

```
> restart;
```

```
> with(LinearAlgebra):with(plots):
```

Warning, the name changecoords has been redefined

Paramètres

```
>
```

```
M1:= << -4 | -r | 0 >,  
      < 1 | -(7+2*r) | 1 >,  
      < -1 | 3 + r | -5 > >;  
omega:= sqrt(r^2 + 2*r + 4);  
a:=(8+3*r)/(32+10*r);  
c:=(8+2*r)/(32+10*r);  
t:=(8+3*r)/(32+10*r);  
g:=(8+2*r)/(32+10*r);  
cg:=2/(32+10*r);  
ca:=(16+7*r)/(8*(32+10*r));  
w:= <<1>, <0>, <-1>>;  
u:= <<r>, <2+r-omega>, <omega-2>>;  
v:= <<r>, <2+r+omega>, <-omega-2>>;
```

$$M1 := \begin{bmatrix} -4 & -r & 0 \\ 1 & -7-2r & 1 \\ -1 & 3+r & -5 \end{bmatrix}$$

$$\omega := \sqrt{r^2 + 2r + 4}$$

$$a := \frac{8 + 3r}{32 + 10r}$$

$$c := \frac{8 + 2r}{32 + 10r}$$

$$t := \frac{8 + 3r}{32 + 10r}$$

$$g := \frac{8 + 2r}{32 + 10r}$$

$$cg := \frac{2}{32 + 10r}$$

$$ca := \frac{16 + 7r}{256 + 80r}$$

$$w := \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$u := \begin{bmatrix} r \\ 2+r-\sqrt{r^2+2r+4} \\ \sqrt{r^2+2r+4}-2 \end{bmatrix}$$

$$v := \begin{bmatrix} r \\ 2+r+\sqrt{r^2+2r+4} \\ -\sqrt{r^2+2r+4}-2 \end{bmatrix}$$

```
> simplify( Multiply(M1,u) + (6 + r - omega)*u );
simplify( Multiply(M1,v) + (6 + r + omega)*v );
simplify( Multiply(M1,w) + 4*w );
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

— Expression de F(C,C) := f11, résolution du système

```
> Y0 := << c >, < cg >, < 0 >>;
Yinf := << c^2 >, < c*cg >, < c*( c -cg ) >>;
```

$$Y0 := \begin{bmatrix} \frac{8+2r}{32+10r} \\ \frac{2}{32+10r} \\ 0 \end{bmatrix}$$

$$Yinf := \begin{bmatrix} \frac{(8+2r)^2}{(32+10r)^2} \\ \frac{2(8+2r)}{(32+10r)^2} \\ \frac{(8+2r) \left(\frac{8+2r}{32+10r} - \frac{2}{32+10r} \right)}{32+10r} \end{bmatrix}$$

> `Y := simplify(Y0 - Yinf);`

$$Y := \begin{bmatrix} \frac{4(4+r)(3+r)}{(16+5r)^2} \\ \frac{4(3+r)}{(16+5r)^2} \\ -\frac{(4+r)(3+r)}{(16+5r)^2} \end{bmatrix}$$

> `eqns := { alpha1*u[1,1] + beta1*v[1,1] + gamm1*w[1,1] = Y[1,1],
alpha1*u[2,1] + beta1*v[2,1] + gamm1*w[2,1] = Y[2,1],
alpha1*u[3,1] + beta1*v[3,1] + gamm1*w[3,1] = Y[3,1]
};`

$$eqns := \left\{ \begin{array}{l} \alpha_1 r + \beta_1 r + \text{gamm1} = \frac{4(4+r)(3+r)}{(16+5r)^2}, \\ \alpha_1 (2+r - \sqrt{r^2+2r+4}) + \beta_1 (2+r + \sqrt{r^2+2r+4}) = \frac{4(3+r)}{(16+5r)^2}, \\ \alpha_1 (\sqrt{r^2+2r+4} - 2) + \beta_1 (-\sqrt{r^2+2r+4} - 2) - \text{gamm1} = -\frac{(4+r)(3+r)}{(16+5r)^2} \end{array} \right\}$$

$$\alpha_1 (2+r - \sqrt{r^2+2r+4}) + \beta_1 (2+r + \sqrt{r^2+2r+4}) = \frac{4(3+r)}{(16+5r)^2},$$

$$\alpha_1 (\sqrt{r^2+2r+4} - 2) + \beta_1 (-\sqrt{r^2+2r+4} - 2) - \text{gamm1} = -\frac{(4+r)(3+r)}{(16+5r)^2}$$

> `solve(eqns, {alpha1, beta1, gamm1});`

$$\left\{ \begin{array}{l} \alpha_1 = \frac{1}{4(256+160r+25r^2)(r^2+2r+4)r} (192+31r^3+110r^2+196r+3r^4 \\ + 23\sqrt{r^2+2r+4}r^2 + 74r\sqrt{r^2+2r+4} + 3\sqrt{r^2+2r+4}r^3 + 96\sqrt{r^2+2r+4}), \beta_1 = \\ \frac{1}{4r(256+160r+25r^2)(r^2+2r+4)} (-74r\sqrt{r^2+2r+4} - 96\sqrt{r^2+2r+4} \\ - 3\sqrt{r^2+2r+4}r^3 + 3r^4 + 196r + 192 + 31r^3 + 110r^2 - 23\sqrt{r^2+2r+4}r^2), \end{array} \right.$$

$$\left. \text{gamm1} = \frac{3+r}{2(16+5r)} \right\}$$

