MR1326617 (96k:32001) 32-02 (32-06 32Cxx 32Dxx 32Fxx 32Jxx 32Lxx)

★**Several complex variables. VII.**

Sheaf-theoretical methods in complex analysis.
A reprint of *Current problems in mathematics. Fundamental directions. Vol. 74* (Russian), Vseross. Inst. Nauchn. i Tekhn. Inform. (VINITI), Moscow.
Edited by H. Grauert, Th. Peternell and R. Remmert.
Encyclopaedia of Mathematical Sciences, 74.
Springer-Verlag, Berlin, 1994. vi+369 *pp*. $99.00. *ISBN* 3-540-56259-1

This volume contains nine chapters arranged so as to provide a systematic introduction to and survey of the theory of complex spaces. Below I discuss each chapter separately, and provide some general comments at the end.

I. R. Remmert, "Local theory of complex spaces", 7–96: Starting from the Weierstrass division and preparation theorems, this chapter develops much of the algebraic and sheaf-theoretic background required to study the theory of complex spaces. The four main results that are considered here are the coherence of structure sheaves, the finite mapping theorem and the Rückert Nullstellensatz, the coherence of ideal sheaves, and the coherence of normalization sheaves. Most results in this chapter have sketches of proofs, except for the results that are purely commutative algebra.

II. Th. Peternell and R. Remmert, "Differential calculus, holomorphic maps and linear structures on complex spaces", 97–144: From the basic structure theorems, it is natural to progress to the behavior of cotangent sheaves as the next level of subtlety. The authors first discuss sheaves of germs of differential forms and criteria for smoothness and for submersions. Then they turn to questions of flatness (although the semicontinuity theorem is postponed until after cohomology has been discussed in Chapter III) and the correspondence $\{\text{vector bundles}\} \leftrightarrow \{\text{locally free sheaves}\}$ and its generalization to arbitrary coherent sheaves, leading eventually to the concept of analytic spectra. The remaining topics here are formal completions, Cohen-Macaulay spaces and dualizing sheaves. The discussions in this chapter are somewhat restricted because sheaf cohomology has not yet been discussed.

III. Th. Peternell, "Cohomology", 145–182: This chapter introduces the main ideas of cohomology theory for complex spaces, using the notions of flabby cohomology (for complexes, and de Rham's theorem) and Čech cohomology (and Dolbeault's theorem). Then the author turns to Stein spaces, Theorems A and B and the solution of the Cousin problems, and then to compact complex spaces, the direct image theorem, the comparison, base change, continuity and Riemann-Roch theorems, and Serre duality. Finally he gives a brief discussion of spectral sequences. Needless to say, this chapter does not contain proofs, although there are many examples, as elsewhere in the book.

IV. G. Dethloff and H. Grauert, "Seminormal complex spaces", 183–220: The first main result in this chapter is that analytically branched coverings are normal complex spaces. The proof of this uses L^2 methods, which are developed in a brief form and applied to other results. Probably the most important result on seminormal complex spaces is the criterion for a quotient space of a

seminormal complex space to be seminormal. Most of the rest of this chapter is spent on developing the necessary abstract information about equivalence relations on complex spaces, so as to enable the reader to understand the sketch of the proof of this main result. Going outside of the realm of holomorphic equivalence relations, the authors turn to meromorphic equivalence relations and their applications to the meromorphic dependence of maps, the reduction of a complex space to a Moishezon space, and other types of non-regular behavior.

V. Th. Peternell, "Pseudoconvexity, the Levi problem and vanishing theorems", 221–257: This chapter contains discussions of plurisubharmonic functions, pseudoconvex domains, 1-convex spaces, the classical Levi problem, and the characterization of exceptional analytic sets. The notion of pseudoconvexity is extended to bundles and the author discusses positive bundles and various vanishing theorems, such as those of Kodaira, Demailly and Grauert-Riemenschneider, and Hodge theory. The author makes a convincing case for treating the case of 1-convex spaces differently from that of q-convex spaces, $q \geq 2$.

VI. H. Grauert, "Theory of q-convexity and q-concavity", 259–284: This chapter contains basic material such as the extension of the notions of q-convexity and q-concavity from domains in $\mathbf{C}^n$ to spaces, and the finite-dimensionality of the cohomology vector space in some cases. These results are applied to such topics as filling in holes in a complex space and the existence of hulls for cohomology classes. Finally, the author proves Serre's duality theorem for q-convex spaces and the q-concavity of the fundamental domain for Siegel modular groups of degree $n > 1$.

VII. Th. Peternell, "Modifications", 285–317: Beginning with the definition of a modification, the author gives a survey of results on the bimeromorphic geometry of complex spaces. He discusses blow-ups and blow-downs, the use of formal objects, the extension of analytic objects, Moishezon spaces, and desingularization.

VIII. F. Campana and Th. Peternell, "Cycle spaces", 319–349: This chapter provides an introduction to one of the most interesting (to the reviewer) parts of complex analysis, the construction of the Douady space $\mathcal{D}(X)$ and Barlet space $\mathcal{C}(X)$ of a complex space X. (These are the analytic analogues of the Hilbert and Chow schemes from the algebraic case.) The authors prove the result of Lieberman and Fujiki that if X is a compact Kähler manifold, then the connected components of the Douady and Barlet spaces are compact. The Barlet space is applied to various structure results, in particular to manifolds of class $\mathcal{C}$, i.e., manifolds bimeromorphic to Kähler manifolds. Finally, convexity results for $\mathcal{C}(X)$ are discussed.

IX. H. Grauert and R. Remmert, "Extension of analytic objects", 351–360: This brief chapter contains an extremely summary treatment of some of the basic results on extension, such as the Remmert-Stein theorems, the Stoll-Bishop theorem and Kneser's Kontinuitätssatz. The authors express the hope that someone will come forth to write a detailed commentary on the results obtained in this field.

Some general remarks: As can be seen from the above summary, this volume contains an enormous amount of material, with at least hints at how the proofs go. There are very many examples and a lot of important remarks which will assist the reader in understanding some of the subtleties. If complete proofs had been included, the volume would probably have been four times its current size. The volume has a subject index, but unfortunately no notation index.

Unfortunately, the exposition is not always clear: important definitions and remarks are buried

in the middle of paragraphs devoted to some other topic (and not always indicated in the index, either), bibliographic references are sometimes to items not included in the bibliographies (at the end of each chapter), and the level of English could have benefited from some serious editing by a native English speaker; there are many places which read as if they have been translated directly from German. (Here is an example from p. 14: “Analytic algebras exist two a penny. The more it will surprise that many of them are hidden in the algebra. . . .”) In addition, the senior authors have kindly included extended passages in German and French, untranslated.

Despite these defects, the book should be very useful for students and for reference.

{Volume VI has been reviewed [MR1095088 (91i:32001)].}

Reviewed by *J. S. Joel*

MR1322991 (96f:32001) 32-02 (14J45 14J60 32J18 32J27 32L07)

Peternell, Thomas (D-BAYRMP);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYRMP)

Neuere Entwicklungen in der komplexen Geometrie. (German. German summary) [Recent developments in complex geometry]

Duration and change, 275–306, *Springer*, *Berlin*, 1994.

This is a survey written by two specialists for beginners or non-experts. They manage to give a short introduction to modern notions, recent results and open problems of an interesting field of mathematics. Although they do not give all possible references, at many places the reader is referred to other more detailed survey articles. This makes this paper comparatively short and easy to read.

The authors report on developments in complex geometry of the previous 20 years. By complex geometry is meant the study of complex manifolds with the methods of algebraic geometry, complex differential geometry and complex analysis. Of course, the authors cannot consider in this paper all aspects of this large and dynamically developing area of modern mathematics. Instead they focus on those parts which are closely related to their own contributions in the field of complex geometry [see, e.g., J.-P. Demailly, T. Peternell and M. H. Schneider, J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037); Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

The main subject of this paper is classification theory, which includes classification up to biholomorphisms or bimeromorphic transformations, up to deformations or up to homeomorphisms. One section explains some basic notions, results and open questions of Mori theory, which deals with the birational classification of projective manifolds. The results and conjectures of this theory suggest investigating Fano manifolds more closely. A manifold is called Fano if it has ample

anticanonical bundle. The section on these manifolds gives a first impression of the theory of Fano manifolds. A central result of importance for the development of higher-dimensional classification theory is S. Mori's solution of the Hartshorne-Fraenkel conjecture [Ann. of Math. (2) **110** (1979), no. 3, 593–606; MR0554387 (81j:14010)] stating that any compact manifold with ample tangent bundle is isomorphic to projective space. This leads to the interesting question of classifying manifolds with "semipositive" tangent bundle. Ideas centered around this problem are discussed in the fourth section. The final part is devoted to the theory of holomorphic vector bundles. After explaining the basic notion of stability and some of its consequences, the usage of moduli spaces of vector bundles over algebraic surfaces to study the differential topology of these manifolds is explained (Donaldson polynomials). Finally, the Kobayashi-Hitchin correspondence is discussed. It forms a bridge between algebraic geometry and complex differential geometry by relating the notion of stability of vector bundles to the existence of a Hermite-Einstein metric.

{For the entire collection see MR1322982 (95i:00037)}

Reviewed by *Bernd Kreussler*

MR1319346 (96k:32012) 32C30 (32J25)

Demailly, Jean-Pierre (F-GREN-F)

Regularization of closed positive currents of type $(1,1)$ by the flow of a Chern connection.

Contributions to complex analysis and analytic geometry, 105–126, *Aspects Math.*, *E*26, *Vieweg*, *Braunschweig*, 1994.

Let X be a compact n-dimensional complex manifold and let T be a closed positive current of bidegree $(1,1)$ on X. In general, T cannot be approximated by closed positive currents of class C^∞: a necessary condition for this is that the cohomology class $\{T\}$ be numerically effective in the sense that $\int_Y \{T\}^p \geq 0$ for every p-dimensional subvariety $Y \subset X$. The author proves that it is always possible to approximate a closed positive current T of type $(1,1)$ by closed real currents admitting a small negative part, and that this negative part can be estimated in terms of the Lelong numbers of T and the geometry of X. Let α be a smooth closed $(1,1)$-form representing the same $\partial\overline{\partial}$-cohomology class as T and let ψ be a quasi-psh function on X such that $T = \alpha + (i/\pi)\partial\overline{\partial}\psi$ (a function is said to be quasi-psh if it is locally the sum of a psh function and a smooth function). Such a decomposition exists even when X is non-Kähler. If ψ_ε is an approximation of ψ, then $T_\varepsilon = \alpha + (i/\pi)\partial\overline{\partial}\psi_\varepsilon$ is an approximation of T. The author proves the following: Theorem. Let T be a closed almost positive $(1,1)$-current and let α be a smooth real $(1,1)$-form in the same $\partial\overline{\partial}$-cohomology class as T, i.e. $T = \alpha + (i/\pi)\partial\overline{\partial}\psi$, where ψ is an almost psh function. Let γ be a continuous real $(1,1)$-form such that $T \geq \gamma$. Suppose that the tangent bundle T_X is equipped with a smooth Hermitian metric ω such that the Chern curvature form $\Theta(T_X)$ satisfies $(\Theta(T_X) + u \otimes$

$\mathrm{Id}_{T_X})(\theta\otimes\xi,\theta\otimes\xi)\geq 0$ for all $\theta,\xi\in T_X$ with $\langle\theta,\xi\rangle=0$, for some continuous nonnegative $(1,1)$-form u on X. Then there is a family of closed almost positive $(1,1)$-currents $T_\varepsilon=\alpha+(i/\pi)\partial\overline{\partial}\psi_\varepsilon$, $\varepsilon\in(0,\varepsilon_0)$, such that ψ_ε is smooth over X, increases with ε, and converges to ψ as ε tends to 0 (in particular, T_ε is smooth and converges weakly to T on X), and such that (i) $T_\varepsilon\geq\gamma-\lambda_\varepsilon u-\delta_\varepsilon\omega$, where (ii) $\lambda_\varepsilon(x)$ is an increasing family of continuous functions on X such that $\lim_{\varepsilon\to 0}\lambda_\varepsilon(x)=\nu(T,x)$ (Lelong number of T at x) at every point, and (iii) δ_ε is an increasing family of positive constants such that $\lim_{\varepsilon\to 0}\delta_\varepsilon=0$.

For the proof, the author uses a smoothing procedure involving a convolution kernel constructed by means of the exponential map associated to the Chern connection on T_X. From a calculation of the Taylor expansion of the exponential map at order 3, a precise estimate of the complex Hessian of the regularized function is derived. Kiselman's singularity attenuation technique is then applied in combination with the above theorem to obtain a family of approximating currents $T_{c,\varepsilon}$ which are smooth in the complement $X\setminus E_c(T)$ of the Lelong sublevel set

$$E_c(T)=\{x\in X\colon\ \nu(T,x)\geq c\}$$

and have Lelong numbers $\nu(T_{c,\varepsilon},x)=\nu(T,x)-c$ along $E_c(T)$. It should be observed that similar results have been proved by the author in a related paper [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)], under slightly different curvature assumptions. Some geometric applications of the smoothing theorem to the study of compact complex manifolds with partially semipositive curvature are given.

{For the entire collection see MR1319341 (95j:32001)}

Reviewed by *Mongi Blel*

MR1319341 (95j:32001) 32-06 (00B30)

★**Contributions to complex analysis and analytic geometry.**

Dedicated to Pierre Dolbeault.

Edited by Henri Skoda and Jean-Marie Trépreau.

Aspects of Mathematics, E26.

Friedr. Vieweg & Sohn, Braunschweig, 1994. xiv+250 *pp*. \$70.00. *ISBN* 3-528-06633-4

Contents: H. Skoda and J.-M. Trépreau, Foreword (Dedication to Pierre Dolbeault, on the occasion of his retirement) (French) (vi–xi); Vincenzo Ancona and Bernard Gaveau, The de Rham complex of a reduced analytic space (1–26); Bo Berndtsson, Some recent results on estimates for the $\overline{\partial}$-equation (27–42); Evgeni M. Chirka [E. M. Chirka] and Edgar Lee Stout, Removable singularities in the boundary (43–104); Jean-Pierre Demailly, Regularization of closed positive currents of type $(1,1)$ by the flow of a Chern connection (105–126); Klas Diederich and Gregor

Herbort, Pseudoconvex domains of semiregular type (127–161); Pierre Dolbeault and Gennadi Henkin [G. M. Khenkin], Surfaces de Riemann de bord donné dans $\mathbf{C}\mathrm{P}^n$ [Riemann surfaces with given boundary in $\mathbf{C}\mathrm{P}^n$] (163–187); Alan Huckleberry, Subvarieties of homogeneous and almost homogeneous manifolds (189–232); Mikael Passare, August Tsikh [A. K. Tsikh] and Oleg Zhdanov, A multidimensional Jordan residue lemma with an application to Mellin-Barnes integrals (233–241); Bernard Shiffman, Separately meromorphic mappings into compact Kähler manifolds (243–250).

