**MR662440 (84k:32011)** 32C30
**Demailly, Jean-Pierre**
**Sur les nombres de Lelong associés à l'image directe d'un courant positif fermé. (French. English summary) [On the Lelong numbers associated with the direct image of a closed positive current]**
*Ann. Inst. Fourier (Grenoble)* **32** (1982), *no. 2*, ix, 37–66.

Generalized Lelong numbers with respect to a logarithmically plurisubharmonic weight are defined for closed positive currents. The invariance properties of these numbers with respect to analytic morphisms give precise bounds for Lelong numbers of direct images.

Reviewed by *Jürgen Leiterer*





**MR679762 (84f:32007)** 32C30 (14C30)
**Demailly, Jean-Pierre**
**Courants positifs extrêmaux et conjecture de Hodge. (French) [Extremal positive currents and the Hodge conjecture]**
*Invent. Math.* **69** (1982), *no. 3*, 347–374.

On a complex manifold $X$, the convex cone $\mathrm{SPC}^p(X)$ of strongly positive closed $(p,p)$-currents is defined (for $0 \le p \le n = \dim_{\mathbf{C}} X$), and it is known that the current $[Z]$ corresponding to integration on an irreducible $p$-dimensional subvariety $Z$ of $X$ is an element of the set $E^p(X)$ of extremal elements of $\mathrm{SPC}^p(X)$. It has been conjectured [cf., e.g., P. Lelong, *Séminaire Pierre Lelong (Analyse), Année 1971–1972*, 112–131, Lecture Notes in Math., 332, Springer, Berlin, 1973; MR0412474 (54 #600)] that, for a Stein manifold $X$, every $T \in E^p(X)$ is a scalar multiple of such a $[Z]$. In this paper the author shows that the conjecture is false for $X = \mathbf{C}^n$ or $\mathbf{P}^n$ and $1 \le p \le n-1$. In fact, he proves that, if $C_d \subset \mathbf{P}^2$ is the curve $Z_0^d + Z_1^d + Z_2^d = 0$, then $((1/d)[C_d])$ converges (in the weak topology on currents) to a counterexample, and he passes to the case of $\mathbf{C}^n$ and $\mathbf{P}^n$ by using an extension theorem for closed positive currents due to H. Skoda [Invent. Math. **66** (1982), 361–376].

In the rest of the paper, the author indicates why the above conjecture is too optimistic (even for Stein or projective manifolds). When $X$ is projective, let $\mathrm{SPC}^p_{\mathbf{Z}}(X)$ consist of those $T \in \mathrm{SPC}^p(X)$ whose cohomology class belongs to the $\mathbf{R}$-span of $(\mathrm{Image}\, H^{2q}(X,\mathbf{Z}) \cap H^{q,q}(X,\mathbf{C}))$, $q = n-p$. Then, the conjecture that the convex cone generated by irreducible $p$-dimensional subvarieties of

$X$ is dense in $\mathrm{SPC}^p_{\mathbf{Z}}(X)$ already implies the Hodge conjecture that $H^{q,q}(X,\mathbf{C})\cap H^{2q}(X,\mathbf{Q})$ is generated by algebraic $p$-cycles, while the original conjecture (density in all of $\mathrm{SPC}^p(X)$) implies a stronger condition which is obviously false in general. The author proves the new conjecture (and its analogue when $X$ is Stein) for $p=n-1$.

Reviewed by *R. R. Simha*





**MR662130 (84d:32014)** 32C30 (10F35)

**Demailly, J.-P.**

**Formules de Jensen en plusieurs variables et applications arithmétiques. (French. English summary) [Jensen formulas in several variables and arithmetic applications]**

*Bull. Soc. Math. France* **110** (1982), *no. 1,* 75–102.

Let $F$ be an entire function on $\mathbf{C}^n$ and $P_1,\cdots,P_N$ be polynomials of degree $\delta$, the maximal homogeneous parts $Q_1,\cdots,Q_N$ of which have a unique common zero at $0\in\mathbf{C}^N$. Let $\varphi(z)=\sum_{j=1}^N|P_j(z)|^2$, $\beta=i\partial\overline{\partial}\varphi$, $T=(i/\pi)\partial\overline{\partial}\mathrm{Log}\,|F|$ and $|F|_r=\sup_{|z|\le r}|F(z)|$.

Using an appropriate generalization of the Poisson-Jensen formula, the author proves the following new variant of the Schwarz lemma in $\mathbf{C}^n$: There exists a constant $C\in(0,1]$, depending only on $P_1,\cdots,P_N$, such that for all $R\ge r\ge 1$ we have

$$\int_{r^{2\delta}}^{CR^{2\delta}}\frac{dt}{t^n}\int_{\varphi(z)<t}T\wedge\beta^{n-1}\le(2\delta)^n\pi^{n-1}\mathrm{Log}\frac{|F|_R}{|F|_r}.$$

This inequality permits the author to give a new proof of E. Bombieri's theorem on algebraic values of meromorphic maps without using $L^2$-estimates for the $\overline{\partial}$-operator.

Some new results concerning zero sets of polynomials in $\mathbf{C}^n$ are also given.

Reviewed by *G. M. Khenkin*

**MR691167 (85d:46004)** 46A06 (32F05 46G20)

**Lelong, Pierre**

**A class of Fréchet complex spaces in which the bounded sets are C-polar sets.**

*Functional analysis, holomorphy and approximation theory* (*Rio de Janeiro*, 1980), *pp.* 255–272, *North-Holland Math. Stud.*, 71, *North-Holland, Amsterdam*, 1982.