```
> alpha1 := factor(
1/4*(192+196*r+110*r^2+3*r^4+23*(r^2+2*r+4)^(1/2)*r^2+74*r*(r^2+2*r+4)^(1/2)+96*(r^2+2*r+4)^(1/2)+3*(r^2+2*r+4)^(1/2)*r^3+31*r^3)/((256+160*r+25*r^2)*(r^2+2*r+4)*r);
gamm1 := 1/2*(3+r)/(16+5*r);
beta1 := factor(
1/4*(-96*(r^2+2*r+4)^(1/2)-3*(r^2+2*r+4)^(1/2)*r^3+192+196*r+3*r^4-74*(r^2+2*r+4)^(1/2)+31*r^3-23*(r^2+2*r+4)^(1/2)*r^2+110*r^2)/(r*(256+160*r+25*r^2)*(r^2+2*r+4));
alpha1 := (3+r)(3r^3+22r^2+3*sqrt(r^2+2r+4)r^2+14r*sqrt(r^2+2r+4)+44r+32*sqrt(r^2+2r+4)+64) / (4(16+5r)^2(r^2+2r+4)r)
gamm1 := (3+r) / (2(16+5r))
beta1 := -(3+r)(-3r^3+3*sqrt(r^2+2r+4)r^2-22r^2-44r+14r*sqrt(r^2+2r+4)+32*sqrt(r^2+2r+4)-64) / (4(16+5r)^2(r^2+2r+4)r)
```

```
> e1 := tau-> exp(-(6+r-omega)*tau);
e2 := tau-> exp(-(6+r+omega)*tau);
e3 := tau-> exp(-4*tau);
```

$$e1 := \tau \rightarrow e^{-(6+r-\omega)\tau}$$

$$e2 := \tau \rightarrow e^{-(6+r+\omega)\tau}$$

$$e3 := \tau \rightarrow e^{-4\tau}$$

```
> f11 := tau-> alpha1*e1(tau)*u[1,1] + beta1*e2(tau)*v[1,1] +
gamm1*e3(tau)*w[1,1] + Yinf[1,1];
f21 := tau-> alpha1*e1(tau)*u[2,1] + beta1*e2(tau)*v[2,1] +
gamm1*e3(tau)*w[2,1] + Yinf[2,1];
f31 := tau-> alpha1*e1(tau)*u[3,1] + beta1*e2(tau)*v[3,1] +
gamm1*e3(tau)*w[3,1] + Yinf[3,1];
f11 := tau -> alpha1 e1(tau) u1,1 + beta1 e2(tau) v1,1 + gamm1 e3(tau) w1,1 + Yinf1,1
f21 := tau -> alpha1 e1(tau) u2,1 + beta1 e2(tau) v2,1 + gamm1 e3(tau) w2,1 + Yinf2,1
f31 := tau -> alpha1 e1(tau) u3,1 + beta1 e2(tau) v3,1 + gamm1 e3(tau) w3,1 + Yinf3,1
```

vérifications

```
> simplify( diff(f11(tau),tau) + 4*f11(tau) + r*f21(tau) - c);
simplify( diff(f21(tau),tau) - f11(tau) + (7+2*r)*f21(tau) - f31(tau)
```

```
);
simplify( diff(f31(tau),tau) + f11(tau) - (3+r)*f21(tau) + 5*f31(tau)
c);
```