{Most of the papers are being reviewed individually.}

MR1318282 (96a:32034) 32F20 (32E10)

Wu, Xiao Qin (PRC-JMU)

The solution of $\overline{\partial}_b$-equation of (p,q)-forms and its L^p & Hölder estimates on a Stein manifold. (English summary)

The collection of theses of Symposium on Real Analysis (Xiamen, 1993).
J. Math. Study **27** (1994), *no. 1,* 174–180.

Let M be a Stein manifold of dimension n and let $\Omega \subset\subset M$ be a strongly pseudoconvex domain with smooth boundary. In this paper, the author uses a method developed by Demailly and Laurent-Thiébaut to construct an integral kernel to solve the tangential Cauchy-Riemann equation $\overline{\partial}_b$ of (p,q)-forms. More precisely, let $T(M)$ be the holomorphic tangent bundle of M and let $\tilde{T}(M\times M)$ be the pullback of $T(M)$. Furthermore, we assume the curvature C of $\tilde{T}(M\times M)$ is 0. Then, for $f\in L^S_{p,q}(\partial\Omega)$ satisfying the compatibility condition, $f=(-1)^p\overline{\partial}_b(T(f)-S(f))$. Using this integral representation for the solution of $\overline{\partial}_b$, the author shows that $\|T(f)-S(f)\|_{\Lambda^r_{1/2}}\lesssim \|f\|_{L^r(\partial\Omega)}$. Here Λ^r_α is the Besov space equipped with the norm

$$\|f\|_{\Lambda^r_\alpha}=\|f\|_{L^r(\partial\Omega)}+\sup_{z(t)\in C,\ 0\le t\le 1}\frac{\|f(\gamma(t))-f(\gamma(0))\|_{L^r}}{|t|^\alpha},$$

where $0<\alpha<1$, $1\le r\le\infty$ and $\gamma\colon (0,1]\to\partial\Omega$ is a C^1 curve with $\|\gamma'(t)\|_{C^1}\le 1$.

{For the entire collection see MR1318248 (95i:00035)}

Reviewed by *Der-Chen E. Chang*

MR1302317 (95i:32022) 32E30 (32H02 32H15 32H20)
Demailly, Jean-Pierre (F-GREN-F); **Lempert, László** (1-PURD);
Shiffman, Bernard (1-JHOP)
Algebraic approximations of holomorphic maps from Stein domains to projective manifolds.
Duke Math. J. **76** (1994), *no. 2,* 333–363.

This paper studies the problem of approximation of holomorphic maps by algebraic maps. The authors show that algebraic approximation is always possible in the case of holomorphic maps to quasiprojective manifolds and of locally free sheaves. In particular, they obtain that any holomorphic map from a Runge domain Ω in an affine algebraic variety S into a quasiprojective algebraic manifold X can be approximated by Nash algebraic maps uniformly on every relatively compact domain $\Omega_0 \subset\subset \Omega$. As an application, they describe how both the Kobayashi-Royden pseudometric and the Kobayashi pseudodistance on projective algebraic manifolds can be given in terms of algebraic curves.

Reviewed by *Min Ru*

MR1299059 (95k:14065) 14K05 (14C20)
Debarre, O. (1-IA); **Hulek, K.** (D-HANN); **Spandaw, J.** (D-BAYRMP)
Very ample linear systems on abelian varieties.
Math. Ann. **300** (1994), *no. 2,* 181–202.

Let X be a “generic” abelian manifold of dimension g and L an ample line bundle. The authors restrict their study to the case in which L is of type $(1,\cdots,1,d)$, which means that L is the pullback of the principal polarization under a cyclic isogeny of degree d. Then they establish that the linear system $|L|$ is base-point-free if and only if $d\geq g+1$, the morphism defined by this linear system being birational if and only if $d\geq g+2$.

These results are part of a wider conjectural picture according to which, for $d>g$, Φ_L (the morphism of X in $\mathbf{P}^{d-1}$ defined by L) should be an embedding outside a set of dimension $2g+1-d$ (in particular, L should be very ample if and only if $d\geq 2g+2$). The authors then show that, for (X,L) generic of type $(1,\cdots,1,d)$, L is very ample for $d>2^g$.

Their results are connected in the case $g=3$ to a conjecture of Griffiths and Harris. In the same case, Ein and Lazarsfeld have also given some very explicit sufficient conditions for a linear

system $|L|$ on X to be base-point-free. Moreover, in the case of arbitrary g, Demailly has given effective bounds on the degree of L, with (X,L) always assumed to be generic, such that $|L|$ is base-point-free or very ample. The authors compare their results with the Ein-Lazarsfeld and Demailly results, respectively.

Reviewed by *Jean-Claude Douai*

MR1299050 (95k:32026) 32J27 (14J40 32J18)

Zhang, Qi [Zhang, Qi[1]] (1-MO)

A note on compact Kähler manifolds with nef tangent bundles. (English summary)

Manuscripta Math. **85** (1994), *no. 1,* 89–96.

Let us assume that X is a compact Kähler manifold of dimension n. A famous conjecture by Frankel and Hartshorne, proved by S. Mori [Ann. of Math. (2) **110** (1979), no. 3, 593–606; MR0554387 (81j:14010)], asserts that, if the sectional curvature of X is positive, or, equivalently, if the tangent bundle T_X is ample, then X is biholomorphic to the projective space $\mathbf{P}^n$. In view of the result of Mori it is natural to ask about a classification of manifolds with tangent bundle satisfying a weaker numerical condition. Namely, one can consider a manifold X with numerically effective (or nef) tangent bundle, which means that the tautological bundle $\mathcal{O}(1)$ on $\mathrm{Proj}(T_X)$ has non-negative degree on any curve $C \subset \mathrm{Proj}(T_X)$. Manifolds of this type have been studied by several authors [see, e.g., J.-P. Demailly, T. Peternell and M. H. Schneider, J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)]. The main theorem of the paper under review is as follows: Suppose that T_X is nef and that some positive multiple $-mK_X$ of the anticanonical line bundle $-K_X = \det(T_X)$ is spanned by global sections. Then the image of the associated map $X \to \mathrm{Proj}(H^0(X, -mK_X))$ is of dimension 1 if and only if there exists an étale covering $\mathbf{P}^1 \times A \to X$, where A is a complex torus. The proof of the theorem depends on results of Demailly, Peternell and Schneider.

Reviewed by *Jarosław A. Wiśniewski*

MR1298871 (95h:32032) 32L20 (32L05)
Marinescu, George [Marinescu, Gheorghe] (R-AOS)
The curvature of numerically effective line bundle and vanishing theorems. (English summary)
Rev. Roumaine Math. Pures Appl. **39** (1994), *no. 1,* 37–42.

In this paper nef line bundles are investigated; in particular, the following theorem is proved: Theorem 1.1. Let E be a big nef line bundle over a compact projective manifold. Then for any Hermitian metric on E the associated curvature form is positive on a nonempty set. The proof makes use of Demailly's asymptotic Morse inequalities. Moreover, vanishing theorems are obtained: Theorem 1.2. Let E be a nef line bundle over a compact Kähler manifold of dimension n. Assume that $\int_{X(\leq 1)}(ic(E))^n$ is positive. Then $H^q(X, E^{-1}) = 0$ for $q < n$. $ic(E)$ is the curvature form of a Hermitian metric on E and $X(\leq 1)$ is the set where $ic(E)$ is nondegenerate and has exactly 0 or 1 negative eigenvalues (§2). Theorem 1.3. Let X be a quasiprojective, weakly 1-complete manifold such that its projective closure has only isolated singularities. Let E be a holomorphic line bundle over X which is positive outside an algebraic subvariety of dimension m. Then $H^q(E, \Omega^p(E))$ vanishes for $p + q > m + n$, where $n = \dim_{\mathbf{C}} X$.

Reviewed by *Antonella Nannicini*

MR1298282 (95f:32040) 32L15 (32F15)
Asserda, Saïd
Convexité holomorphe des domaines pseudoconvexes par rapport à un fibré holomorphe de rang un positif. (French. English, French summaries) [Holomorphic convexity of pseudoconvex domains with respect to a positive holomorphic line bundle]
C. R. Acad. Sci. Paris Sér. I Math. **319** (1994), *no. 6,* 559–562.

Let D be a relatively compact pseudoconvex domain in an n-dimensional complex analytic manifold M and let L be a rank-1 holomorphic line bundle on M. The open subset D is said to be L-convex if for every infinite discrete sequence $(x_m)_m$ of D there exists a holomorphic section $g \in H^0(D, L^q)$ such that the sequence $(g(x_m))_m$ is unbounded. The author proves that if D is a relatively compact pseudoconvex domain in M, there exists an integer p such that, for all $q \geq p$, for all $\varepsilon \in (0, 1]$ and for every infinite discrete sequence $(x_m)_m$ in D, there exists a holomorphic section $g \in H^0(D, L^q)$ such that the sequence $(g(x_m))_m$ is unbounded and satisfies $(*)$ $\int_D \|g\|^2 \delta^{2\varepsilon+3} dV_M < +\infty$, where $\delta(z) = d(z, M \setminus D)$ for all $z \in D$. This theorem generalizes and sharpens a result of K. R. Pinney [Math. Z. **206** (1991), no. 4, 605–615; MR1100844 (92f:32054)] concerning the particular case in which M is compact and ∂D is of class C^2. As-

serda's proof rests essentially on a result of Elencwajg and on L^2-estimates of Hörmander, Skoda and Demailly. Using the Richberg and Greene-Wu approximation theorem as well as the above theorem, the author shows that every relatively compact pseudoconvex domain in a Kähler manifold has a complete Kähler metric. (This result had been obtained by Pinney under the restrictive conditions mentioned above.) From this the author derives the corollary that every locally trivial holomorphic fibration with the unit disk as fiber and a projective manifold as base is meromorphically convex.

In the last part the author considers the submanifold of $\mathbf{P}^n \times \mathbf{P}^{n-1}$ defined by

$$M = \{([z],[t]) \in \mathbf{P}^n \times \mathbf{P}^{n-1}\colon\ z_j t_k - t_j z_k = 0,\ 1 \le j,k \le n\},$$

which is exactly the blowup of $\mathbf{P}^n$ at the point $a = [1,0,\cdots,0]$, with $n \ge 2$, and considers the domain $D = M \setminus S$, where S is the exceptional divisor of the blowup. He constructs a holomorphic section h over D satisfying the inequality $(*)$, with ε as exponent instead of $2\varepsilon + 3$.

Reviewed by *Mongi Blel*

MR1278916 (95d:32001) 32-02 (32H20 32H25 32L05)

Siu, Yum Tong (1-HRV)

Problems and recent results in several complex variables.

Complex analysis and its applications (*Hong Kong*, 1993), 38–49, *Pitman Res. Notes Math. Ser.*, 305, *Longman Sci. Tech.*, *Harlow*, 1994.

This paper is a survey on problems and recent results concerning the construction of holomorphic sections of holomorphic line bundles. Precisely two problems are illustrated: Matsusaka's big theorem and the hyperbolicity of the complement in the complex projective plane of a generic smooth complex curve of sufficiently high degree. Regarding the first problem the main result described here is the following theorem recently obtained by the author. Theorem 2: Let X be a compact complex manifold of complex dimension n and let L be an ample line bundle over X and B be a numerically effective holomorphic line bundle over X. Let C be the Chern number $((n+2)L + B + K_X)L^{n-1}$. Then the line bundle $mL - B$ is very ample over X for $m \ge (24n^n C(1+C)^n)^{n(6n^3)^n}$. The proof of Theorem 2 is obtained by using Demailly's asymptotic strong Morse inequality and Nadel's vanishing theorem.

Regarding the second problem the following conjecture is investigated: Conjecture 2: There exists some effective positive number e with the following property. For $d \ge e$ there exists a Zariski open subset $\mathcal{H}$ in the space of all complex curves of degree d such that any holomorphic map from $\mathbf{C}$ to $\mathbf{P}_2 - C$ is constant for $C \in \mathcal{H}$.

{For the entire collection see MR1278912 (94m:30001)}

Reviewed by *Antonella Nannicini*

MR1257325 (95f:32037) 32J27 (14J45 32L07)

Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYR)

Compact complex manifolds with numerically effective tangent bundles.

J. Algebraic Geom. **3** (1994), *no. 2,* 295–345.

A vector bundle E on a projective variety is said to be numerically effective (nef) if the tautological line bundle $\mathcal{O}_E(1)$ on the associated projective bundle $\mathbf{P}(E)$ is numerically effective. This notion is extended to vector bundles over compact complex manifolds as follows. Let X be a compact complex manifold and let E be a vector bundle on X. Fix a Hermitian metric ω on $\mathbf{P}(E)$. Then E is nef if, for every positive number ε, we can find a Hermitian metric h_ε on $\mathcal{O}_E(1)$ (or a Hermitian Finsler metric on E) such that its curvature Θ_{h_ε} satisfies $\Theta_{h_\varepsilon} \geq -\varepsilon\omega$. This definition does not depend on the choice of the Hermitian metric ω.

The main result of the paper is a structure theorem for Kähler manifolds with nef tangent bundles. Main Theorem: Let X be a compact Kähler manifold with nef tangent bundle. Then there exists an étale finite cover $\tilde{X}$ such that the Albanese mapping $\alpha\colon \tilde{X} \to \mathrm{Alb}(\tilde{X})$ is a surjective, smooth morphism, every fibre of which is a Fano manifold with nef tangent bundle.

The result states that X is essentially constructed by a torus and Fano manifolds. The torus part defines a flat quotient E of T_X, the tangent bundle of X (or, more precisely, of $T_{\tilde{X}}$). Since a Fano manifold is always simply connected, the main theorem implies that the fundamental group of X, a compact Kähler manifold with nef tangent bundle, is an extension of a finite group by a free abelian group of even rank $\mathbf{Z}^{2q}$.

One of the essential steps toward the main theorem is to show the following. Proposition: Let X be an n-dimensional compact Kähler manifold with T_X nef. If $c_1(X)^n = 0$, then (1) $\chi(\mathcal{O}_X) = 0$, (2) there is a nowhere vanishing p-form u for suitable odd p, and, (3) by lifting to a suitable étale cover $\tilde{X}$, the irregularity $q(\tilde{X})$ becomes positive.

The proof of this part depends on a result of J. Tits on subgroups of linear groups [J. Algebra **20** (1972), 250–270; MR0286898 (44 #4105)] and on the authors' result to the effect that $\pi_1(X)$ has subexponential growth [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

Choosing $\tilde{X}$ such that its irregularity attains the maximum, we get a subsheaf $E^* \subset \Omega^1_{\tilde{X}} = (T_X)^*$ generated by global sections. It is easy to show that E^* is a subbundle with trivial Chern classes, and results of Uhlenbeck-Yau and S. Kobayashi tell us that it has a filtration with Hermitian-flat graded pieces. Taking the duals, we get a quotient E of T_X which defines the torus part of $\tilde{X}$, or

the image of the Albanese map. The smoothness of the fibres of the Albanese fibration is proved via the theory of Mori contractions.