Let $P_b(\Omega)$ be the set of plurisubharmonic (p.s.h.) functions bounded above in a domain $\Omega$ of a complex topological vector space $E$; a subset $A$ of $\Omega$ is a control for $P_b(\Omega)$ if there exists a strictly positive function $\gamma(A,x)$ such that (1) $f(x) \leq \gamma(A,x)\cdot\sup\{f(x)\colon x\in A\}$ for all $f\in P_b(\Omega)$.

Such a control is useful for improving bounds for p.s.h. functions. Let $f$ be a given p.s.h. function, less than $M$ in $\Omega$, and less than $m$ in $A$; then $f\leq M(1-\gamma)+m\gamma<M$. Define $g_A$ by $g_A(x)=\sup\{f(x)\colon f\leq 0 \text{ in } \Omega,\ f\equiv -1 \text{ in } A,\ f \text{ is p.s.h. in } \Omega\}$.

The author establishes the equivalence of the following properties: (i) $A$ is a control for $P_b(\Omega)$; (ii) $g_A<0$; (iii) $A$ is not strictly $\mathbf{C}$-polar in $\Omega$. They imply that $g_A$ is the best control associated with $A$.

To obtain uniform bounds in a neighbourhood of a point, the author looks for an upper semicontinuous control. Let $g_A^*$ be the upper regularization of $g_A$. There are only two possibilities: (I) $g_A^*\equiv 0$ and a semicontinuous control does not exist; (II) $g_A^*\not\equiv 0$, in which case $g_A^*$ is the best semicontinuous control.

Whenever $E$ is a Frechet space, one finds by using the polycylinders studied by the reviewer [Ann. Inst. Fourier (Grenoble) 20 (1970), no. 1, 361–432; MR0274804 (43 #564)] that $A$ is a semicontinuous control if and only if $A$ is not strictly $\mathbf{C}$-polar in $\Omega$.

From this result and an extension to infinite-dimensional space of the author's inverse function theorem [Seminaire Pierre Lelong-Henri Skoda (Analyse), Annee 1976/77, 172–195, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522476 (80i:58009)], bounds can be found for a function $M(x,|z|)$ p.s.h. in $E\times\mathbf{C}$. Bounds of this kind have been used by H. Skoda and J. P. Demailly to find a counterexample to Serre's conjecture.

The last section is devoted to the construction of Frechet spaces in which bounded sets are $\mathbf{C}$-polar.

{For the entire collection see MR0691157 (84b:46001)}.

{For the entire collection see MR0691157 (84b:46001)}

Reviewed by *G. Coeuré*

**MR690650 (85d:32057)** 32L15 (32L20)

**Demailly, Jean-Pierre**

**Estimations $L^2$ pour l'opérateur $\bar\partial$ d'un fibré vectoriel holomorphe semi-positif au-dessus d'une variété kählérienne complète. (French) [$L^2$-estimates for the $\bar\partial$-operator of a semipositive holomorphic vector bundle over a complete Kähler manifold]**

*Ann. Sci. École Norm. Sup. (4)* **15** (1982), *no. 3,* 457–511.

Author's review: Let $E$ be a Hermitian vector bundle of rank $r$ over an $n$-dimensional Kahler manifold $X$. The bundle $E$ is said to be $s$-positive if its curvature tensor $K(E)$ identified with a Hermitian form on $TX \otimes E$ takes positive values on tensors of rank $\leq s$ and $\neq 0$. For example, if $E$ is Griffiths positive (i.e. 1-positive) of rank $r \geq 2$, we show that $E^* \otimes (\det E)^s$ is $s$-positive and that $E \otimes \det E$ is Nakano positive (i.e. $n$-positive). In connection with these results, we prove the following vanishing theorem: If $E$ is $s$-positive and $X$ is weakly pseudoconvex, then $H^q(X, \bigwedge^n T^*X \otimes E) = 0$ for $q \geq \sup(1, n - S + 1)$. Given a surjective morphism $E \to Q \to 0$ of Hermitian bundles, we also obtain curvature conditions which imply the surjectivity of the map $H^q(X, E \otimes L) \to H^q(X, Q \otimes L)$, $0 \leq q < n$, where $L$ is a line bundle. All these results are proved in quantitative versions using $L^2$ estimates and plurisubharmonic weights. In order to get rid of continuity assumptions for weights or exhaustion on $X$, a smoothing method is developed for psh functions involving the exponential map $TX \to X$. In particular, if $X$ has an upper semicontinuous exhaustive psh function, then it can be endowed with a complete Kahler metric.

Reviewed by *Autorreferat* (Zbl 507:32021)





**MR658880 (83j:32019)** 32D15 (32L05)

**Demailly, J.-P.**

**Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations $L^2$. (French) [Holomorphic splitting of a morphism of semipositive vector bundles with $L^2$ estimates]**

*Seminar Pierre Lelong-Henri Skoda (Analysis)*, 1980/1981, *and Colloquium at Wimereux, May* 1981, *pp.* 77–107, *Lecture Notes in Math.*, 919, *Springer, Berlin-New York*, 1982.