```
0
0
0
```

```
>
```

Expression de F(A,A) := f0, résolution du système

```
> U0 := << 0 >, < 0 >, < ca >>;
Uinf := << a*g >, < a*cg >, < a*( c -cg ) >>;
```

$$U0 := \begin{bmatrix} 0 \\ 0 \\ \frac{16 + 7r}{256 + 80r} \end{bmatrix}$$

$$Uinf := \begin{bmatrix} \frac{(8 + 3r)(8 + 2r)}{(32 + 10r)^2} \\ \frac{2(8 + 3r)}{(32 + 10r)^2} \\ \frac{(8 + 3r) \left(\frac{8 + 2r}{32 + 10r} - \frac{2}{32 + 10r} \right)}{32 + 10r} \end{bmatrix}$$

```
> U := simplify(U0 - Uinf);
```

$$U := \begin{bmatrix} -\frac{(8 + 3r)(4 + r)}{2(16 + 5r)^2} \\ -\frac{8 + 3r}{2(16 + 5r)^2} \\ \frac{64 + 56r + 11r^2}{16(16 + 5r)^2} \end{bmatrix}$$

```
> eqns := { alpha2*u[1,1] + beta2*v[1,1] + gamm2*w[1,1] = U[1,1],
alpha2*u[2,1] + beta2*v[2,1] + gamm2*w[2,1] = U[2,1],
alpha2*u[3,1] + beta2*v[3,1] + gamm2*w[3,1] = U[3,1]
};
```

$$\text{eqns} := \left\{ \begin{aligned} \alpha_2 (2+r-\sqrt{r^2+2r+4}) + \beta_2 (2+r+\sqrt{r^2+2r+4}) &= -\frac{8+3r}{2(16+5r)^2}, \\ \alpha_2 (\sqrt{r^2+2r+4}-2) + \beta_2 (-\sqrt{r^2+2r+4}-2) - \text{gamm2} &= \frac{64+56r+11r^2}{16(16+5r)^2}, \\ \alpha_2 r + \beta_2 r + \text{gamm2} &= -\frac{(8+3r)(4+r)}{2(16+5r)^2} \end{aligned} \right\}$$

> solve (eqns, {alpha2, beta2, gamm2});

$$\left\{ \begin{aligned} \text{gamm2} &= -\frac{16+7r}{32(16+5r)}, \alpha_2 = -\frac{1}{64r(256+160r+25r^2)(r^2+2r+4)} (384r\sqrt{r^2+2r+4} \\ &+ 512\sqrt{r^2+2r+4} + 1024 + 13\sqrt{r^2+2r+4}r^3 + 13r^4 + 1024r + 154r^3 + 564r^2 \\ &+ 106\sqrt{r^2+2r+4}r^2), \beta_2 = -\frac{1}{64(256+160r+25r^2)(r^2+2r+4)r} (1024 \\ &- 13\sqrt{r^2+2r+4}r^3 + 13r^4 + 154r^3 + 564r^2 + 1024r - 106\sqrt{r^2+2r+4}r^2 \\ &- 512\sqrt{r^2+2r+4} - 384r\sqrt{r^2+2r+4}) \end{aligned} \right\}$$

> alpha2 :=

$$-1/64 * (1024 + 1024*r + 154*r^3 + 512*(r^2+2*r+4)^(1/2) + 106*(r^2+2*r+4)^(1/2)*r^2 + 384*(r^2+2*r+4)^(1/2)*r + 13*(r^2+2*r+4)^(1/2)*r^3 + 13*r^4 + 564*r^2) / (256 + 160*r + 25*r^2) * (r^2 + 2*r + 4) * r;$$

$$\text{gamm2} := -1/32 * (16 + 7*r) / (16 + 5*r);$$

beta2 :=

$$-1/64 * (1024 + 564*r^2 + 13*r^4 - 384*(r^2+2*r+4)^(1/2)*r - 512*(r^2+2*r+4)^(1/2) - 13*(r^2+2*r+4)^(1/2)*r^3 - 106*(r^2+2*r+4)^(1/2)*r^2 + 1024*r + 154*r^3) / ((256 + 160*r + 25*r^2) * (r^2 + 2*r + 4));$$

$$\alpha_2 := -\frac{1}{64r(256+160r+25r^2)(r^2+2r+4)} (384r\sqrt{r^2+2r+4} + 512\sqrt{r^2+2r+4} + 1024 + 13\sqrt{r^2+2r+4}r^3 + 13r^4 + 1024r + 154r^3 + 564r^2 + 106\sqrt{r^2+2r+4}r^2)$$

$$\text{gamm2} := -\frac{16+7r}{32(16+5r)}$$

$$\beta_2 := -\frac{1}{64(256+160r+25r^2)(r^2+2r+4)r} (1024 - 13\sqrt{r^2+2r+4}r^3 + 13r^4 + 154r^3 + 564r^2 + 1024r - 106\sqrt{r^2+2r+4}r^2 - 512\sqrt{r^2+2r+4} - 384r\sqrt{r^2+2r+4})$$

> f12 := tau -> alpha2*e1(tau)*u[1,1] + beta2*e2(tau)*v[1,1] + gamm2*e3(tau)*w[1,1] + Uinf[1,1];