As by-products of the proof, the authors obtain (a) the projectivity of Moishezon manifolds with nef tangent bundles and (b) the classification of nonalgebraic compact 3-folds with nef tangent bundles (up to finite étale coverings) into nonalgebraic tori and some $\mathbf{P}^1$-bundles over nonalgebraic two-dimensional tori.

Reviewed by *Yoichi Miyaoka*

MR1275204 (95f:32035) 32J25 (14C20 14C30 14J60)
Siu, Yum Tong (1-HRV)
An effective Matsusaka big theorem. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **43** (1993), *no. 5,* 1387–1405.

Let us recall the Matsusaka Big Theorem [T. Matsusaka, Amer. J. Math. **94** (1972), 1027–1077; MR0337960 (49 #2729)]: let $P(k)$ be a polynomial in one variable of degree n with rational coefficients whose values are integers at integral values of k. Then there is a positive integer k_0 depending on $P(k)$ such that, for every compact projective algebraic manifold X of complex dimension n and every ample line bundle L over X with $\sum_{v=0}^n (-1)^v \dim H^v(X, kL) = P(k)$ for every k, the line bundle kL is very ample for $k \geq k_0$. It is interesting to find an effective bound for the positive integer k_0. By a result of J. Kollár and Matsusaka [Amer. J. Math. **105** (1983), no. 1, 229–252; MR0692112 (85c:14007)], k_0 can be made to depend only on the coefficients of k^{n-1} and k^n in the polynomial $P(k)$.

In this paper, the author proves an effective version of Matsusaka's Big Theorem (Theorem (0.1)): Let L be an ample holomorphic line bundle over a compact complex manifold X of complex dimension n. Then mL is very ample for any $m \geq$ an explicit positive constant which depends only on $n, L^n, K_X \cdot L^{n-1}$. Moreover, if in addition B is any numerically effective line bundle over X, then (see Theorem (0.2)) $mL - B$ is very ample for any $m \geq$ an explicit positive constant which depends only on H^n, $H^{n-1} \cdot B$, $L^{n-1} \cdot B$, and $L^{n-2} \cdot B \cdot K_X$, where $H := 2(K_X + 3(3n-2)^n L)$.

J.-P. Demailly's recent result [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)] that $12n^nL + 2K_X$ is very ample for any ample line bundle L over X can also be regarded as an effective version of Matsusaka's Big Theorem. But in the paper under review, the ampleness of $mL - B$ does not involve $2K_X$ and $-B$ can be used for any ample line bundle B.

The main idea of the proof of the effective Matsusaka Big Theorem is to show that for the given L and B, there is an effective lower bound m such that $mL - B$ is numerically effective, because of the very ampleness criterion of Demailly and Kollár. This is done in three steps. The first step

is a lemma on the existence of nontrivial holomorphic sections of a multiple of the difference of two ample line bundles whose Chern classes satisfy a certain inequality. Here the strong Morse inequality of Demailly is used. The second step is to produce, for any d-dimensional irreducible subvariety Y of X and any very ample line bundle H of X, a nontrivial holomorphic section over Y of some line bundle related to H. The third step is to use induction to get the numerical effectiveness of $mL - B$.

Reviewed by *Shanyu Ji*

MR1261816 (95e:32033) 32L20 (32C17 32F30)

Hamada, Hidetaka

Cohomology vanishing theorems on a domain exhausted by complete Kähler domains.

Math. Rep. Kyushu Univ. **19** (1993), 17–25.

Let E be a holomorphic vector bundle on a weakly pseudoconvex Kähler manifold X. If E satisfies some positivity condition such as Nakano's positivity, Griffiths' positivity or Demailly's s-positivity [J.-P. Demailly, Ann. Sci. École Norm. Sup. (4) **15** (1982), no. 3, 457–511; MR0690650 (85d:32057)] that generalized the former, we have corresponding cohomology vanishing theorems for the sheaves of holomorphic forms with values in E. The author generalizes these vanishing theorems to the case when X is a complex manifold that is exhausted by a sequence of complete Kähler domains and when the restriction of E on each Kähler manifold satisfies the above-mentioned positivity conditions, but for the cohomologies of order higher than 2. The vanishing of the first cohomology is most important but, as is indicated in the paper, we cannot expect it.

Reviewed by *Tosiaki Kori*

MR1257232 (94i:32047) 32L10 (32J20 58E05)
Bonavero, Laurent (F-GREN-FM)
Inégalités de Morse holomorphes singulières. (French. English, French summaries)
[Singular holomorphic Morse inequalities]
C. R. Acad. Sci. Paris Sér. I Math. **317** (1993), *no. 12,* 1163–1166.

Summary: "We generalize J.-P. Demailly's holomorphic Morse inequalities for a line bundle E equipped with a singular metric on a complex compact manifold X. Our inequalities give an estimate of the cohomology groups with values in the tensor powers $E^{\otimes k}$, twisted by the corresponding sequence of Nadel's multiplier ideal sheaves. The singularities allowed are of the following type: the metric is locally given by a weight $\exp(-\varphi)$, where $\varphi(x) \sim c \cdot \log(\sum |f_j|^2)$ with holomorphic f_j. As a consequence, we obtain a necessary and sufficient condition, invariant under bimeromorphism, for a manifold X to be Moishezon."

MR1256437 (95e:32032) 32L10 (14J15 14J60 32L20)
Takayama, Shigeharu (J-TOKYM)
Ample vector bundles on open algebraic varieties.
Publ. Res. Inst. Math. Sci. **29** (1993), *no. 6,* 885–910.

The main result in this paper is the following: Theorem 1. There exists a function $C(n)$ in $n \in \mathbf{N}$ with the following property: let X be a projective manifold, let D be a reduced effective divisor on X with only simple normal crossings such that $K_X + D$ is nef, let L be a nef line bundle on X which is ample modulo D; then $2(K_X + D) + mL$ is very ample modulo D for any $m \geq C(\dim X)$. In the case $D = 0$ Theorem 1 was proved by J.-P. Demailly [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. The methods used for the proof of Theorem 1 are similar to those used by Demailly. Theorem 1 has interesting consequences, for example, it can be used in the construction of the moduli space of open algebraic varieties of general type.

Reviewed by *Antonella Nannicini*

MR1255695 (95b:32044) 32J27 (14J40 32C17 53C55)
Demailly, Jean-Pierre (F-GREN-F); **Peternell, Thomas** (D-BAYR);
Schneider, Michael [Schneider, Michael Hellmut] (D-BAYR)
Kähler manifolds with numerically effective Ricci class.
Compositio Math. **89** (1993), *no. 2,* 217–240.

The purpose of this paper is to contribute to the solution of the following conjectures: Let X be a compact Kähler manifold with numerically effective (nef) anticanonical bundle $-K_X$; then: Conjecture 1: The fundamental group $\pi_1(X)$ of X has polynomial growth. Conjecture 2: The Albanese map $\alpha\colon X \to \mathrm{Alb}(X)$ is surjective. Section 1 is devoted to proving the following theorem, which is the main contribution to Conjecture 1. Theorem 1: Let X be a compact Kähler manifold with nef anticanonical bundle; then $\pi_1(X)$ has subexponential growth. The main tools used in order to prove Theorem 1 are the solution of the Calabi conjecture and volume bounds for geodesic balls due to Bishop and Gage. It should be mentioned that from the proof of Theorem 1 it follows that Conjecture 1 holds in the case $-K_X$ is Hermitian semipositive (Theorem 2).

In Section 2 the following theorem concerning Conjecture 2 is proved. Theorem 3: Let X be an n-dimensional compact Kähler manifold such that $-K_X$ is nef. Then the Albanese map α is surjective as soon as $\dim \alpha(X)$ is $0, 1$ or n, and, if X is projective, also for $n-1$; moreover, if X is projective and if the generic fibre F of α has $-K_F$ big, then α is surjective. Finally, Section 3 is devoted to the study of the structure of projective 3-folds with nef anticanonical bundles; in particular Conjecture 2 is proved in dimension 3 with purely algebraic methods, except in one very special case.

Reviewed by *Antonella Nannicini*

MR1249407 (94m:32015) 32C30 (32F05)
Blel, M. (TN-TUNISM)
Sur le cône tangent à un courant positif fermé. (French. English, French summaries) [On the tangent cone of a closed positive current]
J. Math. Pures Appl. (9) **72** (1993), *no. 6,* 517–536.

Suppose $1 \le p \le n-1$, and let T be a closed positive current of bidegree (p,p) on an open subset of $\mathbf{C}^n$ containing the origin. The tangent cone to T, if it exists, is the weak limit of dilates of T. Even when the tangent cone fails to exist, every weak limit of dilates of T is a closed, positive, conical current on $\mathbf{C}^n$ with the same Lelong number at the origin as T, as was shown by M. Blel, J.-P. Demailly and M. Mouzali [Ark. Mat. **28** (1990), no. 2, 231–248; MR1084013 (92f:32017)]. The author's main theorem is a characterization of the sets of currents that can arise in this way.

Namely, let K_p denote the set of closed, positive, conical currents on $\mathbf{C}^n$ of bidegree (p,p) with Lelong number equal to one at the origin. If M is a connected, closed subset of K_p, then there exists a closed positive current T of bidegree (p,p) on the unit ball of $\mathbf{C}^n$ for which the set of weak limits of dilates is M. The case $p=1$ is due to C. O. Kiselman [in *International Symposium in Memory of Hua Loo Keng, Vol. II (Beijing, 1988)*, 157–167, Springer, Berlin, 1991; MR1135833 (92i:32011)]. The author also gives a sufficient condition, in terms of limits of the trace measure, for the existence of the tangent cone when $p=1$ or $p=n-1$.

Reviewed by *Harold P. Boas*

MR1247749 (94k:32008) 32C30
Blel, Mongi (TN-TUNISM)
Sur le cône tangent associé à un courant positif fermé. (French. French summary) [On the tangent cone associated with a closed positive current]
Colloque d'Analyse Complexe et Géométrie (Marseille, 1992).
Astérisque No. 217 (1993), 5, 29–38.

Let T be any positive closed current of bidegree (p,p) on an open subset $\Omega\subset\mathbf{C}^n$ containing 0, $1\le p\le n-1$. In 1977 R. Harvey [in *Several complex variables (Williamstown, MA, 1975)*, 309–382, Proc. Sympos. Pure Math., 30, Part 1, Amer. Math. Soc., Providence, RI, 1977; MR0447619 (56 #5929)] raised the question whether there exists an associated "tangent cone" to T, i.e., whether there exists the weak limit of the family $\{(h_r)^*T\}$ as $r\searrow 0$, where $h_r\colon B(0,1)\to B(0,r)$, $z\mapsto rz$. The question has a negative answer by the work of C. O. Kiselman [in *International Symposium in Memory of Hua Loo Keng, Vol. II (Beijing, 1988)*, 157–167, Springer, Berlin, 1991; MR1135833 (92i:32011)] and the author, J.-P. Demailly and M. Mouzali [Ark. Mat. **28** (1990), no. 2, 231–248; MR1084013 (92f:32017)]. In fact, they showed that for any T as above, the set of value limits of the family $\{(h_r)^*T\}$ is a compact connected subset of the set $m\cdot K_p$, where m is the Lelong number of T at 0, and K_p is the set of positive closed conical currents of bidegree (p,p) on $\mathbf{C}^n$ with the Lelong number at 0 equal to 1; conversely, given a connected closed subset M of K_1, there exists a positive closed current T of bidegree $(1,1)$ such that M is equal to the set of limit values of the family $\{(h_r)^*T\}$. In the current paper, the author makes the above result more complete: Given any connected closed subset M of K_p, where $p\ge 2$, there exists a positive closed current of bidegree (p,p) on the unit ball of $\mathbf{C}^n$ such that M is equal to the set of limit values of the family $\{(h_r)^*T\}$. The author also discusses some necessary and sufficient conditions for a current T of bidegree $(1,1)$ or $(n-1,n-1)$ to admit a tangent cone.

{For the entire collection see MR1247746 (94f:32002)}

Reviewed by *Shanyu Ji*

MR1237810 (94h:32005) 32A25 (32E10)
Zhong, Tong De (PRC-XIAM)
A jump formula for the Bochner-Martinelli-Koppelman transform of differential forms on Stein manifolds. (Chinese. English, Chinese summaries)
Xiamen Daxue Xuebao Ziran Kexue Ban **32** (1993), *no. 5,* 525–527.

Let D be a relatively compact domain with C^2-smooth boundary in a Stein manifold M. The author gives a jump formula for C^1-smooth (p,q)-forms on ∂D. This jump formula was asserted by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)].

Reviewed by *Lan Ma*

MR1215287 (95d:14039) 14J45 (32J18)
Nadel, Alan M. (1-IASP)
Relative bounds for Fano varieties of the second kind.
Einstein metrics and Yang-Mills connections (*Sanda*, 1990), 181–191, *Lecture Notes in Pure and Appl. Math.*, 145, *Dekker, New York*, 1993.

From the introduction: “This article is concerned with bounding Fano varieties of the second kind. Recall that a Fano variety is a smooth projective variety with ample anticanonical class. A Fano variety is said to be of the first kind if its Picard number is one, and of the second kind if its Picard number is at least two. Fano varieties of the second kind have nontrivial extremal rays, which can be contracted in accordance with Mori’s program, and are thus sometimes regarded as being less primitive than Fano varieties of the first kind.

“There is a well-known conjecture asserting that there are only finitely many deformation types of Fano varieties in each dimension. There is another well-known conjecture asserting that the anticanonical degree of a Fano variety is bounded from above by a universal constant depending

only on the dimension. These two boundedness conjectures are equivalent, by work of Kollár and Matsusaka or Demailly.

"Boundedness of Fano varieties of dimension three or less follows from the classification. Indeed, the only Fano 1-fold is $\mathbf{P}^1$; the Fano 2-folds are the del Pezzo surfaces; and the classification of Fano 3-folds by Fano, Iskovskikh, Mori, Mukai, and Shokurov implies that there are precisely 104 deformation types of Fano 3-folds. Boundedness of toric Fano varieties was established by Batyrev. Boundedness of Fano varieties of the first kind was established in an earlier paper of ours ["A finiteness theorem for Fano 4-folds", Preprint; per bibl.] in dimension four, and in papers by J. Kollár, Y. Miyaoka and S. Mori [in *Classification of irregular varieties (Trento, 1990)*, 100–105, Lecture Notes in Math., 1515, Springer, Berlin, 1992; MR1180339 (94f:14039)] and us [J. Amer. Math. Soc. **4** (1991), no. 4, 681–692; MR1115788 (93g:14048)] in arbitrary dimensions.

