{y}_k})_+ (-F + \sum \frac{\partial F}{\partial \dot{y}_k}\dot{y}_k)_- = (-F + \sum \frac{\partial F}{\partial \dot{y}_k}\dot{y}_k)_+$$

(where the signs - and + denote before and after the corner), whereas the transversality conditions are written now as

$$[dG + (F - \sum \frac{\partial F}{\partial \dot{y}_k}\dot{y}_k)dx + \sum (\sum_{k=1}^{n} \frac{\partial F}{\partial \dot{y}_k}\dot{y}_k)]_i^f = 0$$

So, nevertheless the existence of corner points (solutions in which one or more $\dot{y}_k$ jump), it is possible to maintain reference trajectories along the kinematic singularities, which act as dynamical attractors. Initial conditions for scape trajectories are given by an infinitesimal version of reflection groups. This infinitesimal version provides the framework for an adaptive control in terms of local symmetries given by vector fields tangent to the escape trajectories. Escape trajectories pass through kinematic singularities, and they can be computed as integral curves of stratified vector fields. So, we hope to improve the stability at phase transitions (take-off and landing) for the human gait.

The hyperredundant character of electromechanical devices for the biped gait makes impossible to obtain unique solutions for motion's equations. The global Hamiltonian arising from the dynamic coupling does not verify the convexity/concavity *sufficient* conditions required for minimum/maximum. However, the application of elementary reflections (based onto action/reaction principles) to pairs of functionals allows us to restore them. The physiologic analogue is given by agonist/antagonistic muscular behavior.

5 Some applications for the assistance to a disabled person

Usual biomechanics analysis of the human gait is based on a decoupling along sagital, coronal and frontal planes which are taken as coordinate planes. Each spherical joint is interpreted as a series of three rotational joints corresponding to the three Euler angles. After performing this decoupling one analyzes the contributions of the muscular-skeletal system connecting the hip, the knee and the ankle which are meaningful for the bipedal walking. The spatial character is often forgotten in benefit of its decomposition in planar rotations holding at coordinate planes. The dynamic coupling includes mechanical effects arising from an interaction with the environment and the body itself which require a more careful study ([8]). In this section, we have put emphasis onto some facts involving the spatial rotations inside the skeletal subsystem as support for the dynamics.

In the human case, femoral rotation within the hip joint is correlated with pelvic rotation. At heel contact the femur is in a neutral position and then rotates internally. It moves back to external rotation before toe-off and remains externally rotated until heel contact. The tibia internally rotates more rapidly than the femur. This assists the unlocking of the knee joint, by assisting in the unscrewing of the knee. Excessive internal rotation may be related to abnormal pronation of the foot. This has implications for abnormal limb and knee function. Our goal to mid-term is to incorporate these facts in advanced mechanical modelling of the human gait to improve the assistance to disabled persons. Nevertheless the importance of foot, its analysis have been discarded in the current approach due to the complexity of its morphological design.

The above analysis displays a complex interaction between the constraints appearing at different levels appearing inside the EMAD. Usual geometric optimization and control devices are based on a feedforward position-force analysis which can be geometrically reformulated in terms of screws and wrenches to include changing orientations and angular moments for an anticipatory and compensatory kinematics. Constraints relative to integrable linear and non-integrable angular momentum at joints are crucial to improve the current reciprocating devices based in a constrained kinematic optimization.

In practice, the most difficult problems for assistance and rehabilitation tasks concern to a non-optimal load location, dissipation effects and the reaction null-space. Thus, the hybrid models must be robust to maintain stability along the biped gait and enough flexible to adapt to uneven terrain. The linearization described above simplifies the decoupling for the augmented Lagrangian approach, and makes easier a centralized control. Dissipation effects along the multichain can be modelled in terms of isometric failures from the base to the end-effector; to achieve it, critical points of the integral functional associated to $J_\ell^T H J_\ell G^{-1}$ are given by harmonic jacobian maps ([2]).

The next step is to improve models for the dynamic coupling of linearized models developed in §3, and to develop electromechanical devices with their corresponding controllers for planar and spherical rotations. Recent advances in MEMS (microelectronic-mechanical systems) support a high-level control based on rotational vector fields. This control is very useful for a decoupled analysis corresponding to the sagital, coronal or frontal planes, and it can be easily implemented from the computational viewpoint. To improve the rehabilitation or the assistance to a disabled person, it is necessary a) to extend such planar study to the $3D$ case, b) to introduce controlled stratified vector fields ([7]) for screws. The reaction null-space and the zero-dynamics will be developed in another paper.

6 Conclusions and future developments

We have proposed a general stratified semianalytic framework to integrate Mechanics, Optimization and Control techniques in Robotics with coarse results in each one of these areas. Most of our results have still a theoretical character, and must be refined, simulated and tested on real bipeds. The application of Differential Analysis of Stratified Spaces to Robotics is still at their beginnings. Furthermore, one requires a bigger effort to integrate optimization and control analysis in more realistic volumetric models. The development of piecewise linear models have been classically formulated in terms of the geometry of lines (screws, twists and wrenches) for Mechanics. Lines support a hierarchised information which must be interpreted in terms of Incidence Varieties (flags) to each geometric, kinematic or dynamic level. The challenge is to integrate 0, 1 and 2-dimensional approaches in a volumetric model by developing some tools of Geometric Algebra, and to implement their corresponding algorithms.

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