```
f22 := tau-> alpha2*e1(tau)*u[2,1] + beta2*e2(tau)*v[2,1] +
gamm2*e3(tau)*w[2,1] + Uinf[2,1];
f32 := tau-> alpha2*e1(tau)*u[3,1] + beta2*e2(tau)*v[3,1] +
gamm2*e3(tau)*w[3,1] + Uinf[3,1];
f12 := tau -> a2 e1(tau) u1,1 + beta2 e2(tau) v1,1 + gamm2 e3(tau) w1,1 + Uinf1,1
f22 := tau -> a2 e1(tau) u2,1 + beta2 e2(tau) v2,1 + gamm2 e3(tau) w2,1 + Uinf2,1
f32 := tau -> a2 e1(tau) u3,1 + beta2 e2(tau) v3,1 + gamm2 e3(tau) w3,1 + Uinf3,1
```

vérifications

```
> simplify( diff(f12(tau),tau) + 4*f12(tau) + r*f22(tau) - a );
simplify( diff(f22(tau),tau) - f12(tau) + (7+2*r)*f22(tau) - f32(tau)
);
simplify( diff(f32(tau),tau) + f12(tau) - (3+r)*f22(tau) + 5*f32(tau)
a );
0
0
0
```

solution particulière pour F(A,A)

```
> fpart := tau -> -r*alpha2*e1(tau) - r*beta2*e2(tau) + r/4*a*cg + a/4;
fpart := tau -> -r a2 e1(tau) - r beta2 e2(tau) + 1/4 r a cg + 1/4 a
```

```
> fpart(tau);
```

$$\frac{1}{64 (256 + 160 r + 25 r^2) (r^2 + 2 r + 4)} \left((384 r \sqrt{r^2 + 2 r + 4} + 512 \sqrt{r^2 + 2 r + 4} + 1024 \right. \\ \left. + 13 \sqrt{r^2 + 2 r + 4} r^3 + 13 r^4 + 1024 r + 154 r^3 + 564 r^2 + 106 \sqrt{r^2 + 2 r + 4} r^2 \right) \\ e \left(- \left(6 + r - \sqrt{r^2 + 2 r + 4} \right) \tau \right) + \frac{1}{64 (256 + 160 r + 25 r^2) (r^2 + 2 r + 4)} \left((1024 \right. \\ \left. - 13 \sqrt{r^2 + 2 r + 4} r^3 + 13 r^4 + 154 r^3 + 564 r^2 + 1024 r - 106 \sqrt{r^2 + 2 r + 4} r^2 \right) \\ \left. - 512 \sqrt{r^2 + 2 r + 4} - 384 r \sqrt{r^2 + 2 r + 4} \right) e \left(- \left(6 + r + \sqrt{r^2 + 2 r + 4} \right) \tau \right) + \frac{r (8 + 3 r)}{2 (32 + 10 r)^2} \\ + \frac{8 + 3 r}{4 (32 + 10 r)}$$