"Here we are interested in the following setup. Let $\pi\colon M \to X$ be a connected (i.e., having connected fibers) surjective morphism from a (smooth) Fano variety M onto a (possibly singular) projective variety X. Let L be an ample (actually, nef and big is enough) line bundle on X. Let F be a general fiber of our morphism; it too is a Fano variety. Let k be the least integer such that K_F^{-k} is very ample on F; according to J.-P. Demailly [J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)], $k \leq 12 f^f$, where $m = \dim M$, $f = \dim F$, and $x = \dim X$. Our main result is as follows. Theorem A. In the above situation, we have the estimate $c_1(M)^m \leq 3(m\tau)^m |(X, L)|$, where $|(X, L)| = \min_{0 \leq \eta \leq x} h^0(X, L^\eta)$, and where $\tau = k^{f-1} c_1(F)^f + 2$ if $f > 0$ and $\tau = 2$ if $f = 0$.

"This article is organized as follows. The first section contains material on coherent sheaves of ideals associated to almost-plurisubharmonic functions, and culminates in a cohomology vanishing theorem. The second section contains various bits and pieces, mainly in the forms of lemmas. The third section contains the actual proof of Theorem A."

{For the entire collection see MR1215274 (93j:53002)}

MR1211891 (94g:14006) 14E25 (14C20)

Beltrametti, Mauro C. (I-GENO); **Sommese, Andrew J.** (1-NDM)

On k-jet ampleness.

Complex analysis and geometry, 355–376, *Univ. Ser. Math., Plenum, New York*, 1993.

The authors previously introduced and studied different notions of higher order embeddings for projective complex manifolds [cf. in Algebraic geometry (L'Aquila, 1988), 24–51, Lecture Notes in Math., 1417, Springer, Berlin, 1990; MR1040549 (91g:14029); in 1988 Cortona Proceedings: projective surfaces and their classification, 33–48, Symposia Math., 32, Academic Press, New York, 1991; per bibl.; M. C. Beltrametti, P. Francia and A. J. Sommese, Duke Math. J. 58 (1989),

no. 2, 425–439; MR1016428 (90h:14021)]. In the paper under review they study the stronger of these notions, k-jet ampleness. A line bundle L on a projective n-fold X is k-jet ample for a nonnegative integer k if, given any r integers $k_1+\cdots+k_r=k$ and any r distinct points x_i on X with maximal ideals m_i, the restriction map on the global section $\Gamma(L)\to\bigoplus_{i=1}^r\Gamma(L/m_i^{k_i})$ is onto.

Relationships with the previously introduced k-very ampleness and k-spannedness are investigated, together with the behaviour of k-jet ampleness under tensor products. A lower bound for the degree of a k-jet ample line bundle is obtained.

A large part of the paper is devoted to obtaining results of k-jet ampleness for adjoint bundles $K+tL$, where K is the canonical bundle of the n-fold. Sharp results relating the k-jet ampleness of L with the τ-jet ampleness of $K+tL$ are obtained by showing that the latter follows from the vanishing of a suitable first cohomology group, obtained via results of Kawamata and Viehweg. On surfaces, conditions on L are relaxed to nef, and under additional assumptions on the dimension of the linear system associated to L and on its self-intersection, assumptions which allow the authors to use Ramanujam vanishing, k-jet ampleness of the adjoint is proved. Further results are given for special surfaces. Some of the results on surfaces were independently obtained by M. Andrea de Cataldo. The authors call attention to a related paper by J.-P. Demailly [J. Differential Geom. 37 (1993), no. 2, 323–374; MR1205448 (94d:14007)].

{For the entire collection see MR1211876 (93j:32001)}.

{For the entire collection see MR1211876 (93j:32001)}

Reviewed by *Gian Mario Besana*

MR1211880 (94k:32009) 32C30 (32F07)

Demailly, Jean-Pierre (F-GREN-F)

Monge-Ampère operators, Lelong numbers and intersection theory.

Complex analysis and geometry, 115–193, *Univ. Ser. Math.*, *Plenum*, *New York*, 1993.

This article surveys the theory of Lelong numbers and their applications for studying intersection theory of analytic cycles. For this point of view, we define a plurisubharmonic (psh) function on a complex manifold X to be an upper semicontinuous function u for which dd^cu is a positive closed current of bidegree $(1,1)$. If u is a locally bounded psh function on X and T is a positive, closed current of bidimension (p,p), the wedge product $dd^cu\wedge T:=dd^c(uT)$ defines a closed positive current; by induction, if $1\le q\le p$ and $u_1,\cdots,u_q$ are locally bounded psh functions on X then

$$dd^cu_1\wedge\cdots\wedge dd^cu_q\wedge T:=dd^c(u_1dd^cu_2\wedge\cdots\wedge dd^cu_q\wedge T)$$

is a closed positive current. If $u_1^k,\cdots,u_q^k$ are decreasing sequences of psh functions converging

pointwise to $u_1, \cdots, u_q$ then, following E. Bedford and B. A. Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)], it is shown in Section 2 that

$$u_1^k dd^c u_2^k \wedge \cdots \wedge dd^c u_q^k \wedge T \to u_1 dd^c u_2 \wedge \cdots \wedge dd^c u_q \wedge T \tag{1}$$

and
$$dd^c u_1^k \wedge \cdots \wedge dd^c u_q^k \wedge T \to dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T \tag{2}$$

weakly.

Section 3 discusses the definition of $dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T$ for certain psh u_i and closed positive currents T even if some of the u_i are not necessarily bounded below. Define the unbounded locus $L(u)$ of u to be the set of points $x \in X$ such that u is unbounded in every neighborhood of x. Modifying the arguments of the previous section, the following result is proved: Theorem 1. Let $u_1, \cdots, u_q$ be psh functions on X. The currents $u_1 dd^c u_2 \wedge \cdots \wedge dd^c u_q \wedge T$ and $dd^c u_1 \wedge \cdots \wedge dd^c u_q \wedge T$ are well-defined and have locally finite mass in X provided $q \le p =$ dimension of T and

$$\mathcal{H}_{2p-2m+1}(L(u_{j_1}) \cap \cdots \cap L(u_{j_m}) \cap \operatorname{Supp} T) = 0$$

for all choices of indices $j_1 < \cdots < j_m$ in $\{1, \cdots, q\}$. Here, $\mathcal{H}_s(E)$ denotes the s-dimensional Hausdorff measure of E. In addition, it is shown that the analogues of the monotone convergence theorems in Section 2 remain valid.

In Section 4, the definition of generalized Lelong numbers is given. For a Stein manifold X, let T be a positive, closed current of bidimension (p, p) and let $\varphi\colon X \to [-\infty, +\infty)$ be a semiexhaustive weight function on $\operatorname{Supp} T$, i.e., φ is continuous and psh on X and there exists R such that $B(R) \cap \operatorname{Supp} T \subset\subset X$, where $B(R) = \{x \in X\colon\ \varphi(x) < R\}$. It follows that $\{\varphi = -\infty\} \cap \operatorname{Supp} T$ is compact and $T \wedge (dd^c\varphi)^p$ is well-defined. For each $r \in (-\infty, R)$, set $\nu(T, \varphi, r) = \int_{B(r)} T \wedge (dd^c\varphi)^p$ and define the generalized Lelong number of T with respect to φ as

$$\nu(T, \varphi) = \int_{\{\varphi=-\infty\}} T \wedge (dd^c\varphi)^p = \lim_{r\to-\infty} \nu(T, \varphi, r). \tag{3}$$

For $X = \mathbf{C}^n$ and $\varphi(z) = \log|z|$, this agrees with the ordinary Lelong number of T at 0,

$$\nu(T, 0) = \lim_{r\to 0} \frac{\sigma_T(B(r))}{\pi^p r^{2p}/p!},$$

where $\sigma_T = T \wedge (dd^c|z|^2)^p$ is the trace measure of T and $B(r)$ is the (Euclidean) ball of radius r centered at 0.

In Section 5, a Lelong-Jensen type formula is proved. Suppose $B(R) \subset\subset X$. Let $\mu_r = (dd^c[\max\{\varphi, r\}])^n - \mathbf{1}_{X-B(r)}(dd^c\varphi)^n$, $r < R$. In the case $X = \mathbf{C}^n$ and $\varphi(z) = \log|z|$, this is just a normalized surface area measure on the sphere of radius e^r. For any psh function V on X, V is μ_r-integrable for each $r < R$ and

$$\mu_r(V) - \int_{B(r)} V(dd^c\varphi)^n = \int_{-\infty}^{r} \nu(dd^cV, \varphi, t)\,dt.$$

This reduces to the classical Jensen formula if $n = 1$ ($X = \mathbf{C}$) and $V = \log|f|$. If $(dd^c\varphi)^n = 0$ on $X - \{\varphi = -\infty\}$, one obtains the formula $\nu(dd^cV, \varphi) = \lim_{r\to-\infty} \mu_r(V)/r$. An interesting case occurs in $\mathbf{C}^n$ if one takes $\varphi(z) = \log\max|z_j|^{\lambda_j}$, where $\lambda_j > 0$. It can be shown that $(dd^c\varphi)^n =$

$\lambda_1 \cdots \lambda_n \delta_0$ and $\mu_r = \lambda_1 \cdots \lambda_n (2\pi)^{-n} d\theta_1 \cdots d\theta_n$ on the distinguished boundary $\{z\colon\ |z_j| = e^{r/|\lambda_j|}\}$ of the polydisk $B(r)$. The Lelong number $\nu(dd^c V, \varphi)$ is then

$$\lim_{r\to-\infty} \frac{\lambda_1\cdots\lambda_n}{r} \int_{\theta_j\in[0,2\pi]} V(e^{r/\lambda_1+i\theta_1}, \cdots, e^{r/\lambda_n+i\theta_n}) \frac{d\theta_1\cdots d\theta_n}{(2\pi)^n},$$

which is the directional Lelong number of $dd^c V$ at 0 with coefficients $\lambda := (\lambda_1, \cdots, \lambda_n)$ introduced by Kiselman. In general, for any current T, define $\nu(T, x, \lambda) = \nu(T, \log\max|z_j - x_j|^{\lambda_j})$; in $\mathbf{C}^n$, taking $\varphi(z) = \log|z - x|$, it follows that the usual Lelong numbers $\nu(T, x)$ agree with the Kiselman numbers $\nu(T, x, (1, \cdots, 1))$.

In Section 6, a new proof is given of Thie's theorem: If A is an analytic set of pure dimension p and $[A]$ is the current of integration over A, then for each $x \in A$, $\nu([A], x)$ is the multiplicity of A at x. Also, a result of Siu's on stability of Lelong numbers for closed, positive currents under slicing along linear subspaces is presented. In Section 7, a generalization of Siu's upper semicontinuity theorem is proved [J.-P. Demailly, Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)]. Section 8 describes the behavior of Lelong numbers under proper morphisms. As a concrete application, it is shown that for T a closed, positive current of bidimension (p, p) and for $0 < \lambda_1 \le \cdots \le \lambda_n$, the directional Lelong numbers of Kiselman satisfy $\lambda_1 \cdots \lambda_p \nu(T, x) \le \nu(T, x, \lambda) \le \lambda_{n-p+1} \cdots \lambda_n \nu(T, x)$. A type of Schwarz lemma relating growth of zeros of entire functions f in $\mathbf{C}^n$ which involves Lelong numbers of the current of integration $[Z_f]$ of the zero variety of f is proved in Section 9. This yields a theorem of Bombieri on algebraic values of meromorphic maps satisfying algebraic differential equations. Finally, in Section 10, a self-intersection inequality for closed positive currents of bidegree (1,1) is given. The motivation behind this inequality is the following. The wedge product of smooth differential forms defines a ring structure on de Rham cohomology, and, for two currents Θ_1, Θ_2 on X, there is a well-defined intersection class $\{\Theta_1\} \cdot \{\Theta_2\}$ in the cohomology ring, even if $\Theta_1 \wedge \Theta_2$ is not defined pointwise as a current. But the wedge product of closed, positive currents cannot always be defined in a reasonable way, and, moreover, such currents cannot necessarily be approximated in the weak topology by smooth closed, positive currents. Indeed, for T a closed, positive current, a necessary condition for such an approximation to be possible is that $\{T\}^p \cdot \{Y\} \ge 0$ for every p-dimensional subvariety $Y \subset X$. The author showed [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] that T can be approximated by closed, real currents with small negative part governed by the curvature of X. Then, by regularizing and taking weak limits, one can compute self-intersections.

{For the entire collection see MR1211876 (93j:32001)}

Reviewed by *Norman Levenberg*

MR1211876 (93j:32001) 32-06

★**Complex analysis and geometry.**

Edited by Vincenzo Ancona and Alessandro Silva.

The University Series in Mathematics.

Plenum Press, New York, 1993. xvi+412 *pp*. $85.00. *ISBN* 0-306-44179-9

{The papers are being reviewed individually.}

MR1207864 (94f:32060) 32L10 (32F10 32L15)

Demailly, Jean-Pierre (F-GREN)

Holomorphic Morse inequalities on q-convex manifolds.

Several complex variables (Stockholm, 1987/1988), 245–257, *Math. Notes*, 38, *Princeton Univ. Press, Princeton, NJ*, 1993.

This note is a report on a paper by T. Bouche [Ann. Sci. École Norm. Sup. (4) **22** (1989), no. 4, 501–513; MR1026747 (91a:32041)] in which holomorphic Morse inequalities for strongly q-convex manifolds were obtained, extending the results of the author [in *Séminaire d'analyse P. Lelong-P.*

Dolbeault-H. Skoda, années 1983/1984, 88–97, Lecture Notes in Math., 1198, Springer, Berlin, 1986; MR0874763 (88f:32069)]. The author carefully describes the main ideas and techniques used by Bouche and illustrates some interesting applications.

{For the entire collection see MR1207850 (93j:32002)}

Reviewed by *Antonella Nannicini*

MR1207850 (93j:32002) 32-06

★**Several complex variables.**

Proceedings of the Special Year held at the Mittag-Leffler Institute, Stockholm, 1987/1988.
Edited by John Erik Fornæss.
Mathematical Notes, 38.
Princeton University Press, Princeton, NJ, 1993. viii+701 *pp*. $39.50. *ISBN* 0-691-08579-X

Range, The $\bar\partial$-Neumann kernel for codimension-one forms on strictly pseudoconvex domains (473–482); Qi Keng Lu and Yi Hong, The heat kernel of the classical domain $\mathfrak{R}_I(m,n)$ (483–506); Guido Lupacciolu and Edgar Lee Stout, Removable singularities for $\bar\partial_b$ (507–518); Per E. Manne, Carleman approximation on totally real submanifolds of a complex manifold (519–528); Alan Noell, Peak functions for pseudoconvex domains in $\mathbf{C}^n$ (529–541); Mikael Passare, On the support of residue currents (542–549); Juhani Riihentaus, Nullsets for BMO functions (550–562); Jean-Pierre Rosay and Walter Rudin, Holomorphic embeddings of $\mathbf{C}$ in $\mathbf{C}^n$ (563–569); Rita Saerens and William R. Zame, The local automorphism group of a CR hypersurface (570–572); A. G. Sergeev, On matrix Reinhardt and circled domains (573–586); Nessim Sibony, On Hölder estimates for $\bar\partial$ (587–599); Edgar Lee Stout, Removable singularities for the boundary values of holomorphic functions (600–629); S. M. Webster, An implicit function theorem of the Nash-Moser type (630–639); H. Wu [Hung Hsi Wu], Old and new invariant metrics on complex manifolds (640–682); Ahmed Zériahi, Inegalités de Markov et développement en série de polynômes orthogonaux des fonctions C^∞ et A^∞ [Markov inequalities and orthogonal polynomial series expansions of C^∞ and A^∞ functions] (683–701).