Let $0 \to S \to E \xrightarrow{g} Q \to 0$ be an exact sequence of holomorphic Hermitian vector bundles over the weakly pseudoconvex complex Kählerian manifold $X$ and let $M$ be a line bundle over $X$. Using a result of H. Skoda [Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)] the author gives geometric conditions in terms of the curvature of the bundles which are sufficient to find for every $f \in \Gamma(X, \mathrm{Hom}(Q, Q \otimes M))$ a holomorphic preimage

$h \in \Gamma(X, \mathrm{Hom}(Q, E \otimes M))$ with $L^2$-estimates. In the natural situation of the exact sequence $0 \to TX \to T\Omega|_X \to NX \to 0$, where $\Omega \subset \mathbf{C}^n$ is a pseudoconvex domain and $X$ is a closed submanifold in $\Omega$, the above results are applied to prove extension theorems for holomorphic functions on $X$ with precise estimates. Similar results have been obtained before by B. Jennane [*Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77* (French), pp. 126–133, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522474 (80m:32016)], C. A. Bernstein and B. A. Taylor [J. Analyse Math. **38** (1980), 188–254; MR0600786 (82h:32002)] and T. Yoshioka [Proc. Japan Acad. Ser. A Math. Sci. **57** (1981), no. 3, 181–184; MR0618087 (82f:32029)].

{For the entire collection see MR0658876 (83d:32001)}

Reviewed by *Peter Pflug*





**MR658879 (83j:32032)** 32L20 (32L05)

**Demailly, J.-P.**

**Relations entre les différentes notions de fibrés et de courants positifs. (French) [Relations between the different notions of positive vector bundles and currents]**

*Seminar Pierre Lelong-Henri Skoda (Analysis)*, 1980/1981, *and Colloquium at Wimereux*, *May* 1981, *pp.* 56–76, *Lecture Notes in Math.*, 919, *Springer*, *Berlin-New York*, 1982.

The author discusses relations among various positivity concepts for Hermitian forms $\Theta$ on vector spaces of the form $T \otimes E$. The main result states: If $\Theta$ is semipositive in the sense of P. A. Griffiths, i.e., $\Theta(x,x) \geq 0$ for the decomposable vectors $x \in T \otimes E$, then for a scalar product $\varphi$ on $E$ the form $\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi$ is strictly semipositive, i.e., for all $x \in T \otimes E$ one has: $(\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi)(x,x) = \sum |x_i^*(x)|^2$, where $x_i^*$ are finitely many decomposable linear forms on $T \otimes E$. In particular, it follows that $(\Theta + \mathrm{Tr}_E\,\Theta \otimes \varphi)(x,x) \geq 0$, i.e., this form is semipositive in the sense of S. Nakano.

This result is then applied to vector bundles and the positivity concepts valid there [cf. the author and H. Skoda, *Séminaire Pierre Lelong- Henri Skoda* (*Analyse*), *Années 1978/79* (French), pp. 304–309, Lecture Notes in Math., 822, Springer, Berlin, 1980; MR0599033 (82h:32028)].

In the last section the author compares a weakly positive $(p,p)$-form $\alpha \in \Lambda^{p,p}\,\mathrm{Hom}_{\mathbf{R}}(T, \mathbf{C})$ (i.e., a form $\alpha$ for which, with every $(k,k)$-form $\beta = \sum_j \varepsilon_k \cdot \beta_j \wedge \overline{\beta}_j$, where the $\beta_j$'s are decomposable $(k,0)$-forms, the form $\alpha \wedge \beta$ is a positive $(n,n)$-form) with forms of the type $L^{p-1} \bigwedge^{p-1} \alpha$. Here, with the existing scalar product on $T$ the operators $L$, $\bigwedge$ are modeled after the usual Kähler operators. For example,

$$C(n,p) \cdot \frac{1}{p!^2} \cdot L^{p-1} \bigwedge{}^{p-1} \alpha - \alpha$$

is a sum of forms $\varepsilon_p \cdot \alpha_j \wedge \overline{\alpha}_j$ with decomposable $(p,0)$-forms $\alpha_j$ and exactly definable constants

$C(n,p)$. An essential tool in the proofs is a summation formula for the $q$th roots of unity.
{For the entire collection see MR0658876 (83d:32001)}

Reviewed by *Peter Pflug*





**MR658876 (83d:32001)** 32-06

★**Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981, et Colloque de Wimereux, Mai 1981. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis), 1980/1981, and Colloquium at Wimereux, May 1981]**

Colloquium on "Plurisubharmonic Functions in Finite or Infinite Dimensions" in honor of Pierre Lelong held in Wimereux, May 1981.
Edited by Lelong and Skoda.
Lecture Notes in Mathematics, 919.
*Springer-Verlag*, *Berlin-New York*, 1982. vii+386 *pp*. ar20.00. *ISBN* 3-540-11482-3

{The previous seminar has been reviewed [ MR0599013 (81j:32003)].}

Contents: Part I. Séminaire d'Analyse (Paris) [Analysis Seminar (Paris)]: Carlos A. Berenstein and B. A. Taylor, On the geometry of interpolating varieties (pp. 1–25); Mongi Blel, Fonctions plurisouharmoniques et idéal définissant un ensemble analytique [Plurisubharmonic functions and the ideal defining an analytic set] (pp. 26–55); J.-P. Demailly, Relations entre les différentes notions de fibrés et de courants positifs [Relations between the different notions of positive vector bundles and currents] (pp. 56–76); J.-P. Demailly, Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations $L^2$ [Holomorphic splitting of a morphism of semipositive vector bundles with $L^2$ estimates] (pp. 77–107); Bernard Gaveau, Intégrales de courbure et potentiels sur les hypersurfaces analytiques de $\mathbf{C}^n$ [Curvature integrals and potentials on analytic hypersurfaces of $\mathbf{C}^n$] (pp. 108–122); B. Gaveau and G. Laville, Fonctions holomorphes et particule chargée dans un champ magnétique uniforme [Holomorphic functions and a charged particle in a uniform magnetic field] (pp. 123–130); Bernard Gaveau and Julian Ławrynowicz, Intégrale de Dirichlet sur une variété complexe. I [The Dirichlet integral on a complex manifold I] (pp. 131–166); Pierre Lelong, Calcul du nombre densité $\nu(x,f)$ et lemme de Schwarz pour les fonctions plurisousharmoniques dans un espace vectoriel topologique [Calculation of the density number $\nu(x,f)$ and Schwarz's lemma for plurisubharmonic functions in a topological vector space] (pp. 167–176); R. Michael Range, Boundary regularity for the Cauchy-Riemann complex (pp. 177–186).

