



# Table ronde APMEP « Vers le prochain centenaire »

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Cédric Villani, médaille Fields 2010

## SPECTRAL METHODS FOR THE NON CUT-OFF BOLTZMANN EQUATION AND NUMERICAL GRAZING COLLISION LIMIT

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#### 1. Introduction

In this paper we are concerned with numerical resolution methods for collisional equations arising both from kinetic theory of rarefied gases and plasma physics. These equations take the form

(1) 
$$\partial_t f + v \cdot \nabla_x f = Q(f, f), \qquad t \ge 0, \quad x, v \in \mathbb{R}^3$$

where f(x, v, t) is assumed to be a nonnegative function that represents the density of particles in position x at time t with velocity v. In (1), Q(f, f) is a quadratic integral operator describing the collisions of particles, whose form we shall make explicit below. Here we will mainly focus our attention to the approximation of Q(f, f).

As it is well known in the physics literature, the Fokker-Planck-Landau (FPL) equation is used to describe the binary collisions, occurring in a plasma, between charged particles and can be written as

(2) 
$$\partial_t f + v \cdot \nabla_x f = Q_L(f, f),$$

(3) 
$$Q_L(f,f) = \nabla_v \cdot \left( \int_{\mathbb{R}^3} dv_* \, a(v - v_*) \left[ f_* \nabla_v f - f(\nabla_v f)_* \right] \right),$$

where a is the matrix-valued function of the form

$$a_{ij}(z) = \Psi(|z|) \left(\delta_{ij} - \frac{z_i z_j}{|z|^2}\right),$$

## Pékin, Yau awards, décembre 2009



## Irene Chen, médaille d'or (16 ans)



## Coordinate-free characterization of homogeneous polynomials with isolated singularities

Irene Chen

November 26, 2009

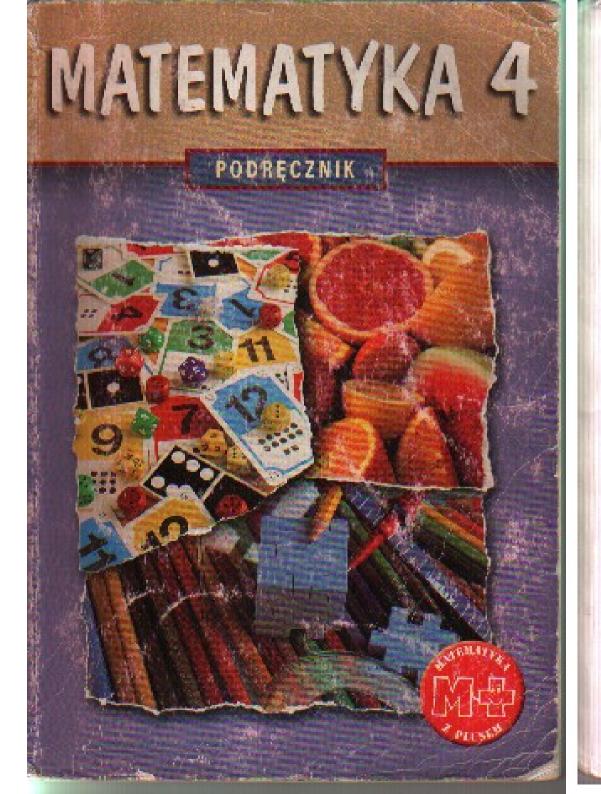
#### Abstract

The Durfee conjecture, proposed in 1978, relates two important invariants of isolated hypersurface singularities by a famous inequality; however, the inequality in this conjecture is not sharp. In 1995, Yau announced his conjecture which proposed a sharp inequality. The Yau conjecture characterizes the conditions under which an affine hypersurface with an isolated singularity at the origin is a cone over a nonsingular projective hypersurface; in other words, the conjecture gives a coordinate-free characterization of when a convergent power series is a homogeneous polynomial after a biholomorphic change of variables. In this project, we prove that the Yau conjecture holds for n = 5. As a consequence, we have proved that  $5!p_g \le \mu - p(v)$ , where  $p(v) = (v-1)^5 - v(v-1) \dots (v-4)$  and  $p_g, \mu$ , and v are, respectively, the geometric genus, the Milnor number, and the multiplicity of the isolated singularity at the origin of a weighted homogeneous polynomial. In the process, we have also defined yet another sharp upper bound for the number of positive integral points within a 5-dimensional simplex.

