MR1782699 (2001c:32002) 32-06

★**Complex analysis and geometry.**

Proceedings of the International Conference held in honor of Pierre Lelong on the occasion of his 85th birthday in Paris, September 22–26, 1997.

Edited by P. Dolbeault, A. Iordan, G. Henkin, H. Skoda and J.-M. Trépreau.

Progress in Mathematics, 188.

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Contents: Henri Skoda, Présence de l'œuvre de Pierre Lelong dans les grands thèmes de recherches d'aujourd'hui [The presence of Pierre Lelong in modern research] (1–30); Bibliographie de Pierre Lelong [Bibliography of Pierre Lelong] (31–37); Józef Siciak, Wiener's type sufficient conditions for regularity in $\mathbf{C}^N$ (39–46); Jean-Pierre Demailly, On the Ohsawa-Takegoshi-Manivel L^2 extension theorem (47–82); Giovanni Bassanelli, A geometrical application of the product of two positive currents (83–90); Michael Christ, Hypoellipticity: geometrization and speculation (91–109); Klas Diederich and Takeo Ohsawa, Moment problems for weighted Bergman kernels (111–122); Gregor Herbort, On the Bergman metric near a plurisubharmonic barrier point (123–132); Ulrich Hiller, On the vanishing order of a holomorphic germ along a complex analytic germ in $\mathbf{C}^n$ (133–142); S. M. Webster, Stationary curves and complete integrability in the complex domain (143–150); C. Laurent-Thiébaut and J. Leiterer, The Malgrange vanishing theorem with support conditions (151–162); Tien-Cuong Dinh, Mesures orthogonales à support compact de longueur finie et applications [Orthogonal measures with compact support of finite length and applications] (163–172); P. Dingoyan, Convexity and Hartogs's theorem in some open subset of a projective manifold (173–181); Klas Diederich and Jeffery D. McNeal, Pointwise nonisotropic support functions on convex domains (183–192); Emmanuel Mazzilli, Un exemple d'obstruction géométrique à l'extension des fonctions holomorphes bornées [Example of geometric obstruction to the extension of bounded holomorphic functions] (193–201); Eric Bedford and Mattias Jonsson, Potential theory in complex dynamics: regular polynomial mappings of $\mathbf{C}^k$ (203–211); Giuseppe Tomassini, Boundaries of Levi-flat hypersurfaces of $\mathbf{C}^2$ (213–219); François Berteloot and Julien Duval, Une démonstration directe de la densité des cycles répulsifs dans l'ensemble de Julia [A direct proof of the density of repulsive cycles in a Julia set] (221–222); Charles L. Epstein and Gennadi M. Henkin, Embeddings for 3-dimensional CR-manifolds (223–236); List of problems (237–241).

{Most of the papers are being reviewed individually.}

MR1782659 (2001m:32041) 32L10 (32D15 32J25 32U05)
Demailly, Jean-Pierre (F-GREN-F)
On the Ohsawa-Takegoshi-Manivel L^2 extension theorem. (English, French summaries)
Complex analysis and geometry (*Paris*, 1997), 47–82, *Progr. Math.*, 188, *Birkhäuser*, *Basel*, 2000.

This paper provides a rather deep insight into the current status of L^2 extension techniques for sections [resp. $(0,q)$-forms] of vector bundles over complex analytic submanifolds. The fundamental extension theorem of T. Ohsawa and K. Takegoshi [Math. Z. **195** (1987), no. 2, 197–204; MR0892051 (88g:32029)], refined in many ways by Ohsawa, and in a more geometric setting by L. Manivel [Math. Z. **212** (1993), no. 1, 107–122; MR1200166 (94e:32050)], is proven here in its most general form. Unfortunately, a gap in the proof of Manivel is pointed out, regarding the regularity of the extension in the case of $(0,q)$-forms when $q>0$. It thus reappears here as a conjecture, which is discussed in detail, but without being settled.

This theorem can yield powerful constructions that have been used in transcendental algebraic geometry. First, any psh function on a pseudoconvex open set in $\mathbb{C}^n$ can be approximated accurately with functions of the form $c\log|f|$ where f is a holomorphic function; this can be applied for instance to approximate the curvature current of a singular metric by divisors with multiplicities controlled by the Lelong numbers of the current. Other implications are detailed, among them a Briançon-Skoda theorem for multiplier ideal sheaves, and an analytical proof of Fujita's approximate Zariski decomposition for big line bundles.

$\{$For the entire collection see MR1782699 (2001c:32002)$\}$

Reviewed by *Thierry Bouche*

MR1782656 (2001h:32001) 32-03 (01A70 32C30 32J25 32U25)
Skoda, Henri (F-PARIS6-MI)
Présence de l'œuvre de Pierre Lelong dans les grands thèmes de recherches d'aujourd'hui. (French. English, French summaries) [The presence of the work of Pierre Lelong in the big picture of modern research]
Complex analysis and geometry (*Paris*, 1997), 1–30, *Progr. Math.*, 188, *Birkhäuser*, *Basel*, 2000.

The goal of this article is to discuss the major lines of the research of Pierre Lelong.

The author begins by outlining the first topics explored by Lelong: plurisubharmonic functions and their approximation by logarithmic functions; closed positive currents and the relation between these currents and plurisubharmonic functions; the potential associated with a closed positive current; the Lelong number of a closed positive current T and of a plurisubharmonic function. He shows that the Lelong number of a closed positive current coincides with that of the potential associated with that current. Hörmander's L^2-estimates allowed Lelong to prove Siu's theorem, namely that for all $c > 0$, the set $E_c = \{z \in \Omega\colon\ \nu_T(z) \geq c\}$ is an analytic subset for every closed positive current on a domain $\Omega \subset \mathbb{C}^n$, where $\nu_T(z)$ denotes the Lelong number of T at the point z.

In the third section Skoda shows how Lelong's methods intersect with those of the geometry of manifolds and algebraic geometry. These methods have been implemented in a clear way by Demailly, for example in his treatment of the Fujita conjecture [J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. However, one can in fact encounter Lelong's ideas in many other fields, for example transcendental number theory, the theory of potentials of several variables, etc.

In the last section the author gives a sketch of the Hodge conjecture, which has not yet been proved, and shows that the Poincaré-Lelong equation has provided partial results. (For a study of the connections between the Hodge conjecture and the theory of closed positive currents see a paper by Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)].)

{For the entire collection see MR1782699 (2001c:32002)}

Reviewed by *Mongi Blel*

MR1772670 (2001m:32042) 32L10 (14C20 32J25)

Demailly, Jean-Pierre (F-GREN-F)

Méthodes L^2 et résultats effectifs en géométrie algébrique. (French. French summary) [L^2-methods and effective results in algebraic geometry]

Séminaire Bourbaki, Vol. 1998/99.
Astérisque No. 266 (2000), *Exp. No.* 852, 3, 59–90.

This paper surveys the recent work that has been done by Demailly, Siu and Nadel among others about effective results in algebraic geometry obtained through Hörmander's L^2-methods for the $\overline{\partial}$ equation with singular metrics. A first section provides good insight into the basic tools, which are defined, and the results, whose proofs are outlined: singular metrics of holomorphic line bundles over complex analytic manifolds, the Bochner-Kodaira-Nakano identity for the antiholomorphic Laplace-Beltrami operator (in the case where the metric is smooth), L^2 estimates with singular metrics, Nadel's multiplier ideal sheaves and the corresponding vanishing theorem. Two important

applications of these techniques are then described in detail: the Fujita conjecture [see Y. T. Siu, in *Modern methods in complex analysis (Princeton, NJ, 1992)*, 291–318, Ann. of Math. Stud., 137, Princeton Univ. Press, Princeton, NJ, 1995; MR1369144 (98f:32032)] and Siu's theorem about the invariance of plurigenera under deformation [see Y. T. Siu, Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035)].

{For the entire collection see MR1772667 (2001b:00028)}

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1772667 (2001b:00028) 00B25
★**Séminaire Bourbaki. Vol. 1998/99. (French) [Bourbaki Seminar. Vol. 1998/99]**
Exposés 850–864.
Astérisque No. 266 (2000).
Société Mathématique de France, *Paris*, 2000. *pp*. *i–iv and* 1–483.

{Vol. 1997/98 has been reviewed [MR1685659 (99m:00026)].}

Contents: Jean-Yves Chemin, Explosion géométrique pour certaines équations d'ondes non linéaires (d'après Serge Alinhac) [Geometric blowup of some nonlinear wave equations (following Serge Alinhac)] (Exp. No. 850, 3, 7–20); Pierre Colmez, Fonctions L p-adiques [p-adic L-functions] (Exp. No. 851, 3, 21–58); Jean-Pierre Demailly, Méthodes L^2 et résultats effectifs en géométrie algébrique [L^2-methods and effective results in algebraic geometry] (Exp. No. 852, 3, 59–90); Tomasz Komorowski, Brownian motion in a Poisson obstacle field (Exp. No. 853, 3, 91–111); Stefano Marmi, Chaotic behaviour in the solar system (following J. Laskar) (Exp. No. 854, 3, 113–136); Michel Boileau, Uniformisation en dimension trois [Uniformization in dimension three] (Exp. No. 855, 4, 137–174); Nicolas Burq, Formules de trace, résonances et quasi-modes (d'après Sjöstrand-Zworski, Stefanov-Vodev et al.) [Trace formulas, resonances and quasimodes (following Sjöstrand-Zworski, Stefanov-Vodev et al.)] (Exp. No. 856, 4, 175–190); Henri Carayol, Preuve de la conjecture de Langlands locale pour GL_n: travaux de Harris-Taylor et Henniart [Proof of the local Langlands conjecture for GL_n: works of Harris and Taylor and of Henniart] (Exp. No. 857, 4, 191–243); Olivier Mathieu, Classification des algèbres de Lie simples [Classification of simple Lie algebras] (Exp. No. 858, 4, 245–286); Michel Talagrand, Verres de spin et optimisation combinatoire [Spin glasses and combinatorial optimization] (Exp. No. 859, 4, 287–317); Christophe Breuil, Intégration sur les variétés p-adiques (d'après Coleman, Colmez) [Integration on p-adic manifolds (following Coleman and Colmez)] (Exp. No. 860, 5, 319–350);

Robert Bryant, Recent advances in the theory of holonomy (Exp. No. 861, 5, 351–374); Yuri I. Manin [Yuriĭ Ivanovich Manin], Classical computing, quantum computing, and Shor's factoring algorithm (Exp. No. 862, 5, 375–404); Joseph Oesterlé, Densité maximale des empilements de sphères en dimension 3 (d'après Thomas C. Hales et Samuel P. Ferguson) [Maximal density of three-dimensional sphere packings (following Thomas C. Hales and Samuel P. Ferguson)] (Exp. No. 863, 5, 405–413); Jean-Pierre Serre, Sous-groupes finis des groupes de Lie [Finite subgroups of Lie groups] (Exp. No. 864, 5, 415–430).