Part II. Colloque de Wimereux, Mai 1981 [Colloquium at Wimereux, May 1981]. Gérard Coeuré, En l'honneur du Professeur Pierre Lelong [In honor of Professor Pierre Lelong] (pp. 189–191); V. Avanissian, Sur les fonctions harmoniques d'ordre quelconque et leur prolongement analytique

dans $\mathbf{C}^N$ [On harmonic functions of arbitrary order and their analytic continuation in $\mathbf{C}^N$] (pp. 192–281); D. Barlet, Développements asymptotiques des fonctions obtenues par intégration sur les fibres [Asymptotic expansions of functions obtained by integration over the fibers] (pp. 282–293); Eric Bedford, The operator $(dd^c)^n$ on complex spaces (pp. 294–323); Christer O. Kiselman, Stabilité du nombre de Lelong par restriction à une sous-variété [Stability of the Lelong number under restriction to a subvariety] (pp. 324–336); Robert E. Molzon and Bernard Shiffman, Capacity, Tchebycheff constant, and transfinite hyperdiameter on complex projective space (pp. 337–357); V. S. Vladimirov, Several complex variables in mathematical physics (pp. 358–386).

{Most of the papers of mathematical interest are being reviewed individually.}





**MR633888 (83m:32002)** 32A22 (10F35 14J17)

**Chudnovsky, G. V.**

**Singular points on complex hypersurfaces and multidimensional Schwarz lemma.**

*Seminar on Number Theory*, *Paris* 1979–80, *pp*. 29–69, *Progr*. *Math*., 12, *Birkhäuser*, *Boston*, *Mass*., 1981.

This is mainly an expository paper concerning the so-called Schwarz lemma in two complex variables, and the related problem of the degree of a hypersurfaces with given singularities. Many results are stated, either as problems or as theorems. Here is an example. Let $S$ be a finite subset of $\mathbf{C}^n$. For each integer $t \geq 1$, let $\omega_t(S)$ be the minimal degree of hypersurface in $\mathbf{C}^n$ having at each point of $S$ a singularity of order at least $t$ (see Chapter 7 of the reviewer's book, *Nombres transcendants et groupes algébriques* [Astérisque, 69–70, Soc. Math. France, Paris, 1979; MR0570648 (82k:10041)]). The author states the inequality $\omega_t(S)/t \geq (\omega_1(S)+n-1)/n$ as a conjecture for general $n \geq 1$, and as a theorem for $n = 2$. Work in this direction, including a proof of this claim for $n = 2$, has been done by D. W. Masser using linear algebra ["A note on multiplicities of polynomials", *Groupe d'étude sur les problèmes diophantiens 1980/81*, Publ. Math. Univ. Pierre et Marie Curie, no. 43, 1981], by G. Wüstholz using commutative algebra ["On the degree of algebraic hypersurfaces with given singularities", ibid.; see also Wüstholz, *Séminaire de Théorie des Nombres Paris 1980–1981*, 359–362, Birkhäuser, Boston, 1982], by J.-P. Demailly using analytic methods [Bull. Soc. Math. France **110** (1982), no. 1, 75–102], by H. Esnault and E. Viehweg using algebraic geometry [Math. Ann. **263** (1983), no. 1, 75–86].

The last part of this paper deals with the study of algebraic values of meromorphic functions of several complex variables (see the reviews of the author's previous works [*Séminaire Delange-Pisot-Poitou, 19é année; 1977–78*, Fasc. 2, Exp. No. 45, Secrétariat Math., Paris, 1978; MR0520333 (80c:10038); Ann of Math. (2) **109** (1979), no. 2, 353–376; MR0528967 (80j:10040)].)

{For the entire collection see MR0633885 (82j:10004)}

Reviewed by *Michel Waldschmidt*





**MR641821 (83f:32014)** 32E10 (30F15 32L05)
**Mok, Ngaiming**
**The Serre problem on Riemann surfaces.**
*Math. Ann.* **258** (1981/82), *no. 2*, 145–168.