{The papers are being reviewed individually.}

MR1206442 (94m:32044) 32J20 (32C10 32C17 32C30 32L07)
Ji, Shanyu (1-HST)
Currents, metrics and Moishezon manifolds. (English summary)
Pacific J. Math. **158** (1993), *no. 2,* 335–351.

In this paper, the author proves the following theorem: Let M be a compact complex manifold of dimension n; then M is Moishezon (bimeromorphic to a projective variety) if and only if there exists a positive definite integral d-closed (1,1)-current (a singular (1,1)-form) ω on M such that ω is smooth outside a proper analytic subset. This is a generalization of Kodaira's embedding theorem. In another paper by the author and B. Shiffman [J. Geom. Anal. **3** (1993), no. 1, 37–61; MR1197016 (93m:32014)] it was proved by a combination of this theorem and a smoothing theorem by J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] that the result is true without the condition on smoothness, which was a conjecture of Shiffman. The proof is based on Demailly's L^2-estimate of the $\bar\partial$-operator on complete Kähler manifolds with noncomplete Kähler metric and with singular Hermitian metric on a line bundle by observing that the open set of smooth points of the current has a complete Kähler metric (Lemma 4.1). The author also proves that M is Moishezon if and only if there is a proper subset of M such that its complement in M admits a complete Kähler-Einstein metric with negative Ricci curvature

(Theorem 1.4).

Reviewed by *Zhuang Dan Guan*

MR1205448 (94d:14007) 14C20 (14E25 32J25 32L10)

Demailly, Jean-Pierre (F-GREN)

A numerical criterion for very ample line bundles. (English summary)

J. Differential Geom. **37** (1993), *no. 2,* 323–374.

Let X be a smooth projective variety of dimension n defined over the field of complex numbers. A divisor (or, equivalently, a line bundle) L on X is ample if, by definition, some positive multiple of L is very ample, that is, it is isomorphic to the restriction of $\mathcal{O}(1)$ for some embedding $X \subset \mathbf{P}^N$. The notion of very ampleness has a definite geometric meaning while, as it follows from well-known Kleiman and Nakai criteria, ampleness has a very numerical character. Therefore, it is natural to ask for a numerical criterion asserting very ampleness of an ample divisor. One observes easily that for curves the answer depends not only on the degree of the divisor L but also on the geometry of X; in particular, it depends on the genus of the curve X. A uniform answer for curves is as follows: if L is an ample divisor on a curve X then $K_X + 3L$ is very ample, where K_X denotes the canonical divisor of X. For surfaces, a result of I. Reider implies very ampleness of $K_X + 4L$. On the other hand, the theory of extremal rays of S. Mori implies ampleness of $K_X + (n+2)L$ for an ample divisor L on a smooth n-fold X; T. Fujita conjectured that $K_X + (n+2)L$ is actually very ample.

The main result of the paper under review is a significant step towards the conjecture of Fujita. Namely, the main theorem of the paper gives numerical conditions for an ample (or, more generally, big and nef) divisor L to imply spannedness or very ampleness (or, more generally, separation of s-jets of sections) of the adjoint divisor $K_X + L$. From the theorem it follows that $2K_X + mL$ is very ample if only $m \geq 12n^n$. This in turn yields an effective version of T. Matsusaka's big theorem [cf. Amer. J. Math. **94** (1972), 1027–1077; MR0337960 (49 #2729); Y. T. Siu, "An effective Matsusaka big theorem", Preprint, 1993; per revr.], and combined with results of Catanese and of Green and Morrison implies an effective bound on the number of irreducible families of n-dimensional smooth polarised varieties (X, L) depending only on the intersection numbers L^n and $K_X \cdot L^{n-1}$ [J. Kollár, Math. Ann. **296** (1993), no. 4, 595–605]. Although the main result of the paper and its applications are formulated in terms of algebraic geometry, the proof of the theorem is analytic and it applies Hörmander L^2 estimates for the operator $\overline{\partial}$, the Aubin-Calabi-Yau theorem and the theory of positive currents and Lelong numbers.

{Reviewer's remarks: Among most recent developments following the paper under review there are the above-mentioned papers of Siu and Kollár as well as a paper of L. M. H. Ein and R. K.

Lazarsfeld [J. Amer. Math. Soc. **6** (1993), no. 4, 875–903; MR1207013 (94c:14016)]. The Kollár and Ein-Lazarsfeld papers involve algebraic settings (Kollár works with varieties having Kawamata log terminal singularities) but, in the end, they depend on the transcendental Kodaira-Kawamata-Viehweg vanishing.}

Reviewed by *Jarosław A. Wiśniewski*

MR1197016 (93m:32014) 32C30 (32J27)
Ji, Shanyu (1-HST); **Shiffman, Bernard** (1-JHOP)
Properties of compact complex manifolds carrying closed positive currents. (English summary)
J. Geom. Anal. **3** (1993), *no. 1,* 37–61.

The authors give an interesting characterization of Moishezon manifolds via currents; it is similar to a previous characterization due to J.-P. Demailly using differential forms. Let M be a compact complex manifold; the authors define a Kählerian current as follows: A current T of bidegree $(1,1)$ is Kählerian if T is d-closed and if there is a strictly positive differential form ω of the same bidegree such that $T' = T - \omega$ is a semipositive current (in the following sense: T' is real and $T'(\eta)$ is nonnegative for any $(n-1, n-1)$ semipositive form η on M). The authors prove M is Moishezon if and only if there exists a Kählerian current in $H^2(M, \mathbf{Z})$; they give another equivalent condition using "singular metrics" on holomorphic line bundles. Moreover, they obtain a sufficient condition so that the intersection of two d-closed semipositive currents of complementary degrees is a positive current.

Reviewed by *Salomon Ofman*

MR1205883 (94e:32051) 32L20 (14F17)
Manivel, Laurent (F-GREN-F)
Théorèmes d'annulation pour les fibrés associés à un fibré ample. (French) [Vanishing theorems for vector bundles associated with an ample vector bundle]
Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **19** (1992), *no. 4,* 515–565.

In this paper the author generalizes a vanishing theorem of J.-P. Demailly [Invent. Math. **91** (1988), no. 1, 203–220; MR0918242 (89d:32066)]. More precisely, the main theorem proved here is the following: Let E be a holomorphic vector bundle of rank d and let L be a line bundle over an n-dimensional compact complex manifold X such that E is ample and L is nef or E is nef and L is ample. Then if $a \in (\mathbf{N}^d_{\geq})$ and $p \geq n-20$, $H^{p,q}(X, \Gamma^a E \otimes (\det\ E)^l \otimes L) = 0$ for $l \geq h(a) + n - p$ and $p+q > n$. Here $\Gamma^a(E)$ is the bundle associated to E and to the representation of $\mathrm{GL}(d, \mathbf{C})$ of weight $a \in (\mathbf{N}^d_{\geq})$, $(\mathbf{N}^d_{\geq})$ is the set of decreasing d-tuplets of natural numbers and $h(a)$ is the number of nonzero components of a. The methods used are similar to those in the above-mentioned paper of Demailly.

Reviewed by *Antonella Nannicini*

MR1195080 (93m:32024) 32F20
Langenbruch, Michael (D-MUNS)
Splitting of the $\overline{\partial}$-complex in weighted spaces of square integrable functions. (English summary)
Rev. Mat. Univ. Complut. Madrid **5** (1992), *no. 2-3,* 201–223.

The author continues the work of Meise, Taylor and collaborators on the splitting of the $\overline{\partial}$-complex in weighted L^2-spaces. Let Ω be a pseudoconvex set in $\mathbf{C}^m$, $\mathcal{B}$ an increasing sequence $\{W_n\}$ of weight functions in Ω, $L^2(\mathcal{B}, \Omega) = \{f \in L^2_{\mathrm{loc}}(\Omega)\colon \int_\Omega |f|^2 e^{-2W_n} < \infty$ for some $n \geq 1\}$. It is shown that the splitting of the $\overline{\partial}$-complex is equivalent to the existence of plurisubharmonic functions Φ_t in Ω $(t \in \Omega)$ and $I\colon \mathbf{N}^* \to \mathbf{N}^*$, $I(n) \geq n$, $A\colon \mathbf{N}^* \to \mathbf{N}^*$, such that, for every $z, t \in \Omega$, $\Phi_t(t) \geq 0$, $\Phi_t(z) \leq W_{I(n)}(z) - W_n(t) + A(n)$. He also provides explicit estimates for the splitting (right inverse) operators R. Applications to the existence of a linear extension from interpolating manifolds are given (this complements results of the reviewer and B. A. Taylor [in *Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981*, *et Colloque de Wimereux, Mai 1981*, 1–25, Lecture Notes in Math., 919, Springer, Berlin, 1982; MR0658877 (83k:32004)] and J.-P. Demailly [ibid., 77–107; MR0658880 (83j:32019)]).

Reviewed by *Carlos A. Berenstein*

MR1178721 (93g:32044) 32L10 (14J60 32L05)
Demailly, Jean-Pierre (F-GREN-F)
Singular Hermitian metrics on positive line bundles. (English summary)
Complex algebraic varieties (*Bayreuth*, 1990), 87–104, *Lecture Notes in Math.*, 1507, *Springer*, *Berlin*, 1992.

In this paper, the author introduces the notion of singular Hermitian metric on a holomorphic line bundle on a complex manifold. A singular Hermitian metric on a holomorphic line bundle L on a complex manifold M is a product of the form $h_0 e^{-\varphi}$, where h_0 is a smooth Hermitian metric on L and φ is a locally L^1-function. The beauty of the definition is that we can take the curvature of a singular Hermitian metric in the sense of a closed current. This extended definition of Hermitian metrics enables us to characterize the line bundle to be pseudoeffective, big or nef in terms of the positivity properties of the curvature of the singular metrics. For the applications of these metrics, the author shows the relation between the Seshadri constant and the singularity of singular Hermitian metrics. This is related to the global generation of nef line bundles. Finally, he uses the singular metrics to prove a new asymptotic estimate for the dimension of the cohomology groups with values in high multiples $\mathcal{O}(kL)$ of a big line bundle L.

{For the entire collection see MR1178715 (93d:14006)}

Reviewed by *Hajime Tsuji*

MR1178715 (93d:14006) 14-06
★**Complex algebraic varieties.**
Proceedings of the conference held in Bayreuth, April 2–6, 1990.
Edited by K. Hulek, T. Peternell, M. Schneider and F.-O. Schreyer.
Lecture Notes in Mathematics, 1507.
Springer-Verlag, *Berlin*, 1992. vi+179 *pp*. $25.00. *ISBN* 3-540-55235-9

Contents: Arnaud Beauville, Annulation du H^1 pour les fibrés en droites plats [Vanishing of H^1 for flat line bundles] (1–15); Mauro C. Beltrametti, Andrew J. Sommese and Jarosław A. Wiśniewski, Results on varieties with many lines and their applications to adjunction theory (16–38); Guntram Bohnhorst and Heinz Spindler, The stability of certain vector bundles on $\mathbf{P}^n$ (39–50); F. Catanese and F. Tovena, Vector bundles, linear systems and extensions of π_1 (51–70); Olivier Debarre,

Vers une stratification de l'espace des modules des variétés abéliennes principalement polarisées [Toward a stratification of the moduli space of principally polarized abelian varieties] (71–86); Jean-Pierre Demailly, Singular Hermitian metrics on positive line bundles (87–104); Takao Fujita, On adjoint bundles of ample vector bundles (105–112); Yujiro Kawamata, Moderate degenerations of algebraic surfaces (113–132); Ulf Persson, Genus two fibrations revisited (a preliminary report) (133–144); Th. Peternell, M. Szurek and J. A. Wiśniewski [Jarosław A. Wiśniewski], Numerically effective vector bundles with small Chern classes (145–156); C. A. M. Peters, On the rank of nonrigid period maps in the weight one and two case (157–165); A. N. Tyurin, The geometry of the special components of moduli space of vector bundles over algebraic surfaces of general type (166–175).

{Most of the papers are being reviewed individually.}

MR1175540 (93f:32012) 32C30 (32-01)
Demailly, Jean-Pierre (F-GREN-F)
Courants positifs et théorie de l'intersection. (French) [Positive currents and intersection theory]
Gaz. Math. No. 53 (1992), 131–159.

This article is a brief exposition of intersection theory from the point of view of positive currents.

Let Y be a complex analytic manifold of dimension n. To any subvariety X of Y, we can associate a current denoted $[X]$ defined by $\langle [X], \varphi\rangle = \int_X \varphi$. After recalling all the requisite definitions, the author gives in the first part of this article the relation between the intersection number of two analytic subvarieties A and B of Y $(\dim A + \dim B = n)$ and the product $[A] \wedge [B]$. The main tools are two theorems of P. Lelong, for which short proofs are given; as an application, Example 5.2 calculates the intersection number of the curves Γ_1 and Γ_2 of $\mathbf{C}^2$ defined, respectively, by the equation $z^2 = w^3$ and $z^3 = w^5$, where (z, w) are the coordinate functions on $\mathbf{C}^2$.

The second part (the last two sections) deals with the "Lelong numbers": to any positive d-closed (p, p)-current T and any $y \in Y$ is associated a number $\nu(T, y) \in \mathbf{R}_+$ called the "Lelong number of T at y", which is related to the "regularity" of T at the point $y \in Y$. Its definition is recalled and generalized in Section 6. The last section deals with more recent results: by using a theorem of Siu, the author states interesting estimations for the degree of the irreducible components of $E_c(T) = \{y \in Y\colon\ \nu(T, y) \geq c\}$.

This article uses mainly elementary results of analytic geometry in the proofs. It is an excellent introduction to the subject.

Reviewed by *Salomon Ofman*

MR1158622 (93e:32015) 32C30 (32C17 32J25)
Demailly, Jean-Pierre (F-GREN)
Regularization of closed positive currents and intersection theory.
J. Algebraic Geom. **1** (1992), *no. 3,* 361–409.