{The papers are being reviewed individually.}

MR1775582 (2001j:14037) 14G40 (11G50 14M25)

Maillot, Vincent (F-ENS-MI)

Géométrie d'Arakelov des variétés toriques et fibrés en droites intégrables. (French. English, French summaries) [Arakelov geometry of toric varieties and integrable line bundles]

Mém. Soc. Math. Fr. (N.S.) No. 80 (2000), *vi*+129 *pp*.

In the present monograph, the author develops the arithmetic intersection theory which involves characteristic classes of line bundles with singular Hermitian metrics and applies it to smooth projective toric varieties.

An arithmetic variety X is a regular scheme which is flat and projective over **Z**. In a remarkable paper H. Gillet and C. Soulé [Inst. Hautes Études Sci. Publ. Math. No. 72 (1990), 93–174 (1991); MR1087394 (92d:14016)] constructed the arithmetic Chow groups $\widehat{\mathrm{CH}}^*(X)$ of X and their intersection product. Let X_∞ be the complex manifold associated to the generic fiber $X_{\mathbf{Q}}$ and $F_\infty\colon X_\infty \to X_\infty$ the complex conjugate of X_∞. For a vector bundle E on X with an F_∞-invariant smooth Hermitian metric h on the generic fiber, the Chern class of (E,h) is defined in $\widehat{\mathrm{CH}}^*(X)$ [H. A. Gillet and C. Soulé, Ann. of Math. (2) **131** (1990), no. 1, 163–203; MR1038362 (91m:14032a)].

In Arakelov geometry, we often need to deal with line bundles with non-smooth Hermitian metrics. But the Chern classes of these bundles cannot be defined in $\widehat{\mathrm{CH}}^*(X)$. One of the purposes of this monograph is to extend the arithmetic Chow groups so that their Chern classes can be defined.

A Hermitian line bundle (L,h) on X is called admissible if L is generated by global sections and if h is continuous, positive and uniformly approximated by positive smooth metrics. (L,h) is called integrable if it is written as a difference of two admissible line bundles. Let $\widetilde{\mathrm{CH}}^p(X)$ denote a group of pairs (Z,g) consisting of a closed subscheme $Z \subset X$ of codimension p and a

real F_∞-invariant $(p-1,p-1)$-current g, subject to rational equivalence. We note that $\widehat{\mathrm{CH}}^p(X)$ is a subgroup of $\widetilde{\mathrm{CH}}^p(X)$. For a rational section s of an integrable line bundle (L,h), the pair $(\mathrm{div}(s), -\log h(s,s))$ determines an element in $\widetilde{\mathrm{CH}}^1(X)$, which is denoted by $\widehat{c}_1(L,h)$.

When the metric h is not smooth, $-\log h(s,s)$ is no longer a Green current of logarithmic type. But for finitely many global sections s_i of admissible line bundles (L_i,h_i) and a Green current g_Z for $Z\subset X$ such that $\mathrm{div}(s_1),\cdots,\mathrm{div}(s_q)$ and Z intersect properly, the $*$-product of g_Z, $-\log h_1(s_1,s_1),\cdots,-\log h_q(s_q,s_q)$, is defined by using the theory of products of positive currents due to E. Bedford and B. A. Taylor [Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)] and J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)]. This generalized $*$-product yields the product $\alpha\widehat{c}_1(L_1,h_1)\cdots\widehat{c}_1(L_q,h_q)$ in $\widetilde{\mathrm{CH}}^p(X)$, where $\alpha\in\widehat{\mathrm{CH}}^{p-q}(X)$ and (L_i,h_i) are integrable line bundles on X. The subgroup of $\widetilde{\mathrm{CH}}^p(X)$ generated by such products as mentioned above is denoted by $\widehat{\mathrm{CH}}^p_{\mathrm{int}}(X)$ and called the generalized arithmetic Chow group of codimension p. It is obvious that $\widehat{\mathrm{CH}}^*(X)\subset\widehat{\mathrm{CH}}^*_{\mathrm{int}}(X)$. The most important property of $\widehat{\mathrm{CH}}^*_{\mathrm{int}}(X)$ is that it possesses a multiplicative structure.

The author applies this generalized arithmetic intersection theory to smooth projective toric schemes over $\mathbf{Z}$. Let Δ be a complete regular fan such that the associated toric scheme $\mathbf{P}(\Delta)$ is smooth and projective over a ground ring. $\mathbf{P}(\Delta)$ has a natural action of a torus T. When the ground ring is a field, the Chow ring of $\mathbf{P}(\Delta)$ is completely known: Let $\Delta(1)$ be the set of all 1-dimensional cones. We can associate to $\sigma\in\Delta(1)$ a T-invariant line bundle L_σ on $\mathbf{P}(\Delta)$. Then the Chow ring of $\mathbf{P}(\Delta)$ over a field is generated by the Chern classes $c_1(L_\sigma)$ for all $\sigma\in\Delta(1)$ and their relations are given in terms of the fan.

In order to seek an analogy of the above result on the arithmetic Chow ring, we have to choose a canonical Hermitian metric on any T-invariant line bundle on $\mathbf{P}(\Delta)$ and to define its arithmetic Chern class. In this paper three equivalent ways to construct the metric are introduced. Although this metric is not smooth, the T-invariant line bundle with this metric, which is denoted by $\overline{L}_\sigma$, becomes integrable. Hence its arithmetic Chern class is defined in $\widehat{\mathrm{CH}}^1_{\mathrm{int}}(\mathbf{P}(\Delta))$.

The author computes the intersection product, $\widehat{c}_1(\overline{L}_{\sigma_1})\cdots\widehat{c}_1(\overline{L}_{\sigma_q})$, of these Chern classes for $\sigma_1,\cdots,\sigma_q\in\Delta(1)$. The main result is the following: If the usual intersection $c_1(L_{\sigma_1})\cdots c_1(L_{\sigma_q})$ is zero in the Chow ring, then the arithmetic intersection $\widehat{c}_1(\overline{L}_{\sigma_1})\cdots\widehat{c}_1(\overline{L}_{\sigma_q})$ is also zero. Furthermore the author shows that the canonical heights of hypersurfaces in $\mathbf{P}(\Delta)$ over $\mathbf{Z}$ are expressed by their Mahler measures. At the end, he proves an arithmetic analogue of the Bernstein-Kushnirenko theorem.

Reviewed by *Yuichiro Takeda*

MR1768171 (2001e:32051) 32U35
Herbort, Gregor (D-WUPP)

The pluricomplex Green function on pseudoconvex domains with a smooth boundary. (English summary)

Internat. J. Math. **11** (2000), *no. 4,* 509–522.

Let D be a bounded hyperconvex domain in $\mathbf{C}^n$ and let $g(z,w)$ denote the pluricomplex Green function on D with a (single) pole at w. In the paper under review, the author studies the behaviour of g as the pole tends to a boundary point. It is well known that for a fixed pole w, $g(z,w)$ tends to zero as z tends to the boundary, but in general g is not symmetric, and in fact it is unknown whether $\lim_{w\to w_0} g(z,w)=0$ for any w_0 in the boundary of an arbitrary bounded hyperconvex domain. This question is interesting not only in itself, but also has consequences for the study of the boundary behaviour of the Bergman metric.

The main result of the paper is that if D admits a Hölder continuous bounded plurisubharmonic exhaustion function, then $\lim_{w\to w_0}\inf_{z\in K} g(z,w)=0$ for every $w_0\in\partial D$ and every compact set $K\subset D$. In particular, this statement holds if D is a bounded pseudoconvex domain with C^2 boundary.

The proof of the main result is rather technical and depends on careful upper and lower bounds of the integral $\mathcal{I}_j=\int|g(\cdot,w_j)|(dd^c\max\{g(\cdot,z_j),-\eta_j\})^n$ (where (z_j) is a sequence in some fixed compact set K and η_j is a suitable sequence of positive numbers tending to ∞). The difficult part is to get a lower bound for $\mathcal{I}_j$ in terms of g itself and a Hölder continuous plurisubharmonic exhaustion function, and this is done by adapting a construction by Demailly.

Reviewed by *Frank Wikström*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1762959 (2001f:32039) 32Q05 (14C30 32M15)
Hwang, Jun-Muk; **To, Wing-Keung** (SGP-SING)
On Seshadri constants of canonical bundles of compact quotients of bounded symmetric domains. (English summary)
J. Reine Angew. Math. **523** (2000), 173–197.

Let L be an ample line bundle over a projective manifold X. To measure the "local positivity" of L at a given point x in X, Demailly introduced the Seshadri number $\varepsilon(L, x)$. Lower bounds of these numbers yield precise results about the generation of s-jets by global sections of $K_X + L$ at x, while upper bounds also carry some local geometric information on the polarized manifold

(X, L).

The paper under review gives lower and upper bounds for $\varepsilon(L, x)$ in terms of metric invariants when L is the canonical line bundle over a smooth compact quotient of a bounded symmetric domain of $\mathbf{C}^n$, endowed with its natural Poincaré-like metric of Ricci curvature -1. The method of proof involves the construction of some singular Hermitian metric on K_X with prescribed pole order at x.

Reviewed by *Thierry Bouche*

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MR1760443 (2001i:32040) 32Q45 (32H02 32J15)

Cantat, Serge (F-ENSLY)

Deux exemples concernant une conjecture de Serge Lang. (French. English, French summaries) [Two examples related to a conjecture of Serge Lang]

C. R. Acad. Sci. Paris Sér. I Math. **330** (2000), *no. 7,* 581–586.

A compact complex analytic space X is hyperbolic if every holomorphic map from the complex plane **C** to X must be constant. M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557 (82h:32026)] conjectured that if X is a pseudo-canonical (Lang's "functorial" terminology for "general type") projective variety, then the image of any holomorphic map from **C** to X will be contained in a proper Zariski subvariety of X. Thus, S. Lang [Bull. Amer. Math. Soc. (N.S.) **14** (1986), no. 2, 159–205; MR0828820 (87h:32051)] conjectured that if X is a projective variety, then every subvariety of X, including X itself, is pseudo-canonical if and only if X is hyperbolic. Lang also conjectured that if X is a projective variety which is not pseudo-canonical, then there will be an abelian variety A and a non-constant rational map from A to X. Thus, Lang conjectured that a projective variety X is hyperbolic if and only if every rational map from every abelian variety to X must be constant. Note that the existence of subvarieties, and hence the projectivity assumption on X, is essential to Lang's reasoning sketched here. However, some authors, for example J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)(Conjecture 2.6)] and J. Winkelmann [Mém. Soc. Math. Fr. (N.S.) No. 72-73 (1998), x+219 pp.; MR1654465 (99g:32058)(Question 4.14.4)], left out the projectivity assumption and asked simply: If X is a compact complex analytic space such that every holomorphic map from a complex torus to X must be constant, then must X be hyperbolic?