This paper is devoted to the famous Serre problem [J. P. Serre, *Colloque sur les fonctions de plusieurs variables* (Brussels, 1953), pp. 57–68; Masson, Paris, 1953; MR0064155 (16,235b)]: If $E$ is a holomorphic fiber bundle with a Stein base and a Stein fiber, is $E$ Stein? Many positive partial results have been obtained, but H. Skoda's counterexample [C. R. Acad. Sci. Paris Sér. A-B **284** (1977), no. 19, A1199-A1202; MR0437802 (55 #10724); Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] shows that the answer is, in general, negative [cf. also J.-P. Demailly, *Séminaire Pierre Lelong-Henri Skoda* (*Analyse*) (année 1976/1977), pp. 15–41, Lecture Notes in Math., 694, Springer, Berlin, 1978; MR0522471 (80e:32008)]. The author solves the Serre problem when the fiber is an open Riemann surface $X$. If $X$ is hyperbolic (in the sense that it admits the Green kernel), the author constructs a continuous strictly subharmonic exhaustion function $\varphi$ on $X$ such that $\varphi - \varphi \circ g$ is bounded for every automorphism $g$ of $X$. Then there exists a continuous strictly plurisubharmonic exhaustion function on $E$ by a result of J.-L. Stehlé on patching plurisubharmonic functions [*Séminaire Pierre Lelong* (*Analyse*) (année 1973–1974), pp. 155–179, Lecture Notes in Math., 474, Springer, Berlin, 1975; MR0399524 (53 #3368)]. Thus $E$ is Stein by the R. Narasimhan theorem [Math. Ann. **146** (1962), 195–216; MR0182747 (32 #229)]. If $X$ does not admit a Green kernel it is said to be parabolic. In this case the author constructs a continuous subharmonic exhaustion function $s$ on $X$ such that $s - s \circ g$ is bounded for any automorphism $x$ of $X$. Now the Stehlé theorem only gives a continuous (but not necessarily strictly) plurisubharmonic exhaustion function $\Psi$ on the total space $E$. Then $E_c = \{z \in E\colon \Psi(z) < c\}$ is an increasing continuous one-parameter family of manifolds with union $E$. In view of the Docquier-Grauert theorem [F. Docquier and H. Grauert, ibid. **140** (1960), 94–123; MR0148939 (26 #6435)], to prove that $E$ is Stein it is enough to prove that each $E_c$ is Stein. The function $(c - \Psi)^{-1}$ is a continuous plurisubharmonic exhaustion function on $E_c$. Thus, applying the Narasimhan theorem, it suffices to construct a continuous strictly plurisubharmonic function on $E_c$. This is done by a partition of unity argument (following J. Brun [Manuscripta Math. **14** (1974), 217–222; MR0364686 (51 #940)]).

A characterisation of the irregular boundary of certain hyperbolic Riemann surfaces and an

improvement of Stehlé's theorem are also given.

Reviewed by *J. T. Davidov*





**MR636112 (83d:32026)** 32L05

**Skoda, H.**

**Morphisme surjectif de fibrés vectoriels semi-positifs. (French) [Surjective morphism of semipositive vector bundles]**

*Conference on Complex Analysis, Nancy* 80 (*Nancy*, 1980), *pp.* 26–32, *Inst. Élie Cartan*, 3, *Univ. Nancy, Nancy*, 1981.

From the text: "Because most of the results in this article have appeared in an earlier publication [the author, Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)], we limit ourselves to a brief summary and refer the reader elsewhere for the proofs [J.-P. Demailly and the author, *Séminaire Pierre LeLong-Henri Skoda (Analyse), Années 1978/79*, pp. 304–309, Lecture Notes in Math., 822, Springer, Berlin, 1980; MR0599033 (82h:32028); the author, ibid., pp. 259–303; MR0599032 (82h:32027); the author, op. cit.; MR0533068 (80j:32047)]."

{For the entire collection see MR0636110 (82i:32004)}





**MR599033 (82h:32028)** 32L20 (32J25)

**Demailly, J.-P.**; **Skoda, H.**

**Relations entre les notions de positivités de P. A. Griffiths et de S. Nakano pour les fibrés vectoriels. (French)**

*Séminaire Pierre Lelong-Henri Skoda (Analyse). Années* 1978/79 (*French*), *pp.* 304–309, *Lecture Notes in Math.*, 822, *Springer, Berlin*, 1980.

La différence entre les deux notions en question est que celle de Nakano se teste sur tous les tenseurs alors que celle de Griffiths se teste sur les tenseurs décomposables. Les auteurs démontrent que si $E$ est un fibré semi-positif au sens de Nakano, alors $E \otimes \det E$ est semi-positif au sens de Griffiths. Cet énoncé, dont la démonstration très simple relève de l'algèbre multilinéaire, est suffisamment précis par exemple pour réduire le théorème d'annulation de Griffiths à celui de Nakano. Il permet

aussi, à partir des résultats de Skoda [Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 4, 577–611; MR0533068 (80j:32047)] concernant la notion de Nakano, d'obtenir des énoncés concernant celle de Griffiths, sensiblement plus fins que ceux obtenus directement par Skoda [see MR0599032 (82h:32027)above].

{For the entire collection see MR0599013 (81j:32003)}

Reviewed by *A. Hirschowitz*





**MR597024 (82f:32007)** 32A15 (32C25)

**Demailly, Jean-Pierre**

**Construction d'hypersurfaces irréductibles avec lieu singulier donné dans $\mathbf{C}^n$. (French)**

*Ann. Inst. Fourier (Grenoble)* **30** (1980), *no. 3*, 219–236.

The author obtains the following interesting result. Theorem: If $S = \{f_1 = \cdots = f_k = 0\} \subset \mathbf{C}^n$ is an analytic subvariety of codimension $\geq 2$, then there exist entire functions $g_1, \cdots, g_k$ of slow growth such that the variety $X = \{\sum f_j g_j = 0\}$ is irreducible, and the singular set of $X$ is contained in $S$. (If $f_j$ is replaced by $f_j^2$, then the singular set of $X$ may be taken to be exactly $S$.)