Let X be a complex manifold of dimension n and let T be a closed current of bidegree $(1,1)$ on X. T is said to be almost positive if there exists a smooth form v of bidegree $(1,1)$ such that $T+v\geq 0$. If T is closed and almost positive, let (T,x) denote the Lelong number of T at x. For every $c>0$, let $E_c(T)=\{x\in X\colon \nu(T,x)\geq c\}$. By Siu's theorem $E_c(T)$ is an analytic subset of X. The main result of the paper under review is an "approximation-regularization" theorem for closed almost positive currents T. It says that T can be approximated, for every $c>0$, by T_c which are smooth outside $E_c(T)$, and such that $\nu(T_c,x)=(\nu(T,x)-c)_+$ at every point $x\in X$. The T_c can be chosen so that they satisfy certain estimates from below. Although the precise statement of the theorem is too long to be described here, it includes the good old regularization theorem of Richberg as a special case, where $T=\partial\overline{\partial}\psi$ for finite and continuous ψ. The proof of the result is based on a combination of three different types of L^2-techniques. Among other things, the reader will be delighted to find an elegant proof of a local approximation theorem for plurisubharmonic functions by logarithms of holomorphic functions. Interesting applications are also given. Some of them are described below. Let $H^{p,q}_{\partial\overline{\partial}}(X)=\{d\text{-closed } (p,q)\text{-forms}\}/\{\overline{\partial}\text{-exact } (p,q)\text{-forms}\}$. A cohomology class $\{\alpha\}\in H^{1,1}_{\partial\overline{\partial}}(X)$ is said to be pseudo-effective (psef) if it can be represented by a closed positive $(1,1)$-current, and nef if for some fixed Hermitian metric ω on X and for every $\varepsilon>0$ there is a smooth form $\alpha_\varepsilon\in\{\alpha\}$ such that $\alpha_\varepsilon\geq-\varepsilon\omega$. Denote respectively by $H^{1,1}_{\rm psef}(X)$ and $H^{1,1}_{\rm nef}(X)$ the cones of pseudo-effective and nef cohomology classes. Then, for compact X, the main result of this article implies that $H^{1,1}_{\rm nef}(X)=H^{1,1}_{\rm psef}(X)$ if the tangent bundle TX of X is nef. Here one says that a vector bundle E is nef if $C_1(\mathcal{O}_E(1))$ is nef over the projectivized bundle $P(E^*)$ of hyperplanes of E. A related new result is that X is Kahler if TX is nef and X is in the Fujiki class. A self-intersection inequality proved in an earlier work of the author's is extended here to an arbitrary closed positive $(1,1)$-current T on a Kahler manifold X. It may be worthwhile to note that this work was strongly motivated by the question of classifying compact Kahler varieties with nef tangent bundle, which is of course of current research interest.

Reviewed by *Takeo Ohsawa*

MR1222208 (94j:32025) 32L20 (14F17 32L10)

Demailly, J.-P. (F-GREN-F)

Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem. (English summary)

Geometrical and algebraical aspects in several complex variables (*Cetraro*, 1989), 81–94, *Sem. Conf.*, 8, *EditEl*, *Rende*, 1991.

Let L be a numerically effective line bundle on a projective algebraic manifold X of dimension n. The main purpose of this paper is to show a vanishing theorem for the cohomology group $H^q(X, \Omega^n(L \otimes D))$ for the effective **Q**-divisor D which may have nonnormal crossings, under a certain natural integrability condition for D. As a corollary, the Kawamata-Viehweg vanishing theorem follows, i.e. if L is as above and $\bigwedge^k c_1(L) \neq 0$, then $H^q(X, \Omega^n(L)) = 0$ for $q > n - k$. The author's proof is analytic in the sense that his method is based on a vanishing theorem of L^2 cohomology for $\overline{\partial}$ on complete Kähler manifolds for line bundles provided with a singular metric which yields a positive curvature in the sense of currents. This theorem is induced from an L^2 estimate for $\overline{\partial}$ by the Bochner-Kodaira-Nakano curvature inequality and a smooth procedure for plurisubharmonic functions on Kähler manifolds by the author. To show the vanishing theorem the problem is reduced to the case that L is ample by standard slicing arguments and a trick of Kawamata in algebraic geometry. Finally, the theorem is shown by using a vanishing theorem on compact Kähler manifolds which follows from the vanishing theorem of L^2 cohomology.

{For the entire collection see MR1222200 (93m:32002)}

Reviewed by *Kensho Takegoshi*

MR1222200 (93m:32002) 32-06 (00B25)

★**Geometrical and algebraical aspects in several complex variables.**

Papers from the conference held in Cetraro, June 1989.
Edited by Carlos A. Berenstein and Daniele C. Struppa.
Seminars and Conferences, 8.
Editoria Elettronica, *Rende*, 1991. x+376 *pp*.

Contents: T. Aoki [Takashi Aoki], Characteristic sets of differential operators of infinite order (1–11); U. Backlund and A. Fällström, A pseudoconvex domain with nonschlicht H^∞-envelope (13–

18); D. Barlet, Vanishing cycles and poles of $\int_X |f|^{2\lambda}\square$ (19–26); Y. Benyamini and Y. Weit, Spaces of continuous functions failing spectral analysis (27–32); C. A. Berenstein and D. C. Struppa, Interpolation and Dirichlet series: a new approach (33–45); C. A. Berenstein and A. Yger, About L. Ehrenpreis fundamental principle (47–61); F. Colonna, Local normality of meromorphic functions (63–80); J.-P. Demailly, Transcendental proof of a generalized Kawamata-Viehweg vanishing theorem (81–94); H. M. Farkas, Identities on compact Riemann surfaces (95–106); R. E. Greene and S. G. Krantz, Invariants of Bergman geometry and the automorphism groups of domains in $\mathbf{C}^n$ (107–136); G. A. Harris [Gary Alvin Harris], Algebra and geometry related to uniqueness sets in $\mathbf{C}^3$ (137–153); A. Kaneko [Akira Kaneko], Analyticité du lieu de singularité de dimension minimale d'une solution analytique réelle [Analyticity of the locus of the singularity of minimal dimension of a real analytic solution] (155–167); M. Kashiwara, T. Kawai [Takahiro Kawai] and Y. Takei [Yoshitsugu Takei], The structure of cohomology groups associated with the theta-zerovalues (169–189); T. Kawai [Takahiro Kawai] and Y. Takei [Yoshitsugu Takei], The complex-analytic geometry of bicharacteristics and the semi-global existence of holomorphic solutions of linear differential equations—a bridge between the theory of partial differential equations and the theory of holomorphic functions (191–199); P. A. Kuchment, On the Floquet theory of periodic difference equations (201–209); P. Lelong, Fonctions plurisousharmoniques de croissance logarithmique dans $\mathbf{C}^n$; partie principale, extension du résultant des polynômes [Plurisubharmonic functions with logarithmic growth in $\mathbf{C}^n$; principal part, extension of the resultant of polynomials] (211–229); R. Meise, B. A. Taylor and D. Vogt, Indicators of plurisubharmonic functions on algebraic varieties and Kaneko's Phragmén-Lindelöf condition (231–250); Y. Okada [Yasunori Okada] and N. Tose, Second micorolocal singularities and boundary values of holomorphic functions (251–263); V. P. Palamodov, A criterion for splitness of differential complexes with constant coefficients (265–291); M. Ru and W. Stoll, The Nevanlinna conjecture for moving targets (293–308); H. S. Shapiro, Global geometric aspects of Cauchy's problem for the Laplace operator (309–324); S. Tajima, Microlocal aspects of Cauchy-Fantappiè kernels associated with certain class of pseudo-convex domains in $\mathbf{C}^2$ and its applications (325–340); Chia Chi Tung, On the degree theory of holomorphic mappings (341–360); L. Ehrenpreis, Extensions of solutions of partial differential equations (361–375).

{The papers are being reviewed individually.}

MR1153703 (93f:32032) 32L10 (58E05 58G25)
Bouche, Thierry (F-GREN-F)
Sur les inégalités de Morse holomorphes lorsque la courbure du fibré en droites est dégénérée. (French) [Holomorphic Morse inequalities when the curvature of the line bundle is degenerate]
Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **18** (1991), *no. 4,* 501–523.

Demailly's holomorphic Morse inequalities for the d''-cohomology of $E^k \otimes G$, E a holomorphic line bundle over a complex manifold X with nondegenerate curvature form, G a holomorphic vector bundle of rank g ([J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)], hereafter referred to as [1]), are extended to the d''-cohomology of $E^k \otimes F^l \otimes G$. Here E is a line bundle with degenerate curvature form and F is a line bundle. Denoting by $c(E)$ the curvature of the Chern connection of E, the author assumes that $\operatorname{rank} ic(E)_x \leq r$ and the kernel fibration of $ic(E)$ is well foliated, that is, there is a codimension r foliation Y of X such that its tangent fibre at any point $x \in X$ is included in $\ker ic(E)_x$ [cf. R. Bott, Ann. of Math. (2) **60** (1954), 248–261; MR0064399 (16,276f)]. Setting $\gamma = ic(E)|NY \oplus ic(F)|TY$, he defines the q-index sets $X(q)$ and $X(\leq q)$ to be $\{x \in X\colon\ \operatorname{ind}\gamma_x = q\}$ and $\bigcup_{1\leq j\leq q} X(j)$, respectively. Then the following inequalities are proved. (1) The asymptotic Morse inequality:

$$\dim H^q(X, E^k \otimes F^l \otimes G) \leq \\ g\frac{k^r}{r!}\frac{l^{n-r}}{(n-r)!}\int_{X(q)}(-1)^q\left(\frac{i}{2\pi}c(E)\right)^r \wedge \left(\frac{i}{2\pi}c(F)\right)^{n-r} + o(k^r l^{n-r}).$$

(2) The strong Morse inequality (Theorem 0.1):

$$\sum_{\nu=0}^{q} \dim H^\nu(X, E^k \otimes F^l \otimes G) \leq \\ g\frac{k^r}{r!}\frac{l^{n-r}}{(n-r)!}\int_{X(\leq q)}\left(\frac{i}{2\pi}c(E)\right)^r \wedge \left(\frac{i}{2\pi}c(F)\right)^{n-r} + o(k^r l^{n-r}).$$

(3) There exists a constant C such that $\dim H^q(X, E^k \otimes G) \leq Ck^r$, $k \to \infty$ (Corollary 0.2). Here $H^q(X, E^k \otimes F^l \otimes G)$ denotes $H^q(X, \mathcal{O}(E^k \otimes F^l \otimes G))$ and $\mathcal{O}(E^k \otimes F^l \otimes G)$ is the sheaf of germs of holomorphic sections of $E^k \otimes F^l \otimes G$. To show these inequalities, the author uses the metric $\omega_{k,l} = k\eta + l\xi$, where η [resp. ξ] is a C^∞-class Hermitian metric on NY [resp. on TY]. By using $\omega_{k,l}$, Demailly's Hermitian Bochner-Kodaira-Nakano identity is extended to the antiholomorphic Laplacian $\Delta''_{k,l}$ of $E^k \otimes F^l \otimes G$ (Lemma 1.2). Thus the method of [1] can be applied to $\Delta''_{k,l}$ and the inequalities are obtained. As an application, the existence of infinitely many tensor products Q^k which admit nontrivial sections, where $Q = F/E$, E is the same as above and the Kodaira dimension of a fibre F is larger than $r+1$, is shown (Theorem 4.4). An alternative proof of these inequalities using heat kernels and an evaluation of the constant C in (3) was given by the author [Bull. Sci. Math. (2) **116** (1992), no. 2, 167–183].

Reviewed by *Akira Asada*

MR1150978 (93h:32021) 32F07 (32F05 35J60)
Klimek, Maciej (IRL-DBLN)
★**Pluripotential theory.**
London Mathematical Society Monographs. New Series, 6.
Oxford Science Publications.
The Clarendon Press, Oxford University Press, New York, 1991. xiv+266 *pp*. $59.95.
ISBN 0-19-853568-6

This book is the first relatively comprehensive book ever written on the subject of the title. Pluripotential theory is, loosely speaking, the study of plurisubharmonic (psh) functions; in this text, all psh functions are defined on domains in $\mathbf{C}^n$. An uppersemicontinuous function $u\colon\Omega\subset\mathbf{C}^n\to[-\infty,\infty)$ is psh on a domain Ω provided $u\not\equiv-\infty$ and its restriction to every complex line l which intersects Ω is either subharmonic or $\equiv-\infty$ on each component of $\Omega\cap l$. P. Lelong's 1969 text [*Plurisubharmonic functions and positive differential forms*, Gordon and Breach, New York, 1969] is classic but much has been accomplished since then; the text of U. Cegrell [*Capacities in complex analysis*, Vieweg, Braunschweig, 1988; MR0964469 (89m:32001)] dealt mainly with capacities in $\mathbf{C}^n$; and P. Lelong and L. Gruman [*Entire functions of several complex variables*, Springer, Berlin, 1986; MR0837659 (87j:32001)] concentrated on growth properties of entire functions. The complex Monge-Ampère operator, $(dd^c(\cdot))^n$, is a nonlinear operator which, in $\mathbf{C}^n$, $n>1$, is the natural replacement for the Laplacian. If $u\colon\Omega\subset\mathbf{C}^n\to\mathbf{R}$ belongs to the class $C^2(\Omega)$, then $(dd^cu)^n=c_n\det H(u)dV_n$ where $H(u)=(\partial^2u/\partial z_j\partial\overline{z}_k)_{j,k=1,\cdots,n}$ is the complex Hessian of u, c_n is a dimensional constant, and $dV_n=$ Lebesgue measure on $\mathbf{C}^n$. More generally, E. Bedford and B. A. Taylor [Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351)] have shown that $(dd^cu)^n$ can be defined as a positive measure (precisely, a positive (n,n) current) for any locally bounded psh function u. Such functions which satisfy the homogeneous complex Monge-Ampère equation $(dd^cu)^n=0$ in Ω are the so-called maximal psh functions in Ω and they play the role of harmonic functions in classical potential theory. This is the point of view of this book. A priori, u is maximal on Ω if u is psh and for any relatively compact subdomain G, if v is psh and $v\le u$ on ∂G, then $v\le u$ in G. A large part of the text is aimed at showing that locally bounded maximal functions satisfy the homogeneous complex Monge-Ampère equation. After a motivational discussion of maximal functions in the preface and a short (essentially notational) chapter on complex differentiation, the author gives all the necessary background material on subharmonic and psh functions in the second (and longest) chapter of the book. No prior knowledge of several complex variables or even classical potential theory is assumed; indeed, the reader who completes the exercises at the end of each of these chapters will have no trouble continuing through the rest of the material.