The paper under review gives examples of non-projective compact complex manifolds that show that this generalization of Lang's conjecture to non-projective manifolds is false, even if one restricts oneself to compact Kähler manifolds. The author's first examples are non-projective $K3$ surfaces, which are Kähler. He shows that if X is a $K3$ surface without any projective curves,

then there are no non-constant holomorphic maps from complex tori into X. Indeed, if there were a non-constant holomorphic map from a complex torus, it would necessarily be surjective by the proper mapping theorem and the absence of one-dimensional subvarieties. This would then contradict the fact that X is simply connected. The fact that hyperbolic $K3$ surfaces form an open subset of the moduli space of all $K3$ surfaces, and the fact that Kummer surfaces are dense in that moduli space, imply that no $K3$ surface is hyperbolic. The author's second examples of non-hyperbolic surfaces with no nontrivial images of complex tori are certain non-Kähler quotients of $\mathbf{D} \times \mathbf{C}$, known as Inoue surfaces [M. Inoue, Invent. Math. **24** (1974), 269–310; MR0342734 (49 #7479)] The paper concludes with a discussion of why one cannot use a construction like Inoue's to produce higher-dimensional projective counterexamples to Lang's original conjecture.

Reviewed by *William A. Cherry*

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MR1758586 (2001j:32037) 32U40 (32C30 32U25 32W20)
Ben Messaoud, Hedi (TN-SFAXS); **ElMir, Hassine** (TN-TUNISM)
Opérateur de Monge-Ampère et tranchage des courants positifs fermés. (French)
[Monge-Ampère operator and slicing of closed positive currents]
J. Geom. Anal. **10** (2000), *no. 1,* 139–168.

For $k \le p \le n$, R a current of bidimension (p,p) in the unit polydisk Δ^n in $\mathbf{C}^n$ ($R \in \mathcal{D}'_{(p,p)}(\Delta^n)$), and $\alpha \ge 0$ a bounded, measurable function with compact support in $\mathbf{C}^k$ such that $\int_{\mathbf{C}^k} \alpha\, d\lambda_k = 1$, the slice of R at $a \in \Delta_k$, denoted by $\langle R, \pi, a\rangle_\alpha$, is the weak limit in $\mathcal{D}'_{(p-k,p-k)}(\Delta^n)$ of

$$R \wedge \pi^*\Big(\frac{1}{\varepsilon^{2k}}\alpha\Big(\frac{z'-a}{\varepsilon}\Big) \cdot \frac{1}{4^k k!}(dd^c|z'|^2)^k\Big)$$

as $\varepsilon \to 0$, provided this limit exists. Here, $z = (z', z'') \in \Delta^k \times \Delta^{n-k}$ and $\pi(z) = z'$ (warning: in the second paragraph of the introduction, where this definition is given, "ε" is mistakenly written "e"). If α is the (normalized) characteristic function of the unit ball in $\mathbf{C}^k$, this agrees with the definition of Federer; if α is a smooth, compactly supported function, this agrees with the notion of R. Harvey and B. Shiffman [Ann. of Math. (2) **99** (1974), 553–587; MR0355095 (50 #7572)] (we remark that Federer, as well as Harvey-Shiffman, allows the projection π to be replaced by an arbitrary C^∞-map of an open set in $\mathbf{C}^n = \mathbf{R}^{2n}$ to $\mathbf{C}^k = \mathbf{R}^{2k}$).

When $R = F + dG$ where F and G have locally integrable coefficients (i.e., R is locally flat), it is well-known that the slice of R exists over each point $a \in \Delta_k$ outside of a set of $2k$-dimensional Lebesgue measure zero. Let T be a positive, closed current of bidimension (p,p) in a neighborhood of the closed unit polydisk $\overline{\Delta^n}$ in $\mathbf{C}^n$. The main result of the paper is Theorem 1.2: there exists a pluripolar set $E \subset \Delta^k$, independent of α, such that for all $a \in \Delta^k \setminus E$, the slice $\langle T, \pi, a\rangle_\alpha$ exists and is independent of α; moreover, for a smooth, compactly supported $(p-k,p-k)$-form φ on Δ^n and a locally bounded plurisubharmonic function v in Δ^k,

$$\int_{\Delta^n} T \wedge (dd^c(v\circ\pi))^k \wedge \varphi = \int_{a\in\Delta^k} \langle T, \pi, a\rangle(\varphi)(dd^c v)^k. \tag{1}$$

The proof uses a special regularization procedure: one considers the regularized currents $T * \alpha_j$, where $\alpha_j(|z|) := j^{2n}\alpha(j|z|)$, and then studies the weak convergence of the sequence $\{(T*\alpha_j)\wedge(dd^c v_j)^k\}_j$ where $\{v_j\}_j$ are plurisubharmonic (psh) functions decreasing to a psh function v whose unbounded locus $L(v)$ avoids supp T in a local sense (Theorem 1.3). A key tool in proving Theorem 1.3, developed in Section 2, is the potential $U = U(\eta, T) = U(\Omega, T)$ associated to a positive, closed current T of bidimension (p,p) in an open set $\Omega_1 \subset \mathbf{C}^n$. For $\Omega \subset\subset \Omega_1$ and $\eta \in \mathcal{D}(\Omega_1)$, $0 \le \eta \le 1$ with $\eta \equiv 1$ on a neighborhood of $\overline{\Omega}$, U is the negative current of

bidimension $(p+1,p+1)$ in $\mathbf{C}^n$ defined by

$$U(z) := \frac{-1}{(n-1)(4\pi)^n} \int_{x\in\mathbf{C}^n} \eta(x)T(x) \wedge \frac{(dd^c(|z-x|^2))^{n-1}}{|z-x|^{2n-2}}.$$

Utilizing techniques developed by J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)], the authors obtain weak convergence of the sequence $\{(T*\alpha_j)\wedge(dd^cv_j)^k\}_j$.

Section 3 begins the discussion on slicing. After the definition and basic properties are given, results on slicing the potential U are proved; it is shown that slices of U exist outside a pluripolar set and (1) holds for U; from this, one deduces the analogous results on the positive, closed current T. Section 4 includes applications of the slicing results; for example, modifications, using (1), are indicated which give a simplification of the proof from [H. Ben Messaoud and H. El Mir, C. R. Acad. Sci. Paris Sér. I Math. **316** (1993), no. 11, 1173–1176; MR1221644 (94e:32020)] of the main result in that paper: if A is a closed, complete pluripolar set in the unit polydisk Δ^n (there exists u psh in Δ^n with $A=\{z\in\Delta^n\colon\ u(z)=-\infty\}$), and if T is a positive, closed current in $\Delta^n\setminus A$ of bidimension (p,p) such that T has finite mass in $\{(z',z'')\in\Delta^n\colon\ r<|z''|\}$, some $r<1$, and $\langle T,\pi,z'\rangle$ exists and has finite mass for every z' in a nonpluripolar subset F of Δ^k, then the trivial extension of T by zero on A is a closed, positive current. At the end of this section, a pair of interesting results are proved. First, a nice sufficient condition for the existence of the slice $\langle T,\pi,a\rangle_\alpha$ at $a\in\Delta_k$ is provided in Theorem 4.4: the function $x\to h_k(a-x')$, where $h_k(x)$ is the standard Newtonian kernel in $\mathbf{C}^k=\mathbf{R}^{2k}$, should be in $L^1_{\mathrm{loc}}(\Delta^n,\sigma'_T)$, where

$$\sigma'_T = T\wedge(dd^c|z'|^2)^{k-1}T\wedge(dd^c|z''|^2)^{p-k+1}$$

(the trace measure of $T\wedge(dd^c|z'|^2)^{k-1}$). Then it is shown that for any pluripolar set $E\subset\Delta^k$, there exists a positive, closed current T of bidimension (p,p) in Δ^n such that the slice of T at $a\in E$ does not exist. Finally, in Section 5, it is shown that the Lelong number is preserved under slicing, outside of an exceptional pluripolar set.

Reviewed by *Norman Levenberg*

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MR1759887 (2001f:32045) 32Q45 (14J29 14J70)
Demailly, Jean-Pierre (F-GREN-F); **El Goul, Jawher** (F-TOUL3)
Hyperbolicity of generic surfaces of high degree in projective 3-space. (English summary)
Amer. J. Math. **122** (2000), *no. 3,* 515–546.

S. Kobayashi [*Hyperbolic manifolds and holomorphic mappings*, Dekker, New York, 1970; MR0277770 (43 #3503)] conjectured that a generic hypersurface of dimension n in the projective space $\mathbf{P}^{n+1}$ is hyperbolic, i.e., every holomorphic map from the affine complex line $\mathbf{C}$ into such a hypersurface is constant. In the paper under review the authors verify the above conjecture for a very generic surface in $\mathbf{P}^3$ of degree $d \geq 21$ (i.e., away from a possible countable union of subvarieties in the moduli space of surfaces of degree d). More precisely, the surfaces for which the claim holds are of general type, have Picard number 1 and their Chern classes satisfy certain inequalities. The methods and techniques developed and used in the paper might be of independent interest but they are far too elaborate to be discussed here.. For the purpose of this review we outline briefly the key ideas of the proof which goes as follows. Using the Riemann-Roch theorem one produces a branched covering Z of X living in the projectivized tangent bundle

of X. If $f\colon \mathbf{C} \longrightarrow X$ is a non-constant holomorphic map, then its first differential extends to a holomorphic map whose image is contained in a leaf of an algebraic foliation on Z. By a recent argument of M. McQuillan [Inst. Hautes Études Sci. Publ. Math. No. 87 (1998), 121–174; MR1659270 (99m:32028)], the resulting curve must be algebraically degenerate, i.e., contained in a proper algebraic subvariety of Z. In order to apply this result one is in fact forced to consider 2-jets. Then the closure of the image of f is either a rational or an elliptic curve. On the other hand, by a result of H. Clemens [Ann. Sci. École Norm. Sup. (4) **19** (1986), no. 4, 629–636; MR0875091 (88c:14037)], a generic surface of degree at least 7 in the projective space contains no rational or elliptic curves which implies that f is in fact constant.

Similar results in a more general context (implying, in particular, the Kobayashi conjecture for generic surfaces of degree at least 36 in $\mathbf{P}^3$) were obtained recently in [M. McQuillan, Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)].

Reviewed by *Tomasz Szemberg*

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MR1453597 (98f:32024)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1750448 (2001b:32006) 32A26 (32E10)
Zhan, Hui Rong (PRC-XIAM); **Yao, Zong Yuan** (PRC-XIAM)
A generalization of the Koppelman formula for differential forms of type (p, q) on Stein manifolds. (Chinese. English, Chinese summaries)
Xiamen Daxue Xuebao Ziran Kexue Ban **39** (2000), *no. 2,* 147–151.