This theorem then is used to construct two noteworthy examples. The first is an irreducible algebraic curve in $\mathbf{C}^2$ of order zero such that the number of singular points in the ball of radius $R$ is larger than any preassigned function $\psi(R)$. The reason for giving the example is that an irreducible algebraic curve of degree $n$ can have at most $\frac{1}{2}(n-1)(n-2)$ double points. Thus this example is related to an earlier example of M. Cornalba and B. Shiffman [Ann. of Math. (2) **96** (1972), 402–406; MR0311937 (47 #499)].

Next, the author considers the Fourier transforms of functions in $\mathcal{D}(\mathbf{R}^n)$ and $\mathcal{E}'(\mathbf{R}^n)$, $n \geq 2$. The elements of $\widehat{\mathcal{D}(\mathbf{R}^n)}$ and $\widehat{\mathcal{E}'(\mathbf{R}^n)}$ are entire functions on $\mathbf{C}^n$ with certain growth conditions. By an application of the theorem above, there exists a function $V = \sum u_j * v_j$, where $u_j, v_j \in \mathcal{D}(\mathbf{R}^n)$, and $V$ is irreducible in $\mathcal{E}'(\mathbf{R}^n)$. As a consequence, it follows that $\mathcal{D}(\mathbf{R}^n) * \mathcal{D}(\mathbf{R}^n) \neq \mathcal{D}(\mathbf{R}^n)$ for $n \geq 2$, a result which was obtained for $n \geq 3$ by L. A. Rubel, W. A. Squires and B. A. Taylor [ibid. (2) **108** (1978), no. 3, 553–567; MR0512433 (80d:32003)] and for $n = 2$ by J. Dixmier and P. Malliavin [Bull. Sci. Math. (2) **102** (1978), no. 4, 307–330; MR0517765 (80f:22005)].

Reviewed by *Eric Bedford*

**MR599013 (81j:32003)** 32-06

**★Séminaire Pierre Lelong-Henri Skoda (Analyse). Années 1978/79. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis). 1978/79]**

Edited by Pierre Lelong and Henri Skoda.
Lecture Notes in Mathematics, 822.
*Springer*, *Berlin*, 1980. viii+356 *pp*. \$23.00. *ISBN* 3-540-10241-8

Contents: Avant-propos (pp. iii-v); D. Barlet, Majoration du volume des fibres génèriques et forme géométrique du théorème d'aplatissement (pp. 1–17); Jorge Alberto Barroso, Comparaison de topologies sur des espaces d'applications holomorphes (pp. 18–32); Urban Cegrell, On product capacities with application to complex analysis (pp. 33–45); J. F. Colombeau, Holomorphy in locally convex spaces and operators on the Fock spaces (pp. 46–60); Hassine El Mir, Fonctions plurisousharmoniques et ensembles polaires (pp. 61–76); Roger Gay, Division des fonctionnelles analytiques. Applications aux fonctions entières de type exponentiel moyennes-périodiques (pp. 77–89); Lawrence Gruman, La géométrie globale des ensembles analytiques dans $\mathbf{C}^n$ (pp. 90–99); B. Jennane, Groupes de cohomologie d'un fibré holomorphe à base et à fibre de Stein (pp. 100–108); Bruno Kramm, $(\mathcal{D}FN)$-analytic spaces, Stein algebras and a "universal" holomorphic functional calculus (pp. 109–128); R. Langevin, Singularités complexes, points critiques et intégrales de courbure (pp. 129–143); Pierre Lelong, Potentiels canoniques et comparaison de deux méthodes pour la résolution du $\partial\overline{\partial}$ à croissance (pp. 144–168); I. Lieb, L'opérateur $\overline{\partial}$ sur une variété $q$-concave (pp. 169–173); Jean-Charles Moreau, Lemmes de Schwarz en plusieurs variables et applications arithmétiques (pp. 174–190); Laurent Schwartz, Martingales conformes sur une variété analytique complexe (pp. 191–198); A. Sebbar, Prolongement des solutions holomorphes de certains opérateurs différentiels d'ordre infini à coefficients constants (pp. 199–220); Nessim Sibony and Pit Mann Wong, Some results on global analytic sets (pp. 221–237); H. Skoda, Diviseurs d'aire bornée dans la boule de $\mathbf{C}^2$: réflexions sur un article de Bo Berndtsson (pp. 238–251); H. Skoda, Remarques à propos des théorèmes d'annulation pour les fibrés semi-positifs (pp. 252–258); Henri Skoda, Relèvement des sections globales dans les fibrés semi-positifs (pp. 259–303); J.-P. Demailly and H.Skoda, Relations entre les notions de positivités de P. A. Griffiths et de S. Nakano pour les fibrés vectoriels (pp. 304–309); M. Valdivia, On certain infinitely differentiable function spaces (pp. 310–316); Jean-Pierre Vigué, Sur la convexité des domaines bornés cerclés homogènes (pp. 317–331); Michel Waldschmidt, Propriétés arithmétiques de fonctions de plusieurs variables, III (pp. 332–356).

{The papers are being reviewed individually.}

**MR533896 (81c:32049)** 32H30 (32A10)

**Demailly, Jean-Pierre**

**Fonctions holomorphes à croissance polynomiale sur la surface d'équation $e^x + e^y = 1$. (French. English summary)**

*Bull. Sci. Math. (2)* **103** (1979), *no. 2,* 179–191.