#### III - Enseignement de spécialité

[À titre indicatif, la répartition horaire entre les différents chapitres peut être : arithmétique : 50 % ; géométrie 50 %.]

Contenus	Modalités de mise en œuvre	Commentaires
Arithmétique []	On fera la synthèse des connaissances acquises dans ce domaine au collège.	[]  \( \times L'arithmétique est un domaine avec lequel l'informatique interagit fortement; on veillera à équilibrer l'usage de divers moyens de calculs: à la main, à l'aide d'une calculatrice ou sur un ordinateur.





#### eozkead ligzby na gzynniki piebwszg.



bwiczenie. Każdą z podanych liczb przedstaw w postac iloczynu liczb pierwszych:
4 6 9 12 15 21 24 30

Każdą liczbę naturalną złożoną można rozłożyć na czynniki pierwsze, czyli przedstawić w postaci iloczynu liczb pierwszych.

Popatrz, jak rozkładamy na czynniki pierwsze na przykład liczbę 84:

Rozkład liczby 84 na czynniki pierwsze: 84 = 2 · 2 · 3 · 7.

#### Zeungt derloren sir, 62

Następujące liczby rozbóż na czynniki pierwsze;

a) 18 b) 42 c) 50 d) 135 el 210 D 400

 Padaj przykład liczby frzycyfrowej, w której rozkładzie na czynnika pierwsze występują trzy różne liczby pierwsze.

#### Superzagadka

Rozszyfruj podany obok rozkład pownej liczby na czynniki pierwsze.



## Examen de « Classe XII » Sciences en Inde

(équivalent indien de la Terminale S ...)

Evaluate 
$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$$

- Q.18. Show that the points A, B, C with position vectors  $2\hat{i} \hat{j} + \hat{k}$ ,  $\hat{i} 3\hat{j} 5\hat{k}$  and  $3\hat{i} 4\hat{j} 4\hat{k}$  respectively, are the vertices of a right triangle. Also find the remaining angles of the triangle.
- Q.19. Evaluate,  $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}$ , dx a, b > 0
- Q.20. Solve the differential equation,  $2xy + y^2 2x^2 \frac{dy}{dx} = 0$ ; y(1) = 2

- **Q.21**. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product.
  - (i) What is the probability that the bolt drawn is defective?
  - (ii) If the bolt is found to be defective find the probability that it is a product of machine B.
- **Q.**22. Two dice are thrown simultaneously. Let X denote the number of sixes, find the probability distribution of X. Also find the mean and variance of X, using the probability distribution table.
- Q.23. Let X be a non-empty set. P(x) be its power set. Let '\*' be an operation defined on elements of P(x) by,

$$A * B = A \cap B$$
  $\forall A, B \in P(X)$ 

Then,

- (i) Prove that \* is a binary operation in P(X).
- (ii) Is \* commutative?
- (iii) Is \* associative?
- (iv) Find the identity element in P(X) w.r.t. \*
- (v) Find all the invertible elements of P(X)
- (vi) If o is another binary operation defined on P(X) as  $A \circ B = A \cup B$  then verify that o distributes itself over \*.

## Roberto di Cosmo (Le Hold-up *planétaire* : la face cachée de Microsoft)





### «Offrons aux jeunes les clés du pouvoir et de la liberté»

## ... mais on peut-on aussi être informaticien sans écran, ni souris, ni ordinateur !!

A l'école, on confond trop utilisation d'outils et apprentissage des concepts. Qui aurait l'idée de dire qu'il suffit d'une calculatrice pour faire des maths? Le projet « Computer Science Unplugged », impulsé par des chercheurs australiens et néo-zélandais, propose des contenus pour faire de l'informatique à l'école sans ordinateur. Les enfants apprennent les rudiments du calcul binaire ou « miment » différentes méthodes de tri de données... sans jamais mettre en marche un PC. L'Institut national de recherche en informatique et automatique (INRIA) est en train de le traduire en français. Les notions de base de l'informatique doivent être étudiées dès le secondaire.

## Classes « expérimentales » SLECC