Summary: "(p, q)-type differential forms on Stein manifolds cannot adopt the Euclidean metric as they can in $\mathbf{C}^n$ because the Euclidean metric on a Stein manifold is not invariant under holomorphic transformations. This article adopts Demailly and Laurent-Thiebaut's methods to solve the problem of the invariant metric by using a Hermitian metric and the Chern connection. A generalization of the Koppelman formula for differential forms of (p, q)-type on Stein manifolds is obtained by introducing a chosen parameter m, a natural number which is greater than or equal

to 2. When m is 2, the formula is just the original Koppelman formula for differential forms of (p,q)-type on Stein manifolds. When m is equal to $3,4,\cdots,N$ $(N<+\infty)$, respectively, a series of Koppelman formulas with different forms can be given."

MR1741779 (2001h:32034) 32M12 (53C56)
Huckleberry, Alan T. (D-BCHMM); **Kebekus, Stefan** (D-BAYR-M8);
Peternell, Thomas (D-BAYR-M1)
Group actions on S^6 and complex structures on $\mathbf{P}_3$.
Duke Math. J. **102** (2000), *no. 1,* 101–124.

It has been known for a long time that the 6-sphere S^6 admits almost complex structures, while the other spheres S^{2n} for $n>1$ have no almost complex structures. But it is not known whether any of these almost complex structures on S^6 are integrable.

In this paper the authors assume that X is S^6 with a complex structure. Under this assumption, it is then proved that X is not an almost homogeneous manifold, i.e., that the group of holomorphic automorphisms (which is a complex Lie group) does not have an open orbit on X.

The proof roughly goes as follows. By a result of F. Campana, J.-P. Demailly and T. Peternell [Compositio Math. **112** (1998), no. 1, 77–91; MR1622747 (99e:32047)] the manifold X does not have any nonconstant meromorphic functions. As a consequence, if X contained an open orbit of its automorphism group it would necessarily be of the form G/Γ, where G is a 3-dimensional complex Lie group and Γ is a discrete subgroup of G. Hence G would be either semisimple or solvable. The first case is easily eliminated. The second is more complicated and proceeds by elimination of the various 3-dimensional solvable complex Lie groups.

This paper also contains some observations about complex structures on $\mathbf{P}_3$ obtained by assuming that S^6 has a complex structure and blowing up a point.

Reviewed by *B. Gilligan*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1748605 (2002e:32046) 32U40 (32C30 32C37 32F10 53C60)

Demailly, Jean-Pierre (F-GREN-F)

Pseudoconvex-concave duality and regularization of currents. (English summary)

Several complex variables (*Berkeley, CA*, 1995–1996), 233–271, *Math. Sci. Res. Inst. Publ.*, 37, *Cambridge Univ. Press*, *Cambridge*, 1999.

The goal of this paper is to investigate some duality properties connecting pseudoconvexity and pseudoconcavity in a certain perspective to obtain a geometric version of the Serre duality theorem. These duality properties are related to several geometric problems, such as the conjecture of Hartshorne asserting that the complement of a q-codimensional algebraic subvariety Y with ample normal bundle N_Y in a projective algebraic variety X is q-convex in the sense of Andreotti-Grauert. M. Schneider proved the conjecture in the case that the normal bundle is positive in the sense of Griffiths. Using Sommese's result, the author proves the conjecture in the case that N_Y^* has a strictly convex plurisubharmonic Finsler metric.

Let X be a complex manifold of dimension n and E a holomorphic vector bundle of rank r. Demailly treats the problem of approximation of closed positive $(1,1)$-currents and the attenuation of their singularities. In general a closed positive current T cannot be approximated in the weak topology by smooth closed positive currents. J.-P. Demailly [Ann. Sci. École Norm. Sup. (4) **15** (1982), no. 3, 457–511; MR0690650 (85d:32057); J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015); in *Contributions to complex analysis and analytic geometry*, 105–126, Vieweg, Braunschweig, 1994; MR1319346 (96k:32012)] proved that this approximation is possible if we allow the regularization T_ε to have a small negative part. The main point is to control the negative part in terms of the global geometry of the ambient geometry X. It turns out that more or less optimal bounds can be described in terms of the convexity of a Finsler metric on the tangent bundle T_X. The author gives an easy proof based on the use of symmetric products of Finsler metrics.

{For the entire collection see MR1748597 (2000k:32002)}

Reviewed by *Mongi Blel*

MR1748597 (2000k:32002) 32-06

★**Several complex variables.**

Papers from the MSRI Program held in Berkeley, CA, 1995–1996.

Edited by Michael Schneider and Yum-Tong Siu.

Mathematical Sciences Research Institute Publications, 37.

Cambridge University Press, Cambridge, 1999. *xii*+564 *pp*. $59.95. *ISBN* 0-521-77086-6

Contents: M. Salah Baouendi and Linda Preiss Rothschild, Local holomorphic equivalence of real analytic submanifolds in $\mathbf{C}^N$ (1–24); Daniel Barlet, How to use the cycle space in complex geometry (25–42); Edward Bierstone and Pierre D. Milman, Resolution of singularities (43–78); Harold P. Boas and Emil J. Straube, Global regularity of the $\overline{\partial}$-Neumann problem: a survey of the L^2-Sobolev theory (79–111); Frédéric Campana and Thomas Peternell, Recent developments in the classification theory of compact Kähler manifolds (113–159); Michael Christ, Remarks on global irregularity in the $\overline{\partial}$-Neumann problem (161–198); John P. D'Angelo and Joseph J. Kohn, Subelliptic estimates and finite type (199–232); Jean-Pierre Demailly, Pseudoconvex-concave duality and regularization of currents (233–271); John Erik Fornæss and Nessim Sibony, Complex dynamics in higher dimension (273–296); John Erik Fornæss and Brendan Weickert, Attractors in $\mathbf{P}^2$ (297–307); Peter Heinzner and Alan Huckleberry, Analytic Hilbert quotients (309–349); Jun-Muk Hwang and Ngaiming Mok, Varieties of minimal rational tangents on uniruled projective manifolds (351–389); Christian Okonek and Andrei Teleman, Recent developments in Seiberg-Witten theory and complex geometry (391–428); Yum-Tong Siu, Recent techniques in hyperbolicity problems (429–508); Domingo Toledo, Rigidity theorems in Kähler geometry and fundamental groups of varieties (509–533); Paul Vojta, Nevanlinna theory and Diophantine approximation (535–564).

{The papers are being reviewed individually.}

MR1724404 (2000i:32057) 32U25 (32C30)

Favre, Charles (S-RIT)

Note on pull-back and Lelong number of currents. (English, French summaries)

Bull. Soc. Math. France **127** (1999), *no. 3,* 445–458.

The main result of this paper is the following: If $f\colon (\mathbf{C}^m,0)\to(\mathbf{C}^n,0)$ is a holomorphic map of maximal rank equal to n, and T a positive closed current of bidegree (1,1), then the pull-back f^*T is well defined and there exists a constant $C>0$ (depending only on f) such that one has the inequalities $\nu(T,0)\le\nu(f^*T,0)\le C\cdot\nu(T,0)$. The author proves a semilocal version of the main result and gives some relations of his result with the results of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)], M. Meo

[C. R. Acad. Sci. Paris Sér. I Math. **322** (1996), no. 12, 1141–1144; MR1396655 (97d:32013)] and C. O. Kiselman ["Le nombre de Lelong des images inverses des fonctions plurisousharmoniques" Bull. Sci. Math., to appear].

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1722815 (2000i:32066) 32W20

Xing, Yang (S-UMEA)

Complex Monge-Ampère equations with a countable number of singular points. (English summary)

Indiana Univ. Math. J. **48** (1999), *no. 2,* 749–765.

As proven by J.-P. Demailly [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)], one can define $(dd^c u)^n$ if u is a plurisubharmonic function such that the set where u is not locally

bounded is relatively compact. In the paper under review the author proves that under certain technical assumptions on a nonnegative Borel measure μ (including the assumption that μ has at most countably many singular points) there exists u with $(dd^c u)^n = \mu$.

Reviewed by *Zbigniew Błocki*

MR1714825 (2000i:14008) 14C20 (14J10)
Küchle, Oliver (D-BAYR-IM); **Steffens, Andreas** (D-BAYR-IM)
Bounds for Seshadri constants.
New trends in algebraic geometry (*Warwick*, 1996), 235–254, *London Math. Soc. Lecture Note Ser.*, 264, *Cambridge Univ. Press*, *Cambridge*, 1999.

In recent years there has been considerable interest in understanding the local positivity of ample line bundles on algebraic varieties. Seshadri constants, introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)], emerged as a natural measure of the local positivity of a line bundle. These numbers are very hard to control and their exact value is known only in very few cases. Therefore, given a polarized variety (X, L) of dimension n, it is interesting to ask for bounds for the Seshadri constant $\varepsilon(L, x)$ at a point $x \in X$. Whereas there is a universal upper bound $\varepsilon(L, x) \leq (L^n)^{1/n}$, the non-existence of a universal lower bound follows from examples given by Miranda. On the other hand, L. M. H. Ein and R. K. Lazarsfeld [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)] showed that if X is a surface then $\varepsilon(L, x) \geq 1$ for all but countably many points $x \in X$ (in fact, all but finitely many provided $L^2 > 1$). It is conjectured that the bound $\varepsilon(L, x) \geq 1$ is valid in any dimension, at least for $x \in X$ very general i.e. away from a countable union of proper subvarieties. A weaker result $\varepsilon(L, x) \geq 1/n$ for x very general was shown by Ein, Küchle and Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)].

In the paper under review the authors refine the study of lower bounds for Seshadri constants. The strategy consists of finding via the Riemann-Roch theorem an effective divisor in $|kL|$, a high multiple of L, having large multiplicity at the given point x. The procedure splits then according to whether the singularity at x is isolated or not, the second case imposing existence of a subvariety on X with unusual low degree with respect to L. The bounds obtained look technically involved but they can be flexibly adjusted to a concrete situation at hand.

The methods presented in the paper combined with the effective very ampleness results due to U. Angehrn and Y. T. Siu [Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)] allow the authors to give bounds valid at arbitrary points of X. These bounds depend of course on the geometry of X.

{For the entire collection see MR1714817 (2000e:14001)}

Reviewed by *Tomasz Szemberg*

MR1713311 (2000i:32034) 32J25 (14C20 32M15 32Q45)
Hwang, Jun-Muk (KR-SNU); **To, Wing-Keung** (SGP-SING)
On Seshadri constants of canonical bundles of compact complex hyperbolic spaces. (English summary)
Compositio Math. **118** (1999), *no. 2,* 203–215.