Let $S \subset \mathbf{C}^2$ be the surface $e^x + e^y = 1$, $(x, y) \in \mathbf{C}^2$. The author proves that if $f(x, y)\colon S \to \mathbf{C}$ is a holomorphic function on $S$ with polynomial growth, then $f(x, y)$ is the restriction of a polynomial on $\mathbf{C}^2$. As a result he deduces that if $f\colon S \to \mathbf{P}^1(\mathbf{C})$ is a meromorphic function on $S$ with finite fibres, then $f$ is constant. In particular, if $f\colon S \to \mathbf{C}$ is a bounded holomorphic function then $f$ is constant. ($\mathbf{P}^1(\mathbf{C})$ is the complex projective "plane".)

Reviewed by *Adib A. Fadlalla*





**MR549547 (80j:32027)** 32E10

**Jennane, B.**

**Groupes de cohomologie d'un fibré holomorphe à base et à fibre de Stein. (French)**

*Invent. Math.* **54** (1979), *no. 1,* 75–79.

Let $X$ and $\Omega$ be complex spaces and $\Pi\colon X \to \Omega$ be a surjective holomorphic map such that $\Omega$ admits a covering by Stein open subsets whose inverse images under $\Pi$ are Stein. The author proves that if $\Omega$ is Stein and has bounded dimension and $\mathcal{F}$ is a coherent sheaf on $X$, then $H^p(X, \mathcal{F})$ vanishes for $p \geq 2$. The proof consists of choosing a suitable Stein covering of $\Omega$ and applying the Mayer-Vietoris sequence. The result is of interest in view of the counterexamples of H. Skoda [same journal **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] and J.-P. Demailly [ibid. **48** (1978), no. 3, 293–302; Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77, pp. 15–41, Lecture Notes in Math., Vol. 694, Springer, Berlin, 1978; MR0522471 (80e:32008)] to the question posed by Serre whether a holomorphic fiber bundle with Stein base and Stein fiber is Stein.

Reviewed by *Y.-T. Siu*

**MR522015 (80e:32002)** 32A10 (32H99)

**Demailly, Jean-Pierre**

**Fonctions holomorphes bornées ou à croissance polynomiale sur la courbe $e^x + e^y = 1$. (French. English summary)**

*C. R. Acad. Sci. Paris Sér. A-B* **288** (1979), *no. 1*, A39–A40.

Author's summary: "We prove a very precise extension theorem for holomorphic functions on the curve $e^x + e^y = 1$, and deduce from it that bounded holomorphic functions are constant, or more generally, that every holomorphic function with polynomial growth extends to a polynomial in $\mathbf{C}^2$. This result immediately applies to meromorphic functions, and can also be used to study some hypersurfaces of $\mathbf{C}^n$."



**MR508989 (81m:32036)** 32L05 (32E10)

**Demailly, Jean-Pierre**

**Un exemple de fibré holomorphe non de Stein à fibre $\mathrm{C}^2$ ayant pour base le disque ou le plan. (French)**

*Invent. Math.* **48** (1978), *no. 3*, 293–302.

The author provides yet another example of a holomorphic fiber space with Stein base, Stein fiber and non-Stein total space in the mainstream originated by H. Skoda's brilliant counterexample to Serre's problem [same journal **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)]. Here the base is any nonempty connected open subset of $\mathbf{C}$ with $\mathbf{C}^2$ as fiber and with transition automorphisms of exponential type, while in Skoda's example the base is multiply connected and the transition automorphisms are locally constant with exponential growth. Moreover the holomorphic functions on such a bundle are constant on each fiber and its Dolbeault group $H^{0,1}$ is non-Hausdorff of infinite dimension.

Reviewed by *Alessandro Silva*

**MR522471 (80e:32008)** 32E10 (32L05)

**Demailly, J.-P.**

**Différents exemples de fibrés holomorphes non de Stein. (French)**

*Séminaire Pierre Lelong-Henri Skoda* (*Analyse*), *Année* 1976/77, *pp*. 15–41, *Lecture Notes in Math.*, 694, *Springer*, *Berlin*, 1978.

L'auteur simplifie notablement l'exemple de H. Skoda [Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] et construit un fibré de base $\mathbf{C}^*$ et de fibre $\mathbf{C}^2$ dont l'espace total n'est pas de Stein. L'automorphisme de transition est constant et polynomial (dans l'exemple de Skoda, les automorphismes de transition étaient à croissance exponentielle). L'exemple peut être décrit en une ligne comme quotient de $\mathbf{C} \times \mathbf{C}^2$ par le groupe cyclique engendré par $\alpha$ avec $\alpha(x, z_1, z_2) = (x + 2i\pi, z_1^k - z_2, z_1)$, $k \geq 2$.

L'auteur montre plus précisément que ce fibré $X$ est de Stein au-dessus de la couronne $\rho_1 < |x| < \rho_2$ si et seulement si $\mathrm{Log}(\rho_2|\rho_1) \leq 2\pi^2/\mathrm{Log}k$; dans le cas contraire, $H^1(X, \mathcal{O}_X)$ est grossier. Toujours en utilisant l'inégalité de Lelong sur la croissance des fonctions plurisousharmoniques, l'auteur construit aussi un fibré de base le disque, de fibre $\mathbf{C}^2$ et dont l'espace total n'est pas de Stein (en particulier, ce fibré n'est pas trivial!).

{For the entire collection see MR0522469 (80a:32001)}

Reviewed by *A. Hirschowitz*





**MR512433 (80d:32003)** 32A15 (30D30)

**Rubel, L. A.**; **Squires, W. A.**; **Taylor, B. A.**

**Irreducibility of certain entire functions with applications to harmonic analysis.**

*Ann. of Math. (2)* **108** (1978), *no. 3*, 553–567.

