Abstracts of talks "Geometric structures in 2 and 3 dimensions" Conference in Autrans

Francesco Bonsante: **Quasi-conformal minimal Lagrangian diffeomorphisms of the hyperbolic plane**
A minimal Lagrangian diffeomorphism between hyperbolic surfaces is an area-preserving diffeomorphism whose graph is a minimal surface. Recently Jean-Marc Schlenker and I proved that every quasi-symmetric homeomorphism of the circle uniquely extends to a quasi-conformal minimal Lagrangian diffeomorphism of the whole plane. In the talk i will present this work, and explain the main tool for the proof that is a non trivial relation between minimal Lagrangian diffeomorphisms and maximal surfaces in Anti de Sitter space.

Stephane Baseilhac: **Some problems of quantum Teichmüller theory**
The Teichmüller space $\mathcal{T}(S)$ of a punctured surface S of negative Euler characteristic is a rational manifold, with charts given by the shear coordinates on any ideal triangulation of S. The quantum Teichmüller space $\mathcal{T}_q(S)$ is an equivalence class of algebras of skew commuting polynomials depending on a complex parameter $q \in \mathbb{C}^*$, that reproduce the algebra of functions on $\mathcal{T}(S)$ with the Weil-Petersson Poisson structure at the limit $q \to 1$.

The representation theory of $\mathcal{T}_q(S)$ is very rich, and has a strong geometric content. We will present some fundamental results and open problems in this field, connected with the determination of the asymptotic behaviour of quantum invariants of 3-manifolds.

Virginie Charette: **Affine deformations of some surfaces with boundary**
Associated to every complete affine three-manifold M with non-solvable fundamental group is a non-compact surface S. We will describe the classification of these complete affine structures when S is homeomorphic to a three-holed sphere. The space of proper affine deformations of S identifies with two opposite octants of $\mathbf{R}^3$ and in fact correspond to actions admitting "crooked" fundamental domains. We will also discuss some recent progress on the case where S is homeomorphic to a one-holed torus. This is joint work with Drumm and Goldman.

Louis Funar: **Quantum representations of mapping class groups**
The image of a quantum representation of the mapping class group of genus at least 2 is:

- large, since the image of any subgroup of the Johnson filtration contains a free non-abelian subgroup, except for a few values of the level;

- not so large, since it is not isomorphic to a lattice in a higher rank Lie group; in particular, if the level is prime, the image is a subgroup of infinite index in a group of unitary matrices with entries in a cyclotomic ring.

This is joint work with Toshitake Kohno.

Olivier Guichard: **Domains of Discontinuity and Anosov Representations**
Anosov representations are representations of a hyperbolic group Γ into a semisimple Lie group G whose limit set satisfies some geometric properties (that we will describe). This notion due to François Labourie generalized the notion of quasifuchsian representation of surface groups into $\mathrm{PSL}(2, \mathbf{C})$.

In this joint work with Anna Wienhard we explain given an Anosov representation ρ how to construct an open ρ-invariant subset U in a projective space associated to G (or more generally in the flag variety associated to G) such that Γ acts properly discontinuously on U with compact quotient. In turn this can be used to give a description of the "higher Teichmüller spaces" as a component of the moduli space of geometric structures of a compact manifold.

Colin Guillarmou: **Invariant eta for 3-dimensional Schottky hyperbolic manifolds**
We define an eta invariant (of type APS) on convex co-compact hyperbolic manifolds of odd dimension. In dimension 3 we study its structure on the deformation space. Joint work with J.Park and S. Moroianu.

*Tobias Hartnick:***Causal representations of surface groups**
We explain how causal structures on Shilov boundaries can be used to define a new class of higher Teichmller representations called "causal representations". These representations (with Hermitian targets) are more general than maximal representations, but still discrete and faithful. Causal representations are already interesting for

$PSL_2(R)$, where they can be used to provide a new characterization of classical Teichmller space. This is joint work with Gabi Ben Simon, Marc Burger, Alessandra Iozzi and Anna Wienhard.

Steve Kerckhoff: **Representations of surfaces that bound 3-manifolds**

Cyril Lecuire: Volume of 3-dimensional hyperbolic convex cores
The convex core is a fundamental set for 3-dimensional hyperbolic manifolds with infinite volume. It contains all the geometrical informations. I will explain how the volume of the convex core is related to others invariants such as the conformal structure on the boundary at infinity.