Seshadri constants were introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)] as a way to measure the local positivity of an ample line bundle. They emerged first in connection with problems revolving around Fujita's conjecture concerning global generation and very ampleness of adjoint line bundles. In the course of time they have constantly gained more and more interest in their own right [L. M. H. Ein, O. Küchle and R. K. Lazarsfeld, J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007); T. Bauer, Math. Ann. **313** (1999), no. 3, 547–583; MR1678549 (2000d:14006)]. Since these numbers are very hard to control and their exact values are known only in very few cases, it is interesting, given a polarized variety (X, L) of dimension n, to ask for bounds for the Seshadri constant $\varepsilon(L, x)$ at a point $x \in X$. Kleiman's nefness criterion provides a universal upper bound $\varepsilon(L, x) \leq (L^n)^{1/n}$. If the actual value of $\varepsilon(L, x)$ is strictly lower than the upper bound, it has strong geometric consequences for X. On the other hand, examples due to Miranda show there is no universal lower bound. However Ein, Küchle and Lazarsfeld [op. cit.] showed that $1/n$ is such a bound if x is sufficiently general. It is conjectured that an ample line bundle behaves as a very ample line bundle at a very general point (i.e. away from a countable union of proper subvarieties); in particular, the actual lower bound at such points is expected to be 1. This conjecture was proved for surfaces by Ein and Lazarsfeld [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)].

For all the above reasons it is interesting and important to study bounds for Seshadri constants for specific classes of polarized varieties. In the article under review the authors provide lower and upper bounds for Seshadri constants of the canonical bundle on compact quotients of the unit ball in $\mathbf{C}^n$. These bounds are expressed in terms of the Poincaré metric invariants. The proof for the lower bound builds upon ideas of Lazarsfeld [Math. Res. Lett. **3** (1996), no. 4, 439–447; MR1406008 (98e:14044)] applied originally to abelian varieties. The proof for the upper bound is more involved. It relies on properties of plurisubharmonic functions which are shown in the second half of the paper.

Reviewed by *Tomasz Szemberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1704301 (2000i:32021) 32F45 (32F17 32U10)
Chen, Bo-Yong (PRC-FUDAN-IM)

Completeness of the Bergman metric on non-smooth pseudoconvex domains. (English summary)

Ann. Polon. Math. **71** (1999), *no. 3,* 241–251.

The author proves that if Ω is either a bounded pseudoconvex domain with Lipschitz boundary in $\mathbf{C}^n$ or a bounded regular domain in $\mathbf{C}$ then it is complete with respect to the Bergman metric. He also gives an example of a bounded domain in $\mathbf{C}$ which is Bergman complete but not regular. As proven by J.-P. Demailly [Math. Z. **194** (1987), no. 4, 519–564; MR0881709 (88g:32034)], pseudoconvex domains with Lipschitz boundary in $\mathbf{C}^n$ are hyperconvex (that is, they admit a bounded plurisubharmonic exhaustion function), whereas regularity of domains in $\mathbf{C}$ is equivalent to hyperconvexity. It has been recently shown by P. Pflug and the reviewer [Nagoya Math. J. **151** (1998), 221–225; MR1650305 (2000b:32065)] and, independently, by G. Herbort [Math. Z. **232** (1999), no. 1, 183–196; MR1714284 (2000i:32020); see the preceding review], that all hyperconvex domains are Bergman complete. In the first of these works the article under review

was used, whereas Herbort did not know about it.

Reviewed by *Zbigniew Błocki*

MR1696762 (2000m:14007) 14C20 (14E25 14M25)
Di Rocco, Sandra (S-RIT)
Generation of k-jets on toric varieties. (English summary)
Math. Z. **231** (1999), *no. 1,* 169–188.

In recent years, there has been interest in understanding higher order embeddings of algebraic varieties. Several notions of higher order embeddings were introduced [M. C. Beltrametti and A. J. Sommese, in *Problems in the theory of surfaces and their classification (Cortona, 1988)*, 33–48, Sympos. Math., XXXII, Academic Press, London, 1991; MR1273371 (95d:14005); in *Complex analysis and geometry*, 355–376, Plenum, New York, 1993; MR1211891 (94g:14006)], two of which—k-very ampleness and k-jet ampleness—attracted quite a lot of attention in the past decade. Whereas by now well understood in the case of surfaces, these notions remain mostly unexplored for polarized varieties of arbitrary dimension. The paper under review contributes towards understanding higher order embeddings of the broad class of varieties: toric varieties. The author shows that for toric varieties both notions mentioned above are equivalent and that they are equivalent to higher convexity for a Δ-support function as defined by the author. This is a nice generalization of strict convexity introduced by Demazure and Oda, which in turn is known to be equivalent to very ampleness of the line bundle in question.

In the last part of the paper the author shows that local and global positivity of a line bundle on a toric variety are closely related. In particular, she shows (Corollary 6.5) that if the Seshadri constant [see J.-P. Demailly, in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)] of a line bundle at a point is at least k then the line bundle is k-jet ample, the converse being true for arbitrary varieties.

Reviewed by *Tomasz Szemberg*

MR1688140 (2000d:32034) 32J27 (32C30 32J15 32Q15)

Lamari, Ahcène

Courants kählériens et surfaces compactes. (French. English, French summaries) [Kähler currents and compact surfaces]

Ann. Inst. Fourier (Grenoble) **49** (1999), *no. 1*, *vii*, *x*, 263–285.

It has been known for a while that every compact complex surface with even first Betti number is Kähler. The classical proof (completed in 1983 by Y. T. Siu's paper [Invent. Math. **73** (1983), no. 1, 139–150; MR0707352 (84j:32036)]) relies on the Kodaira classification, and a case by case examination. The paper under review provides a relatively short and self-contained unified proof of this fact. The strategy, inspired by Harvey-Lawson's work on intrinsic characterization of Kähler manifolds through currents, is to show the existence of a "Kähler current" (closed positive $(1,1)$-current bounded below by a Hermitian metric), which in turn provides a smooth Kähler metric in codimension 2. These constructions are not limited to the surface case; they are interesting in themselves. The main tool here is the regularization theorem of J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)].

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1678549 (2000d:14006) 14C20 (14C21 14E25 14K05)
Bauer, Thomas (D-ERL-MI)
Seshadri constants on algebraic surfaces.
Math. Ann. **313** (1999), *no. 3,* 547–583.

Seshadri constants are invariants introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)]. They encode information on the local positivity of an ample line bundle. The name originates in the Seshadri criterion for ampleness [R. Hartshorne, *Ample subvarieties of algebraic varieties*, Springer, Berlin, 1970; MR0282977 (44 #211)].

Given a smooth variety X of dimension n, a line bundle L on X and a point $x \in X$ we denote by $\varepsilon(L,x)$ the Seshadri constant of L at x. This is the biggest number ε such that the $\mathbf{R}$-line bundle $f^*L - \varepsilon E$ is nef, where f is the blowup of X at x with exceptional divisor E. It is natural to ask what restrictions, if any, can be imposed on that quantity. An upper bound $\varepsilon(L,x) \leq (L^n)^{1/n}$ follows easily from S. L. Kleiman's criterion [Ann. of Math. (2) **84** (1966), 293–344; MR0206009 (34 #5834)]. On the other hand, Miranda gave examples which show that there is no general lower bound in any dimension. Somehow surprisingly, in this context L. M. H. Ein, O. Küchle and R. K. Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)] showed that $\varepsilon(L,x) \geq 1/n$ at a very general point $x \in X$, i.e. outside a countable union of divisors on X. It is conjectured that the actual bound can be improved to $\varepsilon(L,x) \geq 1$, some evidence being provided by earlier work of Ein and Lazarsfeld on surfaces [Astérisque No. 218 (1993), 177–186; MR1265313 (95f:14031)].

The bound $\varepsilon(L,x) \geq 1$ is obvious if X is an abelian variety (and it does not depend on x, as X is homogeneous). M. Nakamaye [Amer. J. Math. **118** (1996), no. 3, 621–635; MR1393263 (97k:14005)] observed that the equality has strong geometric implications, namely X splits as a product of an elliptic curve and an abelian variety of dimension $n-1$. Proceeding along these lines, Lazarsfeld [Math. Res. Lett. **3** (1996), no. 4, 439–447; MR1406008 (98e:14044)] and Bauer [Math. Ann. **312** (1998), no. 4, 607–623; MR1660259 (2000a:14054)] showed that Jacobians, respectively Prym varieties, have small Seshadri constants among principally polarized abelian varieties. This is equivalent to saying that they have a period of unusually short length, as explained by Lazarsfeld, building upon results of P. Buser and P. C. Sarnak [Invent. Math. **117** (1994), no. 1, 27–56; MR1269424 (95i:22018)].

Seshadri constants are very hard to compute in general. They are not known even in the seemingly easy case of surfaces $X \subset \mathbf{P}^3$ of degree $d \geq 5$ (see Bauer [Math. Ann. **309** (1997), no. 3, 475–481; MR1474202 (98i:14009)] for $d \leq 4$). In the paper under review the author restricts his attention to algebraic surfaces and proves a long list of interesting results in this setup.

The paper consists of eight sections. The first one has an introductory character. Every other section could be viewed as a paper on its own.

In the second section a list of possible values for the Seshadri constant of a smooth surface in $\mathbf{P}^3$ is given. The author shows that there are only a few choices for small Seshadri numbers and that they are related to the global geometry of the surface.

In the next section the author gives a lower bound for the Seshadri constant of an ample line bundle L in terms of the canonical slope of L, which is defined as the minimal real number

$\sigma = \sigma(L)$ such that $\sigma L - K_X$ is nef. Furthermore, Miranda's examples, originally constructed on rational surfaces, are generalized to arbitrary surfaces.

The next two sections contain a bound on the degree of curves which cause $\varepsilon(L,x)$ to be small at a very general point $x \in X$. This bound is used to give a quick proof of Nakamaye's result mentioned above in the case of abelian surfaces. The author also discusses the question of how many curves can cause $\varepsilon(L,x)$ to be sub-maximal, i.e. $\varepsilon(L,x) < \sqrt{L^2}$.

The last three sections deal with different aspects of Seshadri constants on abelian surfaces. If (X,L) is a polarized abelian surface with Picard number $\rho(X) = 1$ then the author in fact computes the Seshadri constant of L. Abelian surfaces thus constitute the first nontrivial class of algebraic varieties for which Seshadri constants have been computed. The numbers in effect were conjectured in the previous joint work of the author and the reviewer [T. Bauer, op. cit., 1998 (Appendix)]. The next result presented here is a detailed description of the nef cone of an abelian surface with arbitrary Picard number (it is well known that $1 \le \rho(X) \le 4$). Finally, multiple point Seshadri constants are introduced. These invariants are even harder to control in general; it suffices to say that their computation for $\mathbf{P}^2$ is equivalent to the unsolved Nagata conjecture [M. Nagata, Chinese J. Math. **11** (1983), no. 1, 1–4; MR0692988 (84f:14008)]. In the case of abelian surfaces the author gives interesting lower bounds for multiple point Seshadri constants valid in any points. Previous results along these lines proved by Küchle [Ann. Inst. Fourier (Grenoble) **46** (1996), no. 1, 63–71; MR1385510 (97d:14010)] are valid only for very general points.