In Section II of the book (beginning with Chapter 3), the reader is introduced to the complex Monge-Ampère operator. Chapter 3 begins with a discussion of currents à la Lelong, and the precise definition of the positive (k,k) current $dd^cu_1\wedge dd^cu_2\wedge\cdots\wedge dd^cu_k$ for locally bounded

psh functions $u_1, \cdots, u_k$ is given. Properties of this operator, such as continuity under decreasing limits and comparison theorems, are proved, following another paper of Bedford and Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)]; one goal is to show that locally bounded solutions to $(dd^c u)^n = 0$ are maximal. Given a Borel set E in a domain Ω, the notion of the relative capacity of E in Ω, $C(E, \Omega) \equiv \sup\{\int_E (dd^c u)^n\colon\ 0 < u < 1, u \text{ psh in } \Omega\}$ is introduced to prove the important quasicontinuity property of psh functions: a psh u on Ω is continuous off an open set whose relative capacity can be made arbitrarily small. At this stage in the text, the quasicontinuity theorem is proved for locally bounded psh functions; the general case is deferred until later. In Chapter 4, the author proves existence and uniqueness of the solution to the generalized Dirichlet problem for the complex Monge-Ampère operator for a ball; i.e., given f continuous, real-valued on the boundary ∂B of a ball B, find u psh and locally bounded in B satisfying $(dd^c u)^n = 0$ in B and $u = f$ on ∂B. He states certain potential-theoretic and measure-theoretic preliminaries (Riesz decomposition for subharmonic functions, Poisson-Jensen formula, smoothing by using integral averages of locally integrable functions) and then follows the proof of Bedford and Taylor [op. cit., 1976] (in that paper they solved the inhomogeneous equation but the author does not need this, which makes the exposition a bit simpler). The hard part is verifying maximality of the Perron-Bremermann upper envelope $u(z) \equiv \sup\{v(z)\colon\ v \text{ psh in } B,\ \limsup_{\xi \to \zeta} v(\xi) \le f(\zeta),\ \zeta \in \partial B\}$. The rest of the chapter discusses the relative extremal function $u_{E,\Omega}(z) \equiv \sup\{v(z)\colon\ v \text{ psh in } \Omega,\ v \le 0,\ v|_E \le -1\}$ for a subset E of Ω. This is used, again following Bedford and Taylor [op. cit., 1982], to relate the notions of negligible sets, pluripolarity, and thinness. A set $E \subset \mathbf{C}^n$ is pluripolar if for each point $a \in E$ there exists a neighborhood U of a and a psh function u in U with $E \cap U \subset \{z \in U\colon\ u(z) = -\infty\}$. If A is a family of psh functions on a domain Ω which are locally bounded above, the function $u(z) = \sup\{v(z)\colon\ v \in A\}$ may not be psh because it may not be upper semicontinuous (usc). Define $u^*(z) \equiv \limsup_{\xi \to z} u(\xi)$, the usc regularization of u. The set $N \equiv \{z \in \Omega\colon\ u(z) < u^*(z)\}$ is called a negligible set. Using the relative extremal function $u_{E,\Omega}(z)$ and the notion of relative capacity $C(E, \Omega)$, the author shows, following arguments of Bedford and Taylor, that a pluripolar set can be defined by one global psh function (B. Josefson's theorem [Ark. Mat. **16** (1978), no. 1, 109–115; MR0590078 (58 #28669)]) and that negligible sets are pluripolar; indeed, these sets are characterized as having outer capacity $C^*(E, \Omega) = 0$.

Other than the background material on psh functions and currents, most of the content of Chapters 3 and 4 can be found in two or three papers of Bedford and Taylor; however, the material discussed in the last two chapters is scattered throughout the literature. In Chapter 5, the author studies maximal functions of logarithmic growth; i.e., functions in the class

$$L \equiv \{u \text{ psh in } \mathbf{C}^n\colon\ u(z) - \log|z| \le O(1),\ |z| \to \infty\}.$$

Given a set $E \subset \mathbf{C}^n$, the pluricomplex Green function of E with pole at infinity is the function $V_E \equiv \sup\{u(z)\colon\ u \in L,\ u \le 0 \text{ on } E\}$. The Monge-Ampère measure $\mu_E \equiv (dd^c V_E)^n$ associated with V_E is called the equilibrium measure for E. If $n = 1$ and E is nonpolar and compact, V_E is the usual Green function for the unbounded component of $\widehat{\mathbf{C}} - E$ with logarithmic pole at infinity and μ_E is the usual equilibrium measure for E. Following J. Siciak [Ann. Polon. Math. **39** (1981),

175–211; MR0617459 (83e:32018)], it is shown that if K is compact, then $V_K(z) =$

$$\sup\{(1/\deg p)\log|p(z)|\colon\ p\text{ polynomial},\ \|p\|_K \le 1,\ \deg p \ge 1\}.$$

Thus the pluricomplex Green function can be used in studying problems involving polynomial approximation. The author proves some fundamental properties of these functions and even gives explicit computational examples of V_E for certain compact subsets $E \subset \mathbf{R}^n \subset \mathbf{C}^n$, following M. Baran [Ann. Polon. Math. **48** (1988), no. 3, 275–280; MR0978678 (90j:32019); see also M. Lundin, Michigan Math. J. **32** (1985), no. 2, 197–201; MR0783573 (86h:32030)].

Finally, in the last chapter, a pluricomplex Green function with a logarithmic singularity at a finite point is discussed. Given a domain $\Omega \subset \mathbf{C}^n$ and a point $a \in \Omega$, we call $g_\Omega(z,a) \equiv$

$$\sup\{u(z)\colon\ u\text{ psh in }\Omega,\ u<0, u(z)-\log|z-a| \le O(1),\ z \to a\}$$

the pluricomplex Green function of Ω with pole at a. If Ω is bounded, then $g_\Omega(z,a)$ is maximal in $\Omega - \{a\}$. If $n = 1$, $g_\Omega(z,a)$ is the classical Green function with pole at a (if it exists). After extending the definition of the Monge-Ampère operator so as to include psh functions with logarithmic singularities, the author proves J.-P. Demailly's theorem [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)] that, at least for hyperconvex domains (pseudoconvex domains with bounded psh exhaustions), $g_\Omega(z,a)$ is the unique solution of the degenerate Monge-Ampère equation $(dd^cu)^n = (2\pi)^n\delta_a$, u is psh in Ω, continuous in $\Omega - \{a\}$, $u(z) - \log|z-a| = O(1)$, $z \to a$, $u(z) \to 0$ on $\partial\Omega$. The book ends (modulo a brief appendix on foliations) with some applications of this pluricomplex Green function, including a relationship between $g_\Omega(z,a)$ and the Carathéodory and Kobayashi pseudodistances, and with a pluricomplex Poisson/Riesz decomposition theorem, due to Demailly, for a psh function u on a hyperconvex domain with u continuous up to the boundary.

The beginner should find the book very readable, especially with the long introductory Chapter 2 on subharmonic and plurisubharmonic functions. The experts will be pleased to have the material in Section II of the book all in one place for the first time and will find this book to be a valuable reference tool, especially with a 12–13 page bibliography. There are several exercises at the end of the introductory chapters in Section I. Perhaps it would have been desirable to include exercises at the end of the chapters in Section II which could be designed to encourage the reader to seek out some of the references; on the other hand, the author liberally sprinkles remarks with bibliographic references throughout the text. Certain topics, such as the differential-geometric aspect of the complex Monge-Ampère equation, are not discussed but references are given. All in all, this is a very welcome and much overdue book on an important topic in several complex variables.

Reviewed by *Norman Levenberg*

MR1143476 (93a:32021) 32E30 (32F05)
Zériahi, Ahmed (F-TOUL3-AV)
Fonction de Green pluricomplexe à pôle à l'infini sur un espace de Stein parabolique et applications. (French) [Pluricomplex Green function with pole at infinity on a parabolic Stein space, and applications]
Math. Scand. **69** (1991), *no. 1,* 89–126.

Let X be a complex space of dimension n. E. Bedford [in *Séminaire Pierre Lelong-Henri Skoda (Analyse), Années 1980/1981, et Colloque de Wimereux, Mai 1981*, 294–323, Lecture Notes in Math., 919, Springer, Berlin, 1982; MR0658889 (83i:32025)], J.-P. Demailly [Mém. Soc. Math. France (N.S.) No. 19 (1985), 124 pp.; MR0813252 (87g:32030)] and J. E. Fornæss and R. Narasimhan [Math. Ann. **248** (1980), no. 1, 47–72; MR0569410 (81f:32020)] established basic results on plurisubharmonic (psh) and weakly psh functions and the complex Monge-Ampère operator $(dd^c)^n$. The author begins the article under review by discussing some related results and then defines the natural notions of equilibrium potential $h^*_{K,\Omega}$ and equilibrium measure $\mu_c \equiv (dd^c h^*_{K,\Omega})^n$ associated to a compact subset K contained in a hyperconvex open subset Ω of a pure n-dimensional Stein space X [see Bedford and B. A. Taylor, Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)]. If X is a parabolic Stein space, i.e., if there exists a continuous, psh exhaustion function $g\colon X \to [-\infty,+\infty)$ satisfying $(dd^c g)^n = 0$ on $X - g^{-1}(-\infty)$, one can define a pluricomplex Green function $g_E(x)$ with pole at infinity associated to any $E \Subset X$ by setting $g_E(x) \equiv \sup\{v(x)\colon\ v \in L_g(X),\ v|_E \le 0\}$, where $L_g(X)$ is the class of psh functions v on X satisfying $v(x) \le c_v + g^+(x)$ on X for some constant c_v. In the case $X = \mathbf{C}^n$, this is the usual Siciak extremal function [J. Siciak, Ann. Polon. Math. **39** (1981), 175–211; MR0617459 (83e:32018)].

The author develops the standard properties of g_E à la Siciak and gives many examples. The final two chapters are devoted to applications. He shows that the existence of a parabolic potential (a function g as above) on a Stein space X imposes restrictions on psh functions of minimal increase on X. For example, the space of "generalized" polynomials $P^d_g(X)$ consisting of holomorphic functions f on X satisfying an estimate of the form $|f(x)| \le c_f[1+\exp g(x)]^d$, $x \in X$, has dimension at most $\binom{n+dN}{n}$ for some $N > 1$ independent of d. Finally, if X is an algebraic variety of pure dimension n imbedded in $\mathbf{C}^N$, the Siciak theory carries over nicely in the sense that there exists a "natural" parabolic potential g on X satisfying $c_1 + \log^+|x| \le g(x) \le c_2 + \log^+|x|$, $x \in X$, where $|x|$ is the Euclidean norm on $\mathbf{C}^N$ restricted to X. He then shows that for $K \subset X$ compact, $g_K(x) = \sup\{(1/d)\log|f(x)|\colon\ f \in A_d(X),\ \|f\|_K \le 1,\ d \ge 1\}$, where $f \in A_d(X) =$ regular functions on X such that $\sup(1+|x|)^{-d}|f(x)| < \infty$ (note $A_d(X) \subset P^g_d(X)$). This characterization is used to get a Bernstein-Walsh type theorem using approximation by the spaces $A_d(X)$. See also related results by the author [Ann. Inst. Fourier (Grenoble) **37** (1987), no. 2, 79–104; MR0898932 (88k:32047)].

Reviewed by *Norman Levenberg*

MR1139755 (92k:32055) 32L15 (32J20 32L10 58E05 58G25)
Marinescu, George [Marinescu, Gheorghe]
Asymptotic Morse inequalities. (Romanian. English summary)
Stud. Cerc. Mat. **43** (1991), *no. 5-6,* 243–297.

This survey paper gives an introduction to and a proof of J.-P Demailly's proof [Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147); in *Séminaire d'Analyse P. Lelong–P. Dolbeault –H. Skoda, Années 1985/1986*, 24–47, Lecture Notes in Math., 1295, Springer, Berlin, 1987; MR1047720 (91h:32025)] of Witten's asymptotic Morse inequalities for d''-cohomology and of the Grauert-Riemenschneider conjecture (for a compact irreducible analytic space to be a Moishezon space it is necessary and sufficient that there exist a desingularization $\pi\colon X \to Y$ of Y and that X admit a quasipositive holomorphic line bundle $E \to X$). The last section contains the author's own results on asymptotic Morse inequalities for compact manifolds with isolated cone-like singularities.

Reviewed by *J. S. Joel*

MR1128538 (93b:32048) 32L10 (58G11 58G25)
Demailly, Jean-Pierre (F-GREN)
Holomorphic Morse inequalities.
Several complex variables and complex geometry, *Part* 2 (*Santa Cruz, CA*, 1989), 93–114, *Proc. Sympos. Pure Math.*, 52, *Part* 2, *Amer. Math. Soc.*, *Providence, RI*, 1991.

A simple heat equation proof of the author's holomorphic Morse inequalities is presented. For this purpose, the asymptotic eigenvalue distribution of $\square_k = (1/k)D_k^* D_k - V$ is studied. Here D_k is the associated connection on $E^k \otimes F$, where E, F are complex vector bundles over a smooth manifold M equipped with Hermitian connections, and $V = V \otimes \mathrm{id}_{E^k}$ is a Hermitian endomorphism of F. The asymptotic estimates of the heat kernel $e^{-t\square_k}$ are purely local and the explicit form of the heat kernel in the case of connections with constant curvature, assuming M has a flat metric, is obtained by using Mehler's formula. From these facts, the asymptotic estimate of the heat kernel of $\square_k$ is expressed in terms of $c(E)$, the curvature form of E (Theorem 3.1). To obtain holomorphic Morse inequalities from this estimate, $(2/k)\Delta_k'' = (1/k)\nabla_k^*\nabla_k - V + (1/k)\Theta$ is shown. Here Δ_k''

and ∇_k are the Dolbeault Laplacian and Chern connection on $E^k \otimes F \otimes \bigwedge^{0,q} T^*X$, X a compact complex manifold, E and F are holomorphic Hermitian bundles over X of ranks 1 and r, Θ a Hermitian form independent of k; the eigenvalues of V are easily counted (reviews of complex geometry and Hodge theory are given in Section 2). Thus the asymptotic formula of the heat kernel $e^{-(2t/k)}\Delta''_k$ in bidegree $(0,q)$ is obtained from Theorem 3.1 (Theorem 4.4). Then, according to Witten's idea, a finite-dimensional subcomplex of the Dolbeault complex on $E^k \otimes F$ with the same cohomology is constructed by using the eigenspaces of $(1/k)\Delta''_k$. This allows one to use linear algebraic considerations, and combining these considerations and Theorem 4.4, the strong holomorphic Morse inequality follows. We have $\dim H^0(X, E^k) \geq O(k^n)$ if $ic(E) \geq 0$ and $ic(E) > 0$ at at least one point by this inequality. Hence we have an alternative proof of Siu's theorem (the Grauert-Riemenschneider conjecture, Section 5). In Section 6, a generalization of the holomorphic Morse inequality to q-convex manifolds and its application to an a priori estimate for the Monge-Ampère operator are given (Theorem 6.1 and Corollary 6.6, the author states, were obtained by Bouche and Siu). The case rank $E \geq 2$ is discussed in Section 7 [cf. E. Getzler, C. R. Acad. Sci. Paris Sér. I Math. **304** (1987), no. 16, 475–478; MR0894572 (88j:32040)]. Related open problems are discussed in Section 8, the last section.

