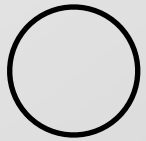
more extra dimensions

What if there is more than one extra dimension ?

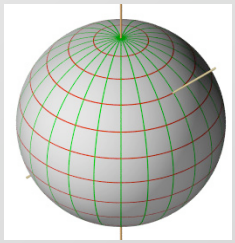
one (compact) extra dimension:



several (compact) extra dimensions:

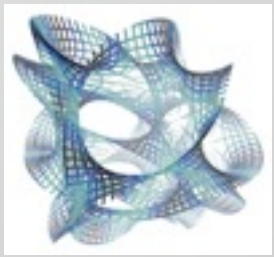
torus

or



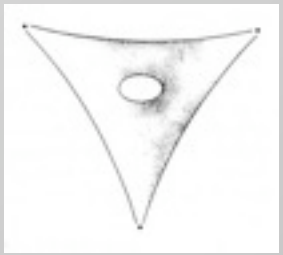
sphere

or



Calabi-Yau

or



orbifolds

or ....

higher-dimensional geometry

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} & g_{04} & g_{05} & g_{06} & g_{07} \\ g_{10} & g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{17} \\ g_{20} & g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} & g_{27} \\ g_{30} & g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} & g_{37} \\ g_{40} & g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} & g_{47} \\ g_{50} & g_{51} & g_{52} & g_{53} & g_{54} & g_{55} & g_{56} & g_{57} \\ g_{60} & g_{61} & g_{62} & g_{63} & g_{64} & g_{65} & g_{66} & g_{67} \\ g_{70} & g_{71} & g_{72} & g_{73} & g_{74} & g_{75} & g_{76} & g_{77} \end{pmatrix}$$

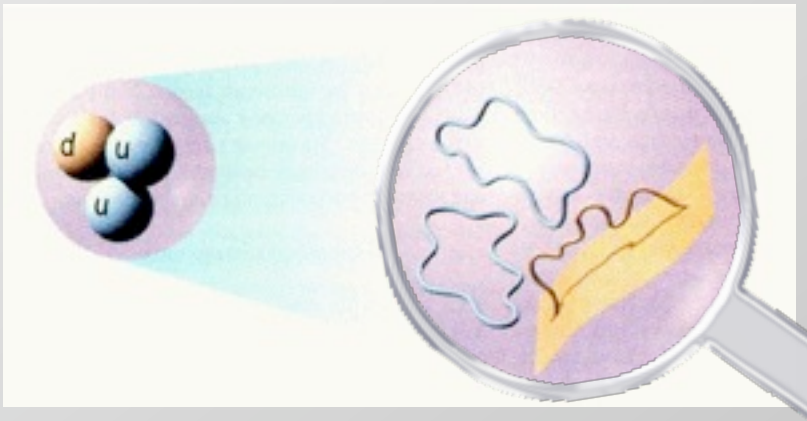
compactification →

various particles & interactions

e.g. weak, strong interactions

string theory: unification of interactions

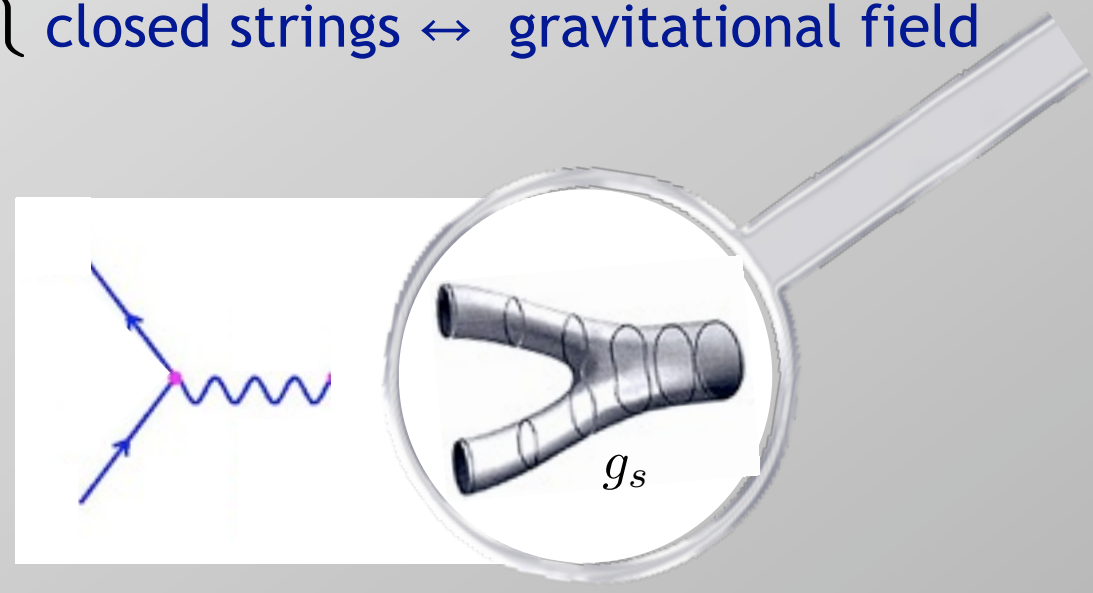
elementary objects are not
point particles, but **extended strings**