One of the upshots of the paper under review is that Seshadri constants of abelian surfaces are rational numbers. Although there is not much other evidence, it is tempting to finish with a conjecture, not addressed directly in the paper, that Seshadri constants are always rational.

Reviewed by *Tomasz Szemberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1678537 (2000b:32017) 32C30 (32E20 32U25)
Guedj, Vincent (F-PARIS11)
Approximation of currents on complex manifolds.
Math. Ann. **313** (1999), *no. 3,* 437–474.

The purpose of this work is the approximation of positive closed currents of bidegree $(1,1)$ on complex projective algebraic manifolds by rational divisors. For X a pseudoconvex domain of $\mathbf{C}^n$ and $H^2(X,\mathbf{R})=0$, in 1972 P. Lelong proved this kind of approximation. In 1982 J.-P. Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)] generalized this result to the case where X is a Stein or projective algebraic manifold modulo some cohomological assumption, and gave a control of the Lelong numbers of the approximation. In 1995 J. Duval and N. Sibony [Duke Math. J. **79** (1995), no. 2, 487–513; MR1344768 (96f:32016)] showed that one can approximate a $(1,1)$ positive current T by rational divisors whose support converges to the support of T in the Hausdorff metric.

The paper under review can be seen as a combination of the result of Demailly, and that of Duval and Sibony. The first main result of this work is the following: Every positive closed current T of bidegree $(1,1)$ on the projective space $\mathbf{C}\mathrm{P}^n$ [resp. the Grassmann manifold $G_{k,m}(\mathbf{C})$ of k-planes of $\mathbf{C}^m$, resp. the hyperquadric $Q_m(\mathbf{C})$ for $m\geq 4$] can be weakly approximated by rational divisors whose support converges to $\operatorname{Supp} T$.

In the second section the author generalizes the notion of rational convexity and gives a generalization of the result of Duval and Sibony. In the last section the author gives the second main

result on the approximation of closed positive currents. He proves the following: Let T be a positive closed current of bidegree $(1,1)$ on a projective algebraic manifold X. Let $\lambda > 0$ be such that $[\lambda T] = c_1(L)$ for some holomorphic line bundle L which we assume is positive. Assume $T = [H] + R$, where $H = \sum_{j=1}^{p} \lambda_j [Z_j]$, where Z_j is an irreducible algebraic hypersurface of X and R is a positive closed current of bidegree $(1,1)$ on X such that the level sets of the Lelong numbers of R are of codimension greater than 2. If T satisfies some condition of convexity ($\forall K \subset\subset X \smallsetminus \mathrm{Supp}\, T$, $\widehat{K}^T \subset\subset X \smallsetminus \mathrm{Supp} T$) then we can approximate T by rational divisors with control of the Lelong numbers of the approximation.

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1676295 (2000e:14057) 14J25 (14H45 32Q45)
Chiantini, Luca (I-SIN); **Lopez, Angelo Felice** (I-ROME3)
Focal loci of families and the genus of curves on surfaces. (English summary)
Proc. Amer. Math. Soc. **127** (1999), *no. 12,* 3451–3459.

In the context of the problem of characterizing which projective algebraic varieties over the complex field are hyperbolic and in view of the Kobayashi-Lang conjecture, the authors pay attention to the intermediate concept of algebraically hyperbolic varieties and their properties [J.-P. Demailly, in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] where the problem is connected with the study of the geometric genus of curves in the varieties.

They obtain several results concerning the genus of curves in general surfaces of $\mathbf{P}^3$ that let them conclude the algebraic hyperbolicity of these surfaces when certain conditions are satisfied. The key point is that they apply the classical theory of focal loci, recently rephrased in modern terms by C. Ciliberto and E. Sernesi [J. Algebraic Geom. **1** (1992), no. 2, 231–250; MR1144438 (92j:14034)] for this purpose.

In this way, they are able to give a short proof of one of the main theorems of G. Xu [J. Differential Geom. **39** (1994), no. 1, 139–172; MR1258918 (95d:14043)] by translating Xu's local analysis with a global property of the focal locus of a family of curves.

With the same method, they also study surfaces in $\mathbf{P}^3$ that are general in a given component of

the Noether-Lefschetz locus and finally they obtain the algebraic hyperbolicity of some general projectively Cohen-Macaulay surfaces in $\mathbf{P}^4$.

Reviewed by *Raquel Mallavibarrena*

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MR1658236 (99k:14030) 14E20 (14C30 14F17 32L20)
Takayama, Shigeharu (J-OSAKEGS)
Nonvanishing theorems on an algebraic variety with large fundamental group.
J. Algebraic Geom. **8** (1999), *no. 1,* 181–195.

Let X be a smooth projective manifold over **C** with infinite fundamental group. J. Kollár's Shafarevich map [Invent. Math. **113** (1993), no. 1, 177–215; MR1223229 (94m:14018)] is an important tool for analyzing the influence of $\pi_1(X)$ on the algebro-geometric properties of X. F. Campana [Bull. Soc. Math. France **122** (1994), no. 2, 255–284; MR1273904 (95f:32036)] gave an independent construction also valid in the Kähler case. Kollár [*Shafarevich maps and automorphic forms*, Princeton Univ. Press, Princeton, NJ, 1995; MR1341589 (96i:14016)(1.8)] refined this construction and defined a quasi-fibration $S: X \to Y$ whose general fiber F is a subvariety of X with finite fundamental group which is maximal among such subvarieties.

This article gives a partial solution to Conjecture (18.9.1) in Kollár's book [op. cit., 1995]. Its main theorem states that, given a Cartier divisor L on X satisfying $h^0(F, K_F + L) \neq 0$ and $L \equiv M + \Delta$, where M is a nef and big **Q**-divisor and (X, Δ) is Kawamata log terminal, then $h^0(X, K_X + L) \neq 0$.

Here is a sketch of the proof. Let $\pi: \tilde{X} \to X$ be the universal covering space of X. Atiyah's L_2-index theorem and the Demailly-Nadel version of the Kawamata-Viehweg vanishing theorem reduces the problem to proving that $\pi^*(K_X + L)$ has a nonzero square-integrable holomorphic section. Let Φ be a generic fiber of the Shafarevich map. Assume, inductively, that $h^0(\Phi, K_\Phi + L) \neq 0$. Then, for all $s > 0$, $\pi^* L$ has a singular Hermitian metric with an isolated pole along Φ of order $\geq s$. Existence of the sought for nonzero L_2 holomorphic section then follows from the Demailly-Nadel theorem.

A slight variant of this theorem (with F replaced by Φ) has been obtained independently by the reviewer using similar arguments [Ann. Inst. Fourier (Grenoble) **49** (1999), no. 1, vi, ix–x,

141–176; MR1688152 (2000d:32036)].

Reviewed by *Philippe P. Eyssidieux*

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17. S. Takayama, *The Levi Problem and the Structure Theorem for Non-Negatively Curved Complete Kähler Manifolds*, preprint (1997). cf. MR 99j:32034

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1707735 (2000k:32020) 32L10 (32C30 32L20)
Bonavero, Laurent (F-GREN-FM)
Inégalités de morse holomorphes singulières. (French. English summary) [Singular holomorphic Morse inequalities]
J. Geom. Anal. **8** (1998), *no. 3,* 409–425.

Let E be a holomorphic line bundle on a compact complex manifold X, and suppose that E is endowed with a smooth Hermitian metric. Demailly's holomorphic Morse inequalities [J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)] give asymptotic estimates for the dimensions of the Dolbeault cohomology groups of $E^{\otimes k}$, in terms of certain curvature integrals depending on the metric.

In the paper under review, these inequalities are extended to the case of singular metrics (with restrictions on the type of the singularities). The estimates are then the same as the original ones of Demailly, provided $E^{\otimes k}$ is twisted by a suitable multiplier ideal sheaf.

One of the main applications of Demailly's inequalities was a solution of the Grauert-Riemenschneider conjecture (another proof was given independently by Y. T. Siu [in *Workshop Bonn 1984 (Bonn, 1984)*, 169–192, Lecture Notes in Math., 1111, Springer, Berlin, 1985; MR0797421 (87b:32055)]) which, in the spirit of the famous projectivity criterion of Kodaira, gave a sufficient condition for a compact complex variety to be Moishezon. As the author points out, this sufficient condition is not necessary: an explicit counterexample is provided. Then he deduces from his singular holomorphic Morse inequalities a complete characterization of Moishezon varieties in terms of the existence, not of a Hermitian line bundle, but of a $(1,1)$-current with certain positivity properties.

Reviewed by *Laurent Manivel* (Saint-Martin-d'Hères)

MR1703223 (2000i:32032) 32J17 (32Q15)
Alessandrini, L. (I-MILANP)
Curves which are obstructions to the existence of Kähler metrics on threefolds. (English, Italian summaries)
Rend. Mat. Appl. (7) **18** (1998), *no. 4,* 683–706 (1999).

In this paper, the author studies complex compact threefolds M containing a smooth curve

C of strictly positive genus such that $M \smallsetminus C$ is Kähler. The main result is the following: put $M_0 = M$ and let by induction M_{n+1} be the manifold obtained by blowing up M_n along the curve C_n of minimal self-intersection in the exceptional divisor E_n of $M_n \to M_{n-1}$ (of course $C_0 = C$). Let $e_n = -C_n \cdot C_n$. Then, under the assumption that $e_n \geq 0$ and $E_n \cdot C_n \geq 0$ for every $n \geq 1$, M is Kähler if and only if C is not homologous to zero in the Aeppli group $V_{\mathbf{R}}^{2,2}(M)$. The proof uses Demailly's regularization theorem of closed positive currents, the Harvey-Lawson criterion and the machinery of ruled surfaces.

Reviewed by *Laurent Bonavero*

MR1689425 (2000d:32041) 32Q10 (32E40 32L20 32Q15 32Q28)

Takayama, Shigeharu (J-NARU)

The Levi problem and the structure theorem for non-negatively curved complete Kähler manifolds.

Analysis and geometry appearing in multivariable function theory (Japanese) (Kyoto, 1997).
Sūrikaisekikenkyūsho Kōkyūroku No. 1058 (1998), 105–113.

The main result is as follows: Let X be a complex manifold with negative canonical bundle K_X. Then X is holomorphically convex if and only if X is pseudoconvex. It generalizes the following result of Ohsawa [T. Ohsawa, Publ. Res. Inst. Math. Sci. **17** (1981), no. 1, 153–164; MR0613939 (82j:32031); supplement; MR0650217 (83h:32021)]: Let X be a 2-dimensional complex manifold with negative canonical bundle K_X. Then X is holomorphically convex if and only if it is weakly 1-complete. The author also proves a structure theorem: Every complete Kähler manifold with non-negative sectional curvature and positive Ricci curvature has a structure of holomorphic fiber bundle over a Stein manifold whose typical fiber is biholomorphic to some compact Hermitian symmetric manifold. The author uses some new techniques from recent developments in complex geometry and analysis on adjoint bundles on projective manifolds by J.-P. Demailly [in *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994)*, 817–827, Birkhäuser, Basel, 1995; MR1403982 (98e:32055)] and Siu [U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)].