```
> simplify( diff( fpart(tau),tau ) + 4*fpart(tau) - r*f22(tau) - a );
0
```

```
> k := simplify( a - fpart(0) );
```

$$k := \frac{31r + 80}{32(16 + 5r)}$$

> f0 := tau -> k*exp(-4*tau) + fpart(tau);

$$f0 := \tau \rightarrow k e^{(-4\tau)} + fpart(\tau)$$

> f0(tau);

$$\begin{aligned} & \frac{(31r + 80)e^{(-4\tau)}}{32(16 + 5r)} + \frac{1}{64(256 + 160r + 25r^2)(r^2 + 2r + 4)} \left((384r\sqrt{r^2 + 2r + 4} + 512\sqrt{r^2 + 2r + 4} \right. \\ & \quad \left. + 1024 + 13\sqrt{r^2 + 2r + 4}r^3 + 13r^4 + 1024r + 154r^3 + 564r^2 + 106\sqrt{r^2 + 2r + 4}r^2 \right) \\ & e^{\left(-(6 + r - \sqrt{r^2 + 2r + 4})\tau \right)} + \frac{1}{64(256 + 160r + 25r^2)(r^2 + 2r + 4)} \left((1024 \right. \\ & \quad \left. - 13\sqrt{r^2 + 2r + 4}r^3 + 13r^4 + 154r^3 + 564r^2 + 1024r - 106\sqrt{r^2 + 2r + 4}r^2 \right) \\ & \quad \left. - 512\sqrt{r^2 + 2r + 4} - 384r\sqrt{r^2 + 2r + 4} \right) e^{\left(-(6 + r + \sqrt{r^2 + 2r + 4})\tau \right)} + \frac{r(8 + 3r)}{2(32 + 10r)^2} \\ & \quad + \frac{8 + 3r}{4(32 + 10r)} \end{aligned}$$

> simplify(diff(fpart(tau), tau) + 4*fpart(tau) - r*f22(tau) - a);
0

Convexité de la courbe paramétrée

alpha1 et gamma1 sont strictement positifs, quand est-il de beta1 ? on voit ci dessous que beta1 > 0.

> simplify((64 + 44*r + 22*r^2 + 3*r^3)^2 - omega^2*(32 + 14*r + 3*r^2)^2);

$$384r^2 + 312r^3 + 156r^4 + 30r^5$$

alpha2 est strictement négatif, quand est-il de beta2 ? on voit ci dessous que beta2 < 0

> simplify((1024 + 1024*r + 564*r^2 + 154*r^3 + 13*r^4)^2 - omega^2*(512 + 384*r + 106*r^2 + 13*r^3)^2);

$$910r^7 + 52152r^5 + 131072r^2 + 129792r^4 + 10972r^6 + 186368r^3$$

La courbe est donc régulière, est-elle birégulière ?

> simplify(diff(f11(tau), tau)*diff(f0(tau), tau, tau) - diff(f11(tau), tau, tau)*diff(f0(tau), tau));

$$\frac{1}{4(256 + 160r + 25r^2)(r^2 + 2r + 4)} \left(\left(-e^{(-2\tau(6+r))} r^3 + 6e^{\left(-\tau(10+r+\sqrt{r^2+2r+4}) \right)} \right) r^3 \right)$$

$$\begin{aligned}
& + 6 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} r^3 - 6 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} \sqrt{r^2 + 2r + 4} r^2 \\
& + 6 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} \sqrt{r^2 + 2r + 4} r^2 + 28 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} r^2 \\
& + 28 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} r^2 - 2 e^{(-2 \tau (6 + r))} r^2 \\
& + 2 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} r \sqrt{r^2 + 2r + 4} \\
& - 2 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} r \sqrt{r^2 + 2r + 4} + 56 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} r \\
& + 56 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} r - 4 e^{(-2 \tau (6 + r))} r \\
& + 32 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} \sqrt{r^2 + 2r + 4} \\
& - 32 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} \sqrt{r^2 + 2r + 4} + 64 e^{\left(-\tau \left(10 + r + \sqrt{r^2 + 2r + 4} \right) \right)} \\
& + 64 e^{\left(\tau \left(-10 - r + \sqrt{r^2 + 2r + 4} \right) \right)} \left. \right) r(3 + r)
\end{aligned}$$

— courbe paramétrée, graphique