{For the entire collection see MR1128530 (92d:32002)}

Reviewed by *Akira Asada*

MR1128530 (92d:32002) 32-06

★**Several complex variables and complex geometry. Part 2.**

Proceedings of the Thirty-seventh Annual Summer Research Institute held at the University of California, Santa Cruz, California, July 10–30, 1989.
Edited by Eric Bedford, John P. D'Angelo, Robert E. Greene and Steven G. Krantz.
Proceedings of Symposia in Pure Mathematics, 52, Part 2.
American Mathematical Society, *Providence*, *RI*, 1991. xvi+625 *pp*. \$219.00 the three volume set. *ISBN* 0-8218-1490-7

Contents: Robert E. Greene, The geometry of complex manifolds: an overview (pp. 1–22); Marco Abate, Angular derivatives in strongly pseudoconvex domains (pp. 23–40); Yukinobu Adachi and Masakazu Suzuki, Degeneracy points of the Kobayashi pseudodistances on complex manifolds (pp. 41–51); Takao Akahori, On the construction of the moduli space for strongly pseudoconvex domains (pp. 53–58); Andrew Balas, On the holomorphic sectional curvature of complete domains in $\mathbf{C}^n$ that are not Stein (pp. 59–63); J. Bland and T. Duchamp, Normal forms for convex domains (pp. 65–81); Ciprian Borcea, Homogeneous vector bundles and families of Calabi-Yau threefolds. II (pp. 83–91); Jean-Pierre Demailly, Holomorphic Morse inequalities (pp. 93–114);

Pierre Dolbeault, Some problems (pp. 115–116); Michael G. Eastwood and C. Robin Graham, Invariants of CR densities (pp. 117–133).

Yakov Eliashberg [Ya. M. Èliashberg] and Mikhael Gromov, Convex symplectic manifolds (pp. 135–162); H. R. Fischer and R. J. Fisher, Jr., Simple partial connections and the Einstein condition (pp. 163–182); Sidney Frankel, Applications of affine geometry to geometric function theory in several complex variables. I. Convergent rescalings and intrinsic quasi-isometric structure (pp. 183–208); Akito Futaki, A Lie algebra character and Kähler-Einstein metrics of positive scalar curvature (pp. 209–215); Bruce Gilligan, On the ends of complex manifolds homogeneous under a Lie group (pp. 217–224); James F. Glazebrook and Franz W. Kamber, Determinant line bundles for Hermitian foliations and a generalized Quillen metric (pp. 225–232); Ian Graham, Sharp constants for the Koebe theorem and for estimates of intrinsic metrics on convex domains (pp. 233–238); Chong-Kyu Han, Rigid immersions of G-structures and analyticity of CR mappings into spheres (pp. 239–249); J. William Helton and Orlando Merino, Optimal analytic disks (pp. 251–262); Marek Jarnicki and Peter Pflug, Some remarks on the product property for invariant pseudometrics (pp. 263–272).

Shanyu Ji, Smoothing of currents and Moišezon manifolds (pp. 273–282); Kang Tae Kim, Biholomorphic mappings between quasicircular domains in $\mathbf{C}^n$ (pp. 283–290); Akio Kodama, Characterizations of certain weakly pseudoconvex domains in $\mathbf{C}^n$ from the viewpoint of biholomorphic automorphism groups (pp. 291–296); Claude LeBrun, Complete Ricci-flat Kähler metrics on $\mathbf{C}^n$ need not be flat (pp. 297–304); Steven Shin-Yi Lu, On meromorphic maps into varieties of log-general type (pp. 305–333); Ngaiming Mok, Aspects of Kähler geometry on arithmetic varieties (pp. 335–396); J. Noguchi, Moduli spaces of holomorphic mappings into hyperbolic spaces and its applications (pp. 397–401); Salomon Ofman, The analytic Radon transform (pp. 403–412); Takeo Ohsawa, Applications of the $\overline{\partial}$ technique in L^2 Hodge theory on complete Kähler manifolds (pp. 413–425); Marius Overholt, Bounds on the derivatives of holomorphic endomorphisms (pp. 427–433).

Giorgio Patrizio and Pit-Mann Wong, Monge-Ampère functions with large center (pp. 435–447); Mathias Peternell, A characterization of affine varieties (pp. 449–453); Thomas Peternell and Michael Schneider [Michael Hellmut Schneider], Compactifications of $\mathbf{C}^n$: a survey (pp. 455–466); T. Ratiu and A. Todorov, An infinite-dimensional point of view on the Weil-Petersson metric (pp. 467–476); Min Ru and Wilhelm Stoll, The Cartan conjecture for moving targets (pp. 477–508); Morihiko Saito, On Kollár's conjecture (pp. 509–517); Leslie Saper and Steven Zucker [Steven M. Zucker], An introduction to L^2-cohomology (pp. 519–534); Vo Van Tan, On the compactification problems for Stein 3-folds (pp. 535–542); Gang Tian, On one of Calabi's problems (pp. 543–556); T. Tonev, Multi-dimensional analytic structures and uniform algebras (pp. 557–561).

J. Varouchas, The $\partial\overline{\partial}$-equation on complex spaces (pp. 563–577); Jean-Pierre Vigué, Fixed points of holomorphic mappings in a bounded convex domain in $\mathbf{C}^n$ (pp. 579–582); L. Waelbroeck, Holomorphic functions taking their values in a q-space and the Cauchy-Fantappiè formula (pp. 583–591); B. Wong [Bun Wong], Schwarz's lemma and Hermitian manifolds with constant holomorphic curvature (pp. 593–600); H. Wu [Hung Hsi Wu], Polynomial functions on complete Kähler manifolds (pp. 601–610); Kichoon Yang, The Chern numbers of projective algebraic

hypersurfaces (pp. 611–617); Shing-Tung Yau, A review of complex differential geometry (pp. 619–625).

{Most of the papers are being reviewed individually.}

MR1100993 (92c:32012) 32C30 (32A25)

Berndtsson, Bo (S-CHAL)

Cauchy-Leray forms and vector bundles.

Ann. Sci. École Norm. Sup. (4) **24** (1991), *no. 3,* 319–337.

In this paper the author obtains integral formulas of Cauchy-Leray-Koppelman type for differential forms on a complex manifold. From the introduction: "The original motivation for our paper was to generalize the constructions of G. M. Khenkin and J. Leiterer [*Theory of functions on complex manifolds*, Birkhäuser, Basel, 1984; MR0774049 (86a:32002)] to forms of arbitrary bidegree. Such a generalization has already been found by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], but they only gave the leading terms in the expansion of the kernels. Still, the idea of Demailly and Laurent-Thiébaut, to use a connection on a bundle, is of fundamental importance in this paper as well."

Let X be a complex manifold and let Y be a complex submanifold of codimension p. Suppose $\pi\colon E \to X$ is a holomorphic vector bundle of rank p, with a holomorphic section $\eta\colon X \to E$, that defines Y, i.e. $Y = \{\eta = 0\}$. The main part of this paper consists in finding a large family of explicit solutions K to the equation $dK = [Y] - c_p[\Theta]$, where $c_p[\Theta]$ is the pth Chern form of the curvature Θ of some connection on E (Theorem 2.4).

Then, setting $X = M \times M$, and $Y = \Delta = \{(\zeta, z) \in M \times M\colon \zeta = z\}$, the author obtains the Cauchy-Leray-Koppelman formula (Theorem 4.1) on the complex manifold M.

Reviewed by *Alexandr M. Kytmanov*

MR1087191 (92f:32023) 32F07 (32J20 35J30)
Foote, Robert L. (1-TXT)
Homogeneous complex Monge-Ampère equations and algebraic embeddings of parabolic manifolds.
Indiana Univ. Math. J. **39** (1990), *no. 4,* 1245–1273.

Let M be an n-dimensional, connected complex manifold with a strictly plurisubharmonic exhaustion $\tau\colon M \to [0, R^2)$ such that the complex homogeneous Monge-Ampère equation $(dd^c \log \tau)^n = 0$ holds on $M \setminus K$, where K is a compact subset of M. This situation has been studied extensively in case K is small and τ (and $\log \tau$) is well behaved near K. For example, it is known that if $\tau \in C^\infty(M)$ and $K = \tau^{-1}(0)$, then, up to biholomorphic maps, M is the ball of radius R in $\mathbf{C}^n$ and τ is the squared Euclidean norm [W. Stoll, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **7** (1980), no. 1, 87–154; MR0577327 (81h:32028)]. The author studies the general setting and his main result is a criterion which guarantees that the Stein manifold M is an affine algebraic submanifold of $\mathbf{C}^{2n+1}$. The result is obtained under the additional hypothesis that τ is at least of class C^6 and locally Reinhardt, i.e., in a neighborhood of every point of M there are holomorphic coordinates $z = x + iy$ such that $\tau(x + iy) = \tau(x)$. This assumption allows the author to study the differential geometry of M with respect to the metric $dd^c\tau$ using the interplay, possible in this case, between the real and the complex homogeneous Monge-Ampère equations. Using previous results of the author on the real equation, an estimate on the Ricci curvature of the metric $dd^c\tau$ is achieved which is the key ingredient for applying and obtaining the proof of the theorem by means of Demailly's algebraicity criterion.

Reviewed by *Giorgio Patrizio*

MR1084013 (92f:32017) 32C30
Blel, Mongi (TN-TUNISM); **Demailly, Jean-Pierre** (F-GREN-F);
Mouzali, Mokhtar (F-GREN-F)
Sur l'existence du cône tangent à un courant positif fermé. (French) [Existence of the tangent cone of a closed positive current]
Ark. Mat. **28** (1990), *no. 2,* 231–248.

Summary: "Let T be a closed positive current in a neighbourhood of 0 in $\mathbf{C}^n$. We show here that T admits a tangent cone (limit of the family of its homotheties) when the projective mass $\nu_T(r)$ converges to $\nu_T(0)$ rapidly enough for the function $(\nu_T(r) - \nu_T(0))/r$ to be locally integrable at 0. This sufficient condition is optimal: we build $(1,1)$ currents without tangent cone such that the integral at $r = 0$ of $(\nu_T(r) - \nu_T(0))/r$ has as small a divergence as one likes. When T is given

by integration on an analytic set, we show that $\nu_T(r) - \nu_T(0) = O(r^\varepsilon)$ and that this recovers the Thie-King theorem on the existence of the tangent cone."

Reviewed by *Daniel Barlet*

MR1077088 (92e:14014) 14F17 (14M15)
Manivel, Laurent (F-GREN-F)
Sur la cohomologie des fibrés associés au fibré quotient universel sur la grassmannienne. **(French. English summary) [On the cohomology of fiber bundles associated with the universal quotient bundle on the Grassmannian]**
Bull. Soc. Math. France **118** (1990), *no. 1,* 67–84.

Summary: "Using a suitable filtration of the inverse image of the tangent bundle of the Grassmannian of a complex vector space, on the variety of its complete flags, we determine, as a continuation of one of J. P. Demailly's papers, certain cohomology groups of the vector bundles associated to the universal quotient bundle on the Grassmannian. In particular, we obtain a vanishing property of the cohomology of the tensor powers of that bundle tensored with large enough powers of its determinant. Finally, the case of symmetric powers leads us to new counterexamples to a conjecture of Faltings and a question raised by Le Potier, both proved false by Peternell, Le Potier and Schneider."

MR1062924 (91e:14016) 14F17 (18G40 32L20)
Manivel, Laurent (F-GREN-F)
Un exemple de non-dégénérescence en E_2 de la suite spectrale de Borel-Le Potier. (French. English summary) [An example of E_2 nondegeneracy of the Borel-Le Potier spectral sequence]
C. R. Acad. Sci. Paris Sér. I Math. **311** (1990), *no. 1,* 31–36.

Summary: "The Borel-Le Potier spectral sequence and its possible degeneracy properties appear as an important tool for the study of the cohomology of the bundles associated with a holomorphic vector bundle on a complex compact variety. We give the example of an ample line bundle on a

variety of incomplete flags, projecting on a Grassmannian, such that the associated Borel-Le Potier spectral sequence does not degenerate at E_2: this allows us to answer in the negative a question raised by J.-P. Demailly."

Reviewed by *Jürgen Leiterer*

MR1056776 (91e:32025) 32J25 (32C30 32F05 32S25)
Azhari, Abdelhak
Sur la conjecture de Chudnovsky-Demailly et les singularités des hypersurfaces algébriques. (French. English summary) [On the Chudnovsky-Demailly conjecture and singularities of algebraic hypersurfaces]
Ann. Inst. Fourier (Grenoble) **40** (1990), *no. 1,* 103–116.

For a finite subset $S \subseteq \mathbf{C}^n$, let $\omega_t(S)$ be the least degree of a hypersurface in $\mathbf{C}^n$ having at least t-fold points at each point of S. It is known that $\omega_{t_1}(S)/(t_1+n-1) \leq \omega_{t_2}(S)/t_2$. The conjecture of Chudnovsky and Demailly is that $(\omega_{t_1}(S)+n-1)/(t_1+n-1) \leq \omega_{t_2}(S)/t_2$. Here it is shown that $(\omega_{t_1}(S)+n-1-a_{t_2})/(t_1+n-1) \leq \omega_{t_2}(S)/t_2$, where a_{t_2} is the minimal dimension of the locus of worse than normal crossings singularities among all hypersurfaces of degree ω_{t_2} having t_2-fold points at S. Some related results are also obtained. The methods are complex analytic and depend, via a theorem of Demailly, on L^2 estimates due to Hörmander.

Reviewed by *G. K. Sankaran*

MR1055992 (91e:32014) 32F10 (32L10)
Demailly, Jean-Pierre (F-GREN-F)
Cohomology of q-convex spaces in top degrees.
Math. Z. **204** (1990), *no. 2,* 283–295.

Highly elegant proofs of the following three theorems are given. Theorem 1: Let Y be an analytic subvariety in a complex space X. If Y is strongly q-complete, then Y has a fundamental family of strongly q-complete neighbourhoods in X. Theorem 2: Let X be a complex space such that all irreducible components have dimension $\leq n$. Then: (a) X is always strongly $(n+1)$-complete.

(b) If X has no compact irreducible component of dimension n, then X is strongly n-complete. (c) If X has only finitely many irreducible components of dimension n, then X is strongly n-convex. Theorem 3: Let (M,ω) be an n-dimensional Kähler manifold. Suppose that M is absolutely q-convex, i.e. admits a smooth plurisubharmonic exhaustion function that is strongly q-convex on $M \setminus K$ for some compact set K in M. Set $\Omega^r = \mathcal{O}(\Lambda^r T^* M)$. Then the de Rham cohomology groups with arbitrary [resp. compact] support have decompositions $H^k(M,\mathbf{C}) \simeq \bigoplus H^s(M,\Omega^r)$, $H^r(M,\Omega^s) \simeq H^s(M,\Omega^r)$, $k \geq n+q$, $H^k_c(M,\mathbf{C}) \simeq \bigoplus H^s_c(M,\Omega^r)$, $H^r_c(M,\Omega^s) \simeq H^s_c(M,\Omega^r)$, $k \leq n-q$, and these groups are finite-dimensional. Moreover, there is a Lefschetz isomorphism $\omega^{n-r-s} \wedge \cdot \colon H^s_c(M,\Omega^r) \to H^{n-r}(M,\Omega^{n-s})$, $r+s \leq n-q$.

Reviewed by *Takeo Ohsawa*