{For the entire collection see MR1689414 (2000a:00014)}

Reviewed by *Shanyu Ji*

MR1690918 (2000e:32043) 32Uxx (32-06 32C30)
Lelong, Pierre
★**Positivity in complex spaces and plurisubharmonic functions/Positivité dans les espaces complexes et fonctions plurisousharmoniques. (French summary)**
Edited and with a note by Paulo Ribenboim.
Queen's Papers in Pure and Applied Mathematics, 112.
Queen's University, Kingston, ON, 1998. *x*+243 *pp. ISBN* 0-88911-828-0

This book is a collection of previously published works by the author on the theory of closed positive currents and plurisubharmonic functions.

The first article originally appeared as a book [*Fonctions plurisousharmoniques et formes différentielles positives*, Gordon & Breach, Paris, 1968; MR0243112 (39 #4436)] that served as a reference text in the theory of closed positive currents and plurisubharmonic functions. In particular, the author outlines the connections among holomorphic functions, plurisubharmonic functions and closed positive currents. Finally he introduces the integration of a differential form on an analytic subset.

In the second article [Bull. Soc. Math. France **85** (1957), 239–262; MR0095967 (20 #2465)] Lelong shows that if X is an analytic subset of pure dimension p in a complex-analytic manifold, then the integration current $[X]$ on X defines a closed positive (p, p) current. This current is defined as the continuation of the integration current on the regular points of X. In particular, Lelong proves that the mass of this current near the singular points is finite.

In the third article [in *Les probabilités sur les structures algébriques (Actes Colloq. Internat. CNRS, No. 186, Clermont-Ferrand, 1969)*, 251–263, Éditions Centre Nat. Recherche Sci., Paris, 1970; MR0409897 (53 #13649)] the author studies the frequency of obtaining certain functions in the algebra $\mathcal{A}(\Omega)$ of holomorphic functions on Ω, a domain of holomorphy in $\mathbf{C}^n$, with $n \geq 2$. He shows that if η is a subset of functions of $\mathcal{A}(\Omega)$ that can be continued outside Ω, then η is a thin set. The notions of negligible and polar sets are introduced.

The fourth article [in *Séminaire Pierre Lelong (Analyse), Année 1971-1972*, 112–131. Lecture Notes in Math., 332, Springer, Berlin, 1973; MR0412474 (54 #600)] is a study of the extremal elements of the cone of closed positive currents on a complex-analytic manifold Ω that is countable at infinity. In particular, Lelong proves that if X is an irreducible analytic subset of pure dimension p, then the integration current $[X]$ on X is an extremal current in the cone of closed positive (p, p) currents on Ω. This kind of problem is associated with the Hodge conjecture, as has been studied by J.-P. Demailly [Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007)]. Lelong shows that in a pseudoconvex domain Ω in $\mathbf{C}^n$ for which $H^2(\Omega, \mathbf{C}) = 0$, the cone of integration currents on analytic subsets of pure dimension $n-1$ is dense in the cone of closed positive $(n-1, n-1)$ currents.

In the fifth article [in *Séminaire Pierre Lelong (Analyse) (année 1972–1973)*, 97–106. Lecture Notes in Math., 410, Springer, Berlin, 1974; MR0372904 (51 #9108)] the author proves a support

theorem for currents T in a manifold M such that T and ∂T are of order zero and $\operatorname{Supp} T$ is in a C^∞ submanifold that is smoothly embedded in M.

In the sixth article [in *Séminaire Pierre Lelong (Analyse) (année 1975/76)*, 136–156. Lecture Notes in Math., 578, Springer, Berlin, 1977; MR0486608 (58 #6328)] Lelong associates with every closed positive (p,p) current T on a pseudoconvex domain a closed positive $(1,1)$ current that has the same Lelong number at every point of the domain. This current is obtained from a potential associated with T which is an almost plurisubharmonic function. This result allows the author to give a simpler proof of Siu's theorem on the analyticity of the density set of a closed positive current.

The seventh article [Exposition. Math. **3** (1985), no. 2, 149–164; MR0816400 (87f:32001)] is a discussion of three important directions of research: the representation of analytic sets as density sets, the continuation of closed positive currents and the notion of capacity in complex analysis (see the article for details).

In the eighth article [Exposition. Math. **3** (1985), no. 2, 187–191; MR0816405 (87m:32002)] the author notes the priority of S. Lang and E. Bombieri's 1970 article [Invent. Math. **11** (1970), 1–14; MR0296028 (45 #5089)] over a paper by Bombieri of the same year [Invent. Math. **10** (1970), 267–287; MR0306201 (46 #5328)].

The ninth article [in *Geometrical and algebraical aspects in several complex variables (Cetraro, 1989)*, 211–229, EditEl, Rende, 1991; MR1222216 (94g:32016)] concerns the existence of a principal part $\tilde{f}_\xi(x)$ at every point ξ in the class of plurisubharmonic functions of minimal (logarithmic) growth for every plurisubharmonic function f in a locally convex complex vector space (see the paper for details).

In the last article [Math. Ann. **299** (1994), no. 4, 673–695; MR1286891 (95g:32025)] Lelong studies plurisubharmonic functions of logarithmic type on $\mathbf{C}^n$. He considers the existence of constants independent of R for the inequalities (A) $0 \le m(f,0,R) - \lambda(f,0,R) \le \sigma C_n$, (B) $0 \le M(f,0,R) - \lambda(f,0,R) \le \sigma\gamma_n$. Moreover, assuming that f_k is of logarithmic type σ_k for $k = 1,\cdots,m$, and that therefore $f = \sum f_k$ is of type $\sigma = \sum \sigma_k$, he studies the existence of constants δ_n independent of m and R such that

$$\sum_1^m M(f_k,0,R) \le M\Big(\sum_1^m f_k,0,R\Big) + \sigma\delta_n$$

(see the paper for further details).

Reviewed by *Mongi Blel*

MR1677100 (2000b:14020) 14E30 (14J30)
Peternell, Thomas (D-BAYR-IM); **Serrano, Fernando**
Threefolds with nef anticanonical bundles. (English summary)
Dedicated to the memory of Fernando Serrano.
Collect. Math. **49** (1998), *no. 2-3,* 465–517.

Mori theory, the cornerstone of birational classification theory, aims at finding a minimal model of a non-singular variety, i.e., a birational model with only "mild" singularities and with a nef canonical bundle. At this time the theory is complete in dimension 3 only, but the framework is laid out in all dimensions, except for one (albeit very hard) step.

The subject of the paper under review is a class of varieties at the other end of the spectrum: varieties with a nef anticanonical bundle. The main goal of the paper is to confirm the conjecture that for smooth varieties with a nef anticanonical bundle the Albanese map is a surjective submersion. The authors succeed in proving this in dimension 3 with the help of Mori theory.

Considerable work had been done on questions related to this one before this article. The first author along with J.-P. Demailly and M. Schneider [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)] had proved the same statement for Kähler manifolds with a semi-positive Ricci curvature. They also proved surjectivity for this larger class of varieties in dimension 3, while Q. Zhang [J. Reine Angew. Math. **478** (1996), 57–60; MR1409052 (97m:14039)] proved surjectivity in all dimensions. Therefore the interesting part of the statement remaining was the smoothness of the Albanese map, which is proved in the present article in dimension 3 and is still open in higher dimensions.

The main idea of the proof is the following: First one can assume that the canonical bundle is not nef, otherwise the statement would follow by the Beauville-Bogomolov-Kobayashi decomposition theorem [A. Beauville, J. Differential Geom. **18** (1983), no. 4, 755–782 (1984); MR0730926 (86c:32030)]. Now if the canonical bundle is not nef, then Mori theory produces an extremal ray that can be contracted and either one gets a fibration over a smaller-dimensional variety or the second Betti number drops.

Certainly not everything is so easy. First of all, in order to run Mori theory one has to allow singularities, but this is not a major concern, as the singularities appearing are indeed very mild and have been extensively studied. The big drawback is that working with them makes the arguments longer and more technical (and of course it is somewhat harder to work with them than with smooth varieties).

The fibration case is relatively well understood, but the other one poses some problems. It is not at all clear that the resulting variety will still have a nef anticanonical bundle. The authors' way of dealing with this problem is to further enlarge the category, namely, they study varieties with an almost nef canonical bundle, i.e., they allow the canonical bundle to be negative on finitely many curves. This actually does the trick, and the induction works.

The authors prove a theorem that has evaded researchers for a long time and they find solutions to many problems along the way; yet one hopes that there is a simpler proof. Indeed the proof given here is dependent on the dimension restriction in more than one way. The dependency does not stop at the one coming from Mori theory. The way the fibration case is handled seems at times ad hoc and uses the benefits of low dimension every now and then. One hopes that perhaps these parts

could be replaced with arguments working in all dimensions and one could confine the dimension restriction to the one coming from Mori theory.

{For the entire collection see MR1673613 (99i:00032)}

Reviewed by *Sándor J. Kovács*

MR1660941 (99i:32035) 32L10 (32G13 32J18)

Siu, Yum-Tong (1-HRV)

Invariance of plurigenera.

Invent. Math. **134** (1998), *no. 3,* 661–673.

FEATURED REVIEW.

Starting with the works of Euler and Abel on integrals on algebraic curves it has become clear that all the analytic information about a compact Riemann surface is contained in global holomorphic 1-forms. These are objects that locally can be written as $f(z)dz$, where z is a local coordinate and f is a holomorphic function. Riemann established that the dimension of the vector space of global holomorphic 1-forms is the same as the topological genus of the underlying surface. This is one of the earliest results establishing the topological nature of certain analytically defined numbers.

In the past 100 years much effort was directed towards finding higher-dimensional versions of the result of Riemann. One direction is to replace the dimension of the vector space of global holomorphic 1-forms with the Euler characteristic of the line bundle of holomorphic n-forms where n is the dimension. The Hirzebruch-Riemann-Roch theorem expresses this number in terms of the Chern classes of the complex manifold. The situation becomes murkier if we would like to study the vector space of global holomorphic n-forms. Hodge theory shows that this is naturally a subspace of one of the topological cohomology groups, but it is still unknown how to convert this information into an explicit formula. The theory does, however, imply the following weaker statement: If X_t is a family of smooth, complex, projective varieties depending continuously on a parameter t then the dimension of the vector space of global holomorphic n-forms on X_t is a locally constant function of t.