```
> r:=10;
```

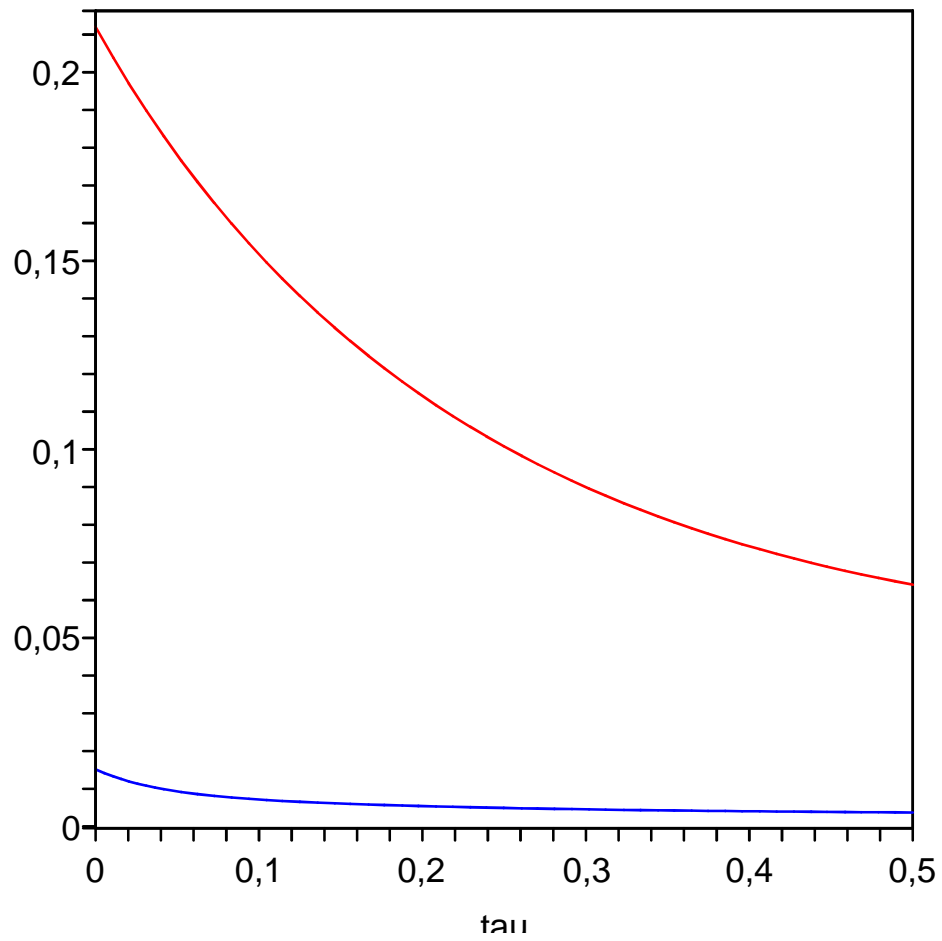
```
r := 10
```

```
> F1:=plot( f11(tau), tau=0..0.5, color=red):
```

```
G1:=plot( f21(tau), tau=0..0.5, color=blue):
```

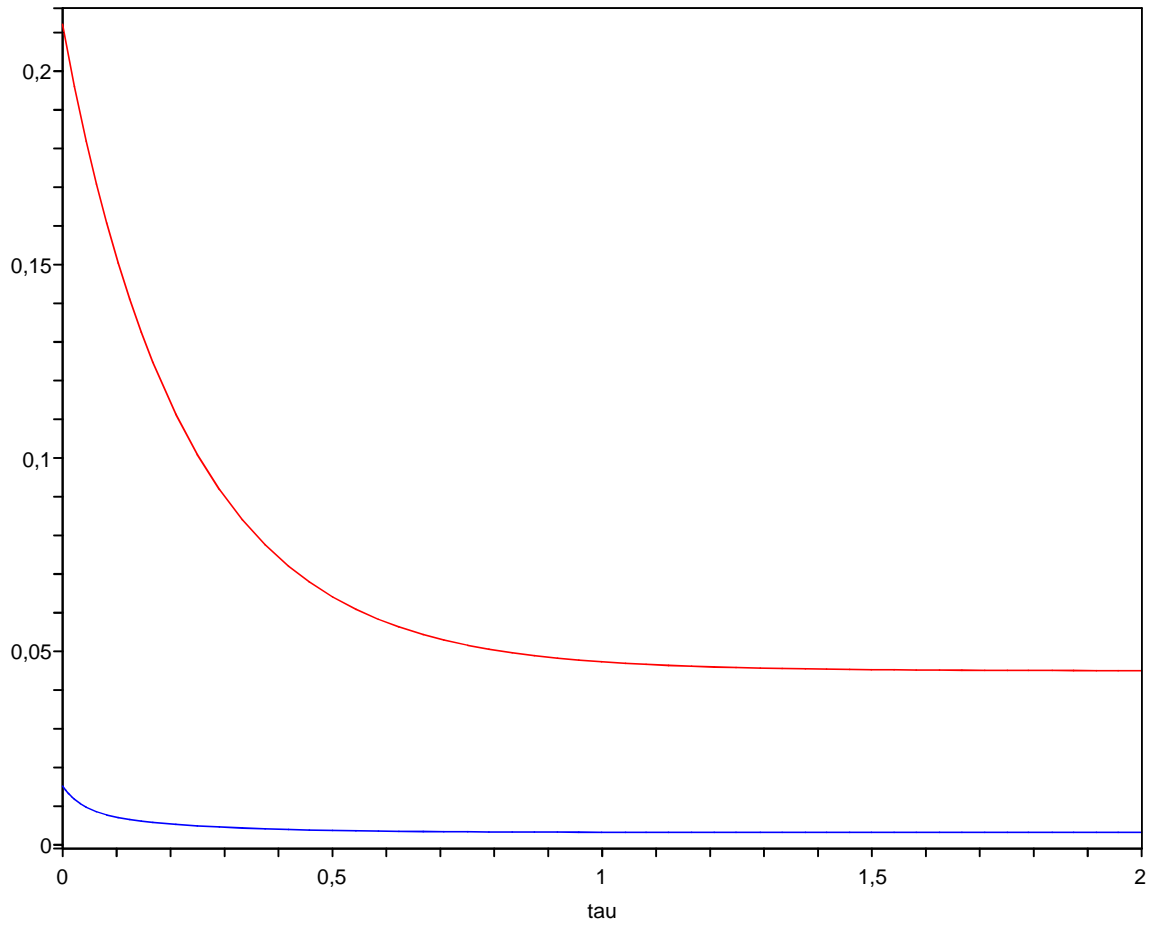
```
display({F1, G1}, axes=boxed, title=`F(C,C) en rouge and F(C*,CG) en
bleu `);
```

F(C,C) en rouge and F(C*,CG) en bleu