The authors show that if $f_1, \cdots, f_n$ $(n \geq 3)$ are nonconstant meromorphic functions of one variable and $g(z_1, \cdots, z_n) = f_1(z_1) + \cdots + f_n(z_n)$ then the local variety $V$ of $g$, $V = \{z \in \mathbf{C}^n \colon g$ is analytic at $z$ and $g(z) = 0\}$, is irreducible. An immediate corollary is that if $f_1, \cdots, f_n$ are entire functions, then $g$ is irreducible in the ring of entire functions in $\mathbf{C}^n$; this generalizes a known result for polynomials [cf. J. W. S. Cassels, Proceedings of the Fifteenth Scandinavian Congress (Oslo, 1968), pp. 1–17, Lecture Notes in Math., Vol. 118, Springer, Berlin, 1970; MR0268161 (42 #3060); erratum, MR **42**, p. 1825].

The following result in harmonic analysis follows also from the above theorem: $(*)$ $C_0^\infty(\mathbf{R}^n) * C_0^\infty(\mathbf{R}^n) \neq C_0^\infty(\mathbf{R}^n)$ if $n \geq 3$, which answers a question of L. Ehrenpreis [Amer. J. Math. **82** (1960), 522–588; MR0119082 (22 #9848)]. Recently, J. Dixmier and P. Malliavin have proved $(*)$ when $n = 2$ by a different method [Bull. Sci. Math. (2) **102** (1978), no. 4, 307–330; MR0517765 (80f:22005)]. The case $n = 1$ is still open.

Recent work of J.-P. Demailly [ibid. (2) **103** (1979), no. 2, 179–191] on analysis on the variety $V$ defined by the functions $f_1(z) = f_2(z) = e^z$, $f_3(z) = 1$, is of related interest.

Reviewed by *Carlos A. Berenstein*





**MR522469 (80a:32001)** 32-06

★**Séminaire Pierre Lelong-Henri Skoda (Analyse). Année 1976/77. (French) [Seminar Pierre Lelong-Henri Skoda (Analysis). Year 1976/77]**

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*Springer*, *Berlin*, 1978. iii+334 *pp*. $17.60. *ISBN* 3-540-09101-7

From the foreword: "The present volume of the 1976–1977 seminar continues the series of volumes previously published as Lecture Notes [71 (1968), 116 (1969), 205 (1970), 275 (1971), 332 (1972), 410 (1973), 474 (1974), 524 (1975), 578 (1976)]. Certain articles have been edited, we must admit, with a certain delay and in fact several were not in final form before the beginning of 1978."

Table of Contents: Pierre Lelong and Henri Skoda, Préface (p. i); Exposés faits au Séminaire d'Analyse dont les résultats paraitront ailleurs (p. iii); Jacek Bochnak, Sur le 17ème problème de Hilbert pour les fonctions de Nash (pp. 1–4); J.-P. Demailly, Différents exemples de fibrés holomorphes non de Stein (pp. 15–41); G. Dloussky, Prolongements d'applications analytiques (pp. 42–95); François Gramain, Fonctions entières arithmétiques (pp. 96–125); B. Jennane, Extension d'une fonction définie sur une sous-variété avec contrôle de la croissance (pp. 126–133); Paul Krée, Méthodes fonctorielles en analyse de dimension infinie et holomorphie anticommutative (pp. 134–171); Pierre Lelong, Un théorème de fonctions inverses dans les espaces vectoriels topologiques complexes et ses applications à des problèmes de croissance en analyse complexe (pp. 172–195); Leopoldo Nachbin, Sur la densité des sous-algèbres polynomiales d'applications continûment différentiables (pp. 196–202); Philippe Noverraz, Sur la mesure gaussienne des ensembles polaires en dimension infinie (pp. 203–213); P. Raboin, Le problème du $\overline{\partial}$ sur un espace de Hilbert (pp. 214–227); J.-P. Ramis, Géométrie analytique et géométrie algébrique (variations sur le thème "GAGA") (pp. 228–289); Henri Skoda, Morphismes surjectifs et fibrés linéaires semi-positifs (pp. 290–324); Hiroshi Yamaguchi, Fonctions entières paraboliques dans $\mathbf{C}^2$ (pp. 325–334).

{The papers are being reviewed individually.}





**MR0508091 (58 #22657)** 32E10
**Skoda, H.**
**Fibrés holomorphes à base et à fibre de Stein. (French)**
*Invent. Math.* **43** (1977), *no. 2*, 97–107.

Author's review: There exist holomorphic locally trivial fiber spaces, with Stein base and Stein fiber, which are not Stein manifolds. Therefore we give a negative answer to a problem of J.-P. Serre [*Colloque sur les fonctions de plusieurs variables* (Brussels, 1953), pp. 57–68, Thone, Liège, 1953; MR0064155 (16,235b)]. In this example, the base is an open set in $\mathbf{C}$, the fiber is $\mathbf{C}^2$, the transition automorphisms are locally constant. It is possible to choose the fundamental group of the base free with two generators. The counterexample is based on a Lelong type inequality concerning the growth of a holomorphic function on a fiber space with fiber $\mathbf{C}^n$. Using this inequality, we prove that holomorphic functions on this fiber space are constant on the fiber. Recently, J.-P. Demailly [cf. *Séminaire Pierre Lelong-Henri Skoda (Analyse), Année 1976/77*, pp. 15–41, Lecture Notes in Math., Vol. 694, Springer, Berlin, 1978] has given another example of a fiber space, with fiber $\mathbf{C}^2$, base the open unit disc, but which is not a Stein space.