particles ↔ string vibration modes

light degrees of freedom
- open strings ↔ gauge fields
- closed strings ↔ gravitational field

string interactions



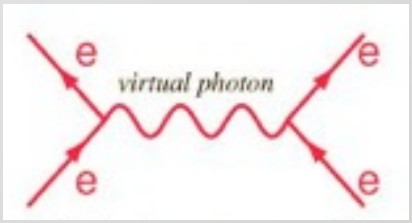
standard model of particle physics

- physics at the microscopic scale
  origin and structure of matter

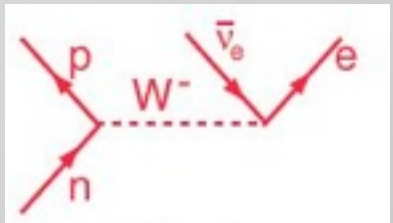
- quantum field theory (QED, Yang-Mills)
  describing the
  - constituents of matter (fermions)
  - fundamental interactions (bosons)

$$D^\mu F_{\mu\nu} = J_\nu$$

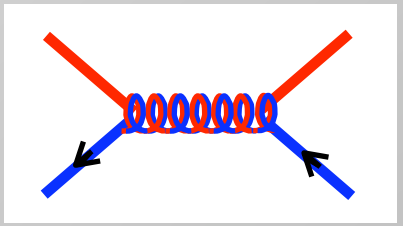
$$A_\mu = \begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{pmatrix}$$



electro: photon



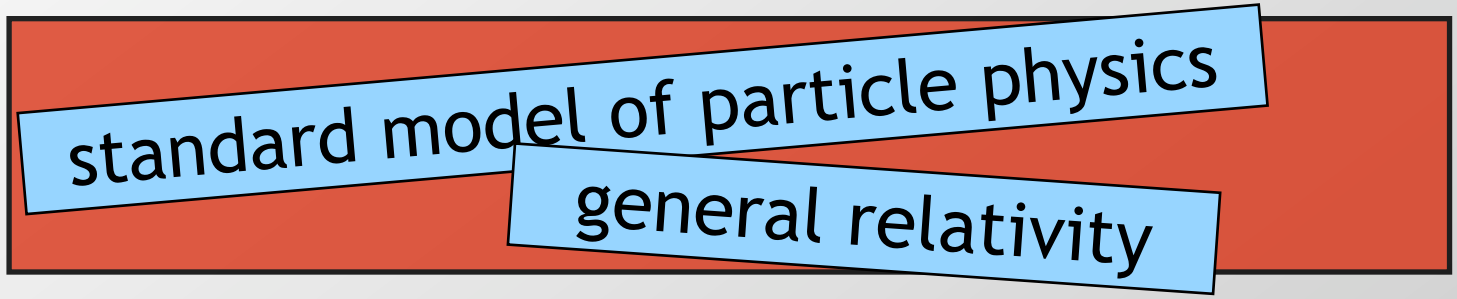
weak: W,Z-boson



strong: gluon

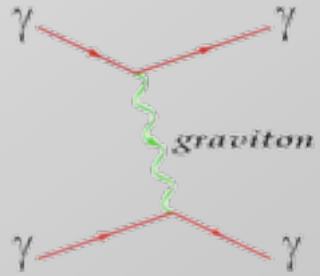
and gravity ..?

fundamental physics — unification of interactions



- GR cannot be cast into standard quantum theory framework

  non-renormalizable as QFT (graviton is spin-2)



- standard model / general relativity

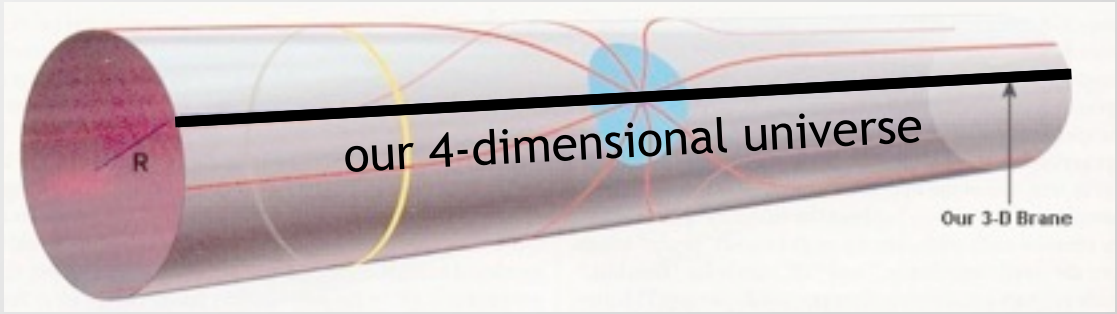
  considered as effective theories, valid on their respective scales

search for a fundamental theory

- contains GR / SM in the respective limits
- explanation of parameters: number of paricles, masses, couplings, symmetries, cosmological constant, etc...