```
> F2:=plot( f11(tau), tau=0..2, color=red):  
G2:=plot( f21(tau), tau=0..2, color=blue):  
display({F2, G2}, axes=boxed, title=`F(C,C) en rouge et F(C*,CG) en  
bleu `);
```

F(C,C) en rouge et F(C*,CG) en bleu



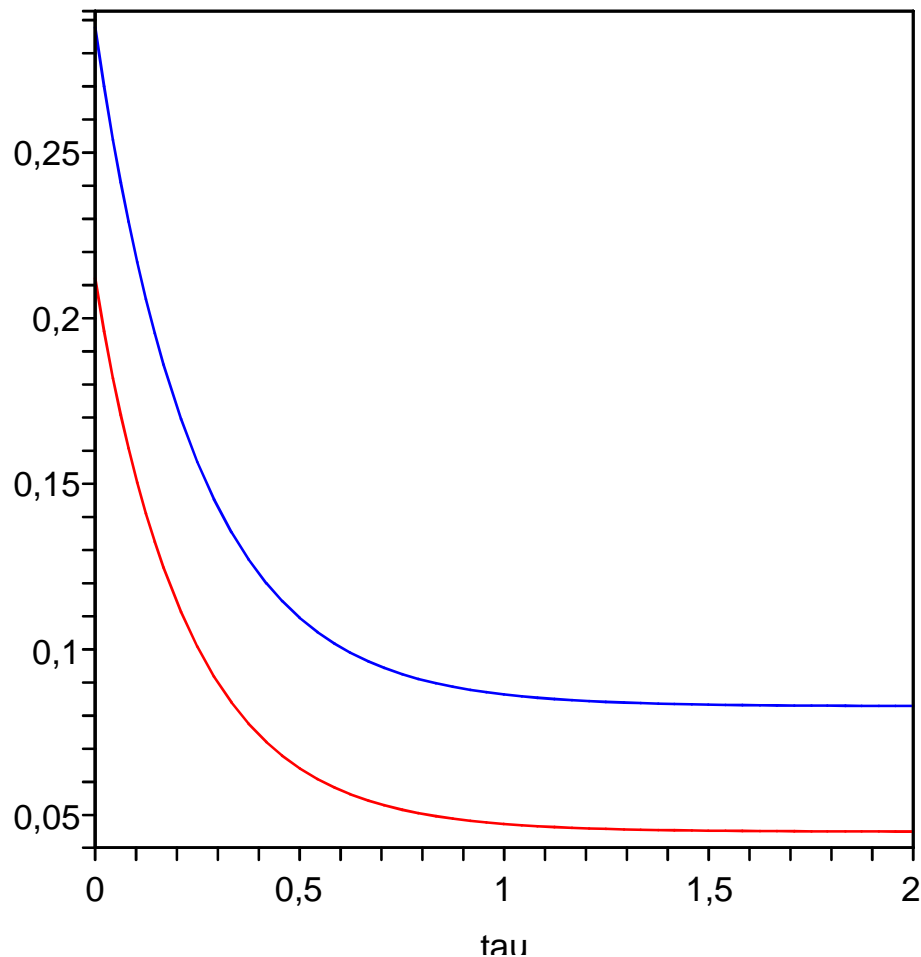
```
> f11(tau);  
f0(tau);
```

$$\begin{aligned}
& 10 \left(\frac{299}{87120} + \frac{767}{2700720} \sqrt{124} \right) e^{(-16 - \sqrt{124}) \tau} + 10 \left(\frac{299}{87120} - \frac{767}{2700720} \sqrt{124} \right) e^{(-16 + \sqrt{124}) \tau} \\
& + \frac{13}{132} e^{(-4 \tau)} + \frac{49}{1089} \\
& \frac{65}{352} e^{(-4 \tau)} - 10 \left(-\frac{1747}{21605760} \sqrt{124} - \frac{709}{696960} \right) e^{(-16 - \sqrt{124}) \tau} \\
& - 10 \left(-\frac{709}{696960} + \frac{1747}{21605760} \sqrt{124} \right) e^{(-16 + \sqrt{124}) \tau} + \frac{361}{4356}
\end{aligned}$$