*John Loftin:***Towards a Compactification of the Moduli Space of Convex $\mathbb{RP}^2$ Surfaces**
There is a canonical identification, due independently to Labourie and the speaker, of a convex $\mathbb{RP}^2$ structure on a closed oriented surface of genus $g \geq 2$ and a pair (Σ, U) consisting of a conformal structure and a holomorphic cubic differential. Allow Σ to degenerate to a nodal curve in the Deligne-Mumford compactification of the moduli space of Riemann surfaces and U to degenerate to a regular cubic differential on the nodal curve. This forms a partial compactification of the moduli space of convex $\mathbb{RP}^2$ structures. Using analytic techniques, we construct convex $\mathbb{RP}^2$ structures on the noncompact surfaces corresponding to this singular data and relate the holonomy to earlier work of Goldman. We also consider other limits $(\Sigma, \lambda U)$ as $\lambda \to \infty$ and relate the limiting projective holonomy to a description of limit points of convex $\mathbb{RP}^2$ structures due to Parreau.

Duc Manh Nguyen: **Trees, triangulations, and volume form on moduli space of flat surfaces**
We study some deformation of translation surfaces in the framework of flat surfaces with cone singularities. We show that the moduli space of such surfaces can be equipped with a structure of flat complex affine orbifold, together with a 'natural' parallel volume form, which agree with the usual one in the case of translation surfaces, up to a multiplicative constant. We also prove that the volume of these moduli spaces with respect to this volume form, normalized by some energy function, is finite. From this, we get new proofs for some classical results due to Masur-Veech, and Thurston.

Frederic Palesi: **Connected components of moduli spaces of $PSL(2, \mathbb{R})$ representations of non-orientable surface groups**
In this talk, we discuss the connected components of moduli spaces of representations of surface groups into a Lie group G, including the case of non-orientable surfaces. When G is compact, the number of connected components only depends on the fundamental group of G, and the mapping class group acts ergodically on each component. When G is PSL(2,R), a classical result states that for an orientable surface of genus g, there are 4g-3 connected components indexed by the Euler class. For a non-orientable surface, we show that there are only two connected components, indexed by a reduction modulo 2 of the Euler class.

Anne Parreau: **Complete reducibility and quotients of representation spaces**
Let Γ be a finitely generated group and G be the group of rational points of a reductive group over a local field. Complete reducibility is a geometric property in spherical builings, introduced by J-P Serre, characterizing reductive representations of Γ in G in characteristic zero. This notion is related to various natural CAT(0) geometric properties, via the action of G on his associated symmetric space or affine building. One can prove that the space of completely reducible conjugacy classes is the maximal Hausdorff quotient for the action of G on $\mathrm{Hom}(\Gamma, G)$, in a direct and rather elementary way, and arbitrary characteristic.

Richard Wentworth: **Bosonisation formulas for Riemann surfaces**
This will be mostly an introductory talk on determinants of laplace operators on holomorphic hermitian line bundles over Riemann surfaces. I will recall the formulas of Polyakov-Alvarez and Belavin-Knizhnik which describe the variation of the determinant with the metric in conformal and Teichmueller directions. Determinants for line bundles of nonzero degree are related to scalar determinants through the bosonization formulas, and these appear in perturbative string theory. The variational proof of these formulas leaves an undetermined constant, and I will describe how factorization properties of determinants can be used to compute this constant exactly.

Maxime Wolff: **Compactifications of moduli spaces of $PSL(2, \mathbf{R})$ representations**

Mike Wolf: **The Weil-Petersson Hessian of Length**
We present a brief and self-contained proof of a formula for the Weil-Petersson Hessian of the geodesic length of a closed curve (either simple or not simple) on a hyperbolic surface. The formula is the sum of the integrals of two naturally defined positive functions over the geodesic, proving convexity of this functional over Teichmuller space (due to Wolpert (1987)), and permitting several other immediate applications. For example, Wolperts result that the Thurston metric is a multiple of the Weil-Petersson metric directly follows on taking a limit of the formula over an appropriate sequence of curves