Kaluza's fifth dimension

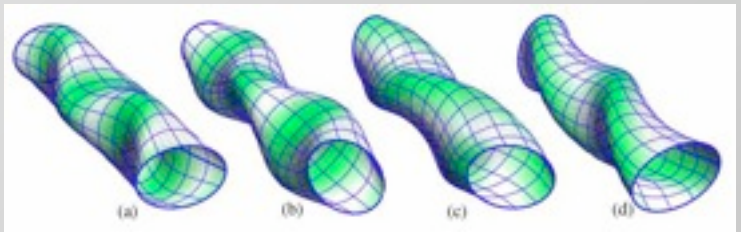
How does Einstein's general relativity look like,
if the universe has five instead of four dimension ?



a 5-dimensional universe

Einstein gravity : $R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R \;=\; 0$

now gravitational waves can propagate along the fifth dimension



describe a four-dimensional
electromagnetic field !
(T. Kaluza, 1921)

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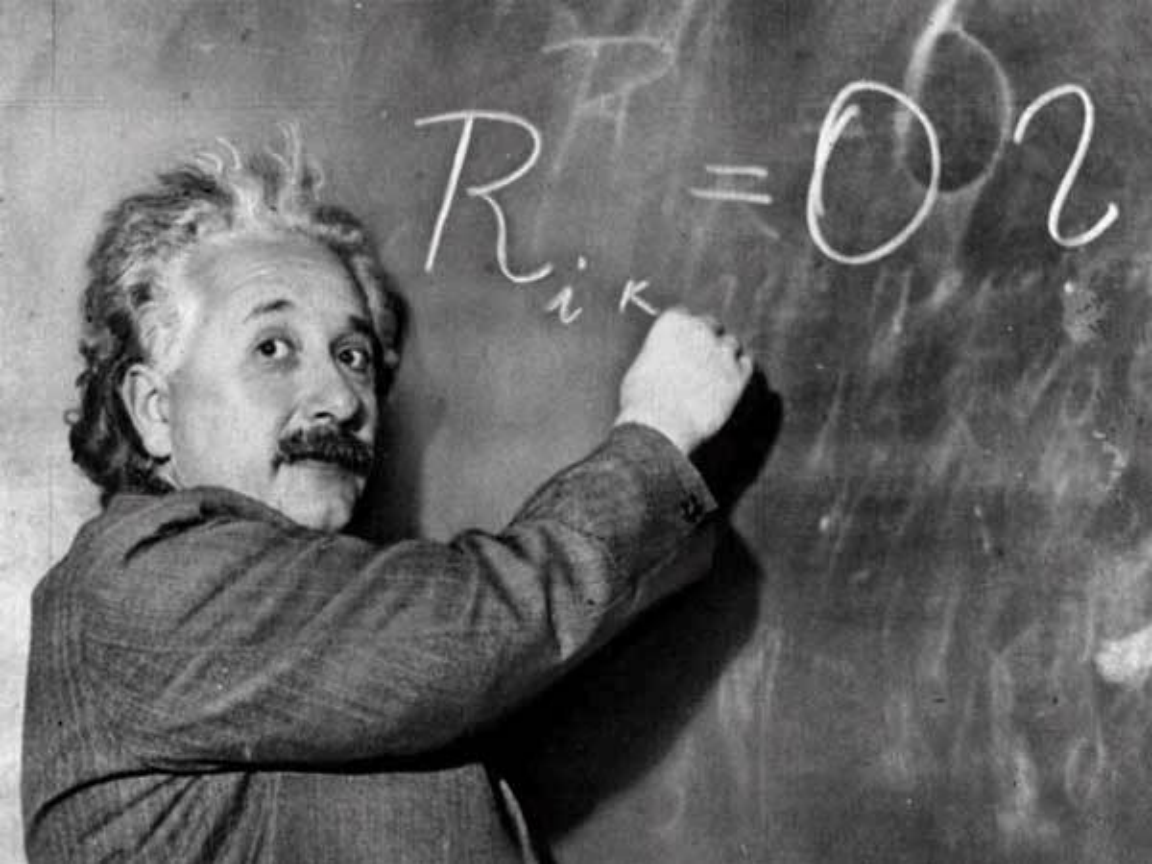
$R_{ik} = 0$