```

> F:=plot( f11(tau), tau=0..2, color=red):
G:=plot( f0(tau), tau=0..2, color=blue):
display({F, G}, axes=boxed, title=`F(C,C) en rouge et F(A,A) en bleu`)
F(C,C) and F(A,A)

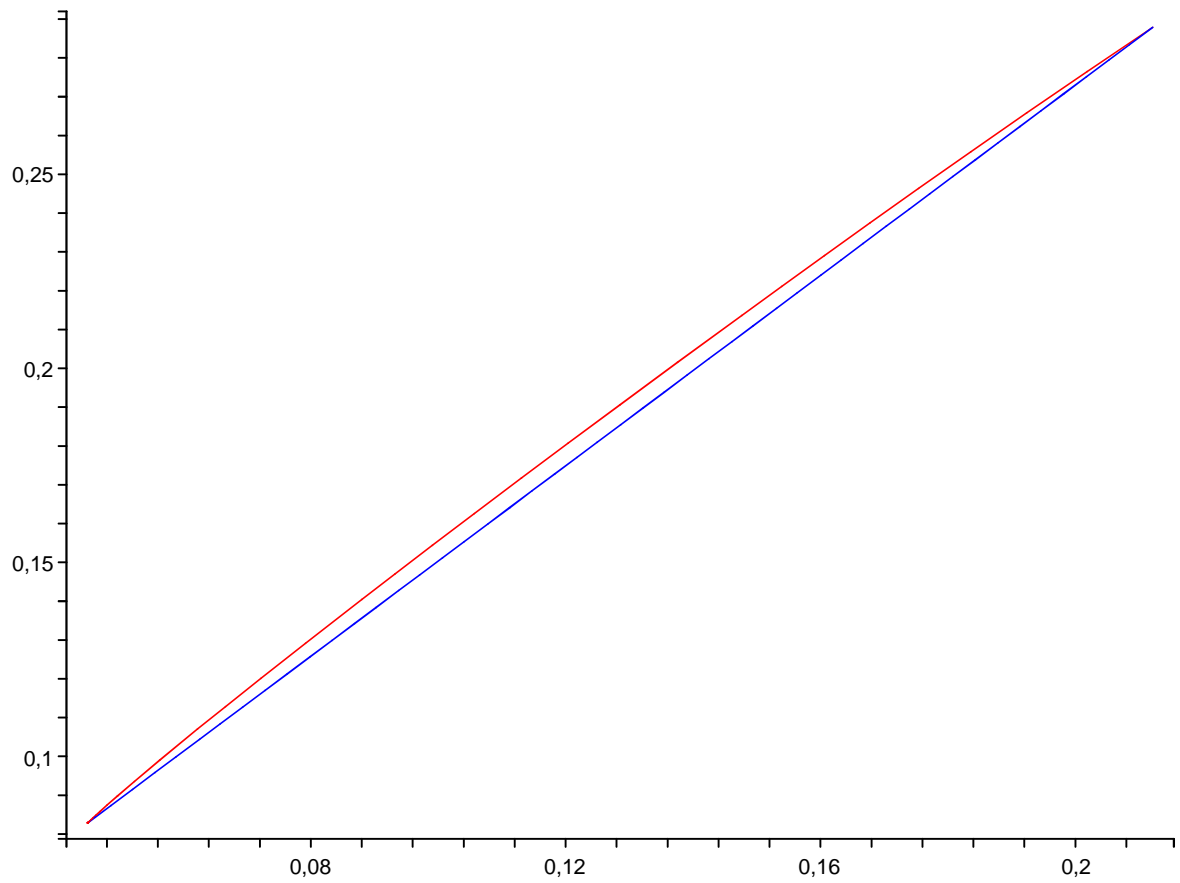
```



```

> plot([[f11(tau), f0(tau), tau=0..5], [c + s*(c^2 - c), a + s*(a^2 - a)
s=0..1]], color=[red,blue]);

```



en bleu le segment reliant $(F(C,C)(0), F(A,A)(0))$ à $(F(C,C)(\text{inf}), F(A,A)(\text{inf}))$, en rouge la courbe paramétrée $\tau \rightarrow (F(C,C)(\tau), F(A,A)(\tau))$ pour $r:= 100$

>