It has also been realized that in higher dimensions this single number does not carry enough information about a variety. Instead, one should look at global sections of tensor powers of the line bundle of n-forms. This line bundle is frequently denoted by $K_X^{\otimes m}$ and its local sections can be written as

$$f(z_1, \cdots, z_n)(dz_1 \wedge \cdots \wedge dz_n)^{\otimes m}.$$

The dimension of the vector space of global sections of $K_X^{\otimes m}$ is called the mth plurigenus of X and it is denoted by $P_m(X)$. A very natural question is: are the numbers $P_m(X)$ topological in nature? Very little is known about this problem. The following weaker conjecture received much

attention because it is especially important in moduli problems: Let X_t be a family of smooth, complex, projective varieties depending continuously on a parameter t. Are the $P_m(X_t)$ locally constant functions of t?

The traditional approach is to convert $P_m(X_t)$ into an Euler characteristic and then use the Hirzebruch-Riemann-Roch theorem. Unfortunately, the higher cohomology groups of $K_X^{\otimes m}$ are usually nonzero and they vary wildly within the birational equivalence class of X. This, however, led to the first approaches to the conjecture. In many cases one can find a suitable birational model X^* such that $P_m(X) = P_m(X^*) = \chi(K_{X^*}^{\otimes m})$. If such a model can be found for every X_t in a family in a continuous manner then we arrive at a solution of the conjecture. This approach has been successfully carried out for surfaces [S. Iitaka, J. Math. Soc. Japan **22** (1970), 247–261; MR0261639 (41 #6252)] and for threefolds [J. Kollár and S. Mori, J. Amer. Math. Soc. **5** (1992), no. 3, 533–703; MR1149195 (93i:14015)]. Some cases were settled in all dimensions by M. N. Levine [Invent. Math. **74** (1983), no. 2, 293–303; MR0723219 (85d:32054)].

The paper under review approaches the question differently. Instead of changing the variety X, we would like to change the line bundle $K_X^{\otimes m}$ in such a way that we do not change the space of global sections but we do eliminate higher cohomology groups. (In general this is only possible if $K_X^{\otimes m}$ is replaced by a sheaf which is not locally free.) The key step is to introduce a metric on the line bundle $K_X^{\otimes m}$ which blows up along a subvariety in a carefully controlled manner. These ideas were introduced in algebraic geometry by Iitaka and his school [cf. Y. Kawamata, K. Matsuda and K. Matsuki, in *Algebraic geometry, Sendai, 1985*, 283–360, Adv. Stud. Pure Math., 10, North-Holland, Amsterdam, 1987; MR0946243 (89e:14015)] and in complex manifold theory by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)].

In the paper under review the author proves the deformation invariance of plurigenera for varieties where sections of $K_X^{\otimes m}$ separate points over a dense open set. These are usually called varieties of general type. For such varieties there is a natural choice of the singular metric. Let g_i be a basis of the global sections of $K_X^{\otimes m}$; then we can declare that $(\sum |g_i|^2)^{1/2m}$ is a section of K_X which has norm 1 everywhere. We get a singular metric which depends on the choice of the basis and m. The proof now depends on two key observations. First, all global sections of $K_X^{\otimes m}$ are L^2 in these metrics and this remains so after small perturbations of the metric. In particular, the dependence on m is not a serious problem. Second, the extension theorems of T. Ohsawa and K. Takegoshi [Math. Z. **195** (1987), no. 2, 197–204; MR0892051 (88g:32029)] and L. Manivel [Math. Z. **212** (1993), no. 1, 107–122; MR1200166 (94e:32050)] can be globalized to extend L^2 sections from X_0 to nearby X_t with a suitable perturbation of the metric. The perturbations required for the second part are just small enough that they do not matter if all choices are made carefully.

Besides solving a long-standing problem, the author's method is applicable to several other problems concerning deformations of varieties. Two such results are in papers by Kawamata [J. Amer. Math. Soc. **12** (1999), no. 1, 85–92; MR1631527 (99g:14003)] and N. Nakayama ["Invariance of the plurigenera of algebraic varieties", Preprint, Res. Inst. Math. Sci., Kyoto Univ., Kyoto, 1998; per revr.].

Reviewed by *János Kollár*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1647555 (2000c:32067) 32L20 (32L10)

de Cataldo, Mark Andrea A. (D-MPI)

Singular Hermitian metrics on vector bundles. (English summary)

J. Reine Angew. Math. **502** (1998), 93–122.

The purpose of the paper under review is to extend the techniques related to singular Hermitian metrics on line bundles to the higher-rank case. This generalization is carefully carried out so that it yields natural extensions to works of Nadel, Demailly and Siu, among which the following are well known: multiplier ideal sheaves, Nadel's version of the Kawamata-Viehweg vanishing

theorem, and effective very ampleness.

(See [Y. T. Siu, Ann. Inst. Fourier (Grenoble) **43** (1993), no. 5, 1387–1405; MR1275204 (95f:32035); J.-P. Demailly, Invent. Math. **124** (1996), no. 1-3, 243–261; MR1369417 (97a:32035); G. Fernández del Busto, J. Algebraic Geom. **5** (1996), no. 3, 513–520; MR1382734 (98d:14007)] for related work.)

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1643933 (99k:14062) 14J30 (14J40 32J18 32J25)

Peternell, Thomas (D-BAYR-IM)

Moishezon manifolds and rigidity theorems. (English summary)

Bayreuth. Math. Schr. No. 54 (1998), 1–108.

Let X be a complex compact manifold of dimension n; X is said to be a Moishezon manifold if the transcendence degree of the field of meromorphic functions over $\mathbf{C}$ is equal to n. B. Moishezon studied these manifolds and also proved that they become projective after a finite number of monoidal transformations (i.e. blow-ups) with nonsingular center. In the present paper the author discusses some central questions relative to these manifolds with special emphasis on dimension 3.

He starts by recalling some projectivity criteria: a Moishezon manifold is projective if and only if it is a Kähler manifold (this is due to Moishezon) or if and only if it has a line bundle whose curvature is semipositive and positive in at least one point (this is due to Siu and Demailly).

Then he proves a new projectivity criterion for Moishezon 3-folds X which says that X is projective if and only if there is no irreducible curve $C \subset X$ homologous to zero and $\overline{\mathrm{NE}(X)} \cap -\overline{\mathrm{NE}(X)} = 0$, where $\mathrm{NE}(X)$ is the cone of effective curves in the vector space of 1-cycles modulo numerical equivalence.

The proof uses the so-called Mori theory; it is not known whether the closure can be omitted, and also, at the moment, there is not a clear generalization to higher dimension.

Among others things, in the rest of the paper the author proves that a Moishezon 3-fold X homeomorphic to $\mathbf{P}^3$ is actually (isomorphic to) $\mathbf{P}^3$ and that every nonprojective Moishezon 3-fold contains a rational curve. The latter is a conjecture in higher dimension.

The interested reader can find similar results and arguments in [J. Kollár, in *Surveys in differential geometry (Cambridge, MA, 1990),* 113–199, Lehigh Univ., Bethlehem, PA, 1991; MR1144527 (93b:14059)].

Reviewed by *Marco Andreatta*

MR1645092 (99h:32035) 32L20

Koziarz, Vincent (F-NANCS-IE)

Annulation de la cohomologie pour les fibrés semi-positifs. (French. English, French summaries) [Vanishing theorems for semipositive bundles]

C. R. Acad. Sci. Paris Sér. I Math. **327** (1998), *no. 2,* 143–148.

This paper presents a general technique for proving vanishing theorems for holomorphic vector bundles over some special Hermitian complex-analytic manifolds. The main idea, which goes back

to J.-P. Demailly [in *Séminaire d'analyse P. Lelong-P. Dolbeault-H. Skoda, années 1983/1984*, 88–97, Lecture Notes in Math., 1198, Springer, Berlin, 1986; MR0874763 (88f:32069)], is to describe these groups as an inductive limit of L^2 weighted cohomology groups, and reduce to L^2-cohomology vanishing theorems. This only requires a sufficient collection of weights so that any C^∞ section of E eventually becomes L^2. This is formalized here under the term "having enough metrics", further arguments boiling down to Demailly's. The rest of the paper presents various consequences of this technique, among them a relative Nakano vanishing theorem over irreducible Kähler analytic spaces. Unfortunately, the required collection of weights is not made explicit here.

Reviewed by *Thierry Bouche*

MR1642740 (99f:14017) 14E05 (14J99)

Tsai, I-Hsun (RC-NTAI)

Chow varieties and finiteness theorems for dominant maps.

J. Algebraic Geom. **7** (1998), *no. 4,* 611–625.

The Iitaka-Severi conjecture states that the number of smooth complex projective varieties of general type that arise as a dominant rational image of X is finite up to birational equivalence.

This article is one of many of the author's that are related to this topic. The central goal of the present article is giving effective estimates in the case when the target is assumed to be a surface.

The estimates heavily depend on effective very ampleness of adjoint bundles (as in Fujita's conjecture) of J.-P. Demailly, J. Kollár, L. Ein-R. Lazarsfeld, and Y. T. Siu. Therefore, although these estimates are probably far from being sharp, as soon as there are better estimates for very ampleness they should yield better ones in this case.

Besides using effective very ampleness results, the paper studies Chow varieties and uses results of F. M. E. Catanese [J. Algebraic Geom. **1** (1992), no. 4, 561–595; MR1174902 (93j:14005)].

Reviewed by *Sándor J. Kovács*

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MR1639560 (99m:58204) 58G26 (32L07 32S20)
Yoshikawa, Ken-ichi (J-NAGO-GM)
Smoothing of isolated hypersurface singularities and Quillen metrics. (English summary)
Asian J. Math. **2** (1998), *no. 2,* 325–344.

The author considers the following. Let $\pi\colon X \to S = \{t \in \mathbf{C};\ |t| < 1\}$ be a proper surjective holomorphic map of complex manifolds. Suppose that π is of maximal rank outside of a finite number of points in $X_0 = \pi^{-1}(0)$; this family (π, X, S) is then said to be a smoothing of isolated hypersurface singularities (IHS). Let g_X be a Kähler metric of X and $g_{X/S}$ be the induced metric on TX/S. Let (E, h) be a holomorphic Hermitian vector bundle on X. Write $\lambda(E)$ for the determinant line bundle associated with cohomologies, and $\|\cdot\|_Q$ for its Quillen metric relative to $g_{X/S}$ and h. Assume that (π, X, S) is projective over S. Then the main theorem of this paper computes the curvature current of $\|\cdot\|_Q$:

$$c_1(\lambda(E), \|\cdot\|_Q) = \frac{(-1)^{n+1}}{(n+2)!} r(E)\mu(X_0)\delta_0 + \pi_*(\mathrm{Td}(TX/S, g_{X/S})\mathrm{ch}(E, h))^{(1,1)}.$$

Here $\dim X = n+1$, $r(E)$ is the rank of E, δ_0 the Dirac measure at 0 and $\mu(X_0)$ is the Milnor number of the singular fiber. Moreover the second term on the right-hand side lies in $L^p_{\mathrm{loc}}(S)$ for some $p > 1$ depending only on $\mathrm{Sing}\, X_0$. (See [J.-M. Bismut and J.-B. Bost, Acta Math. **165** (1990), no. 1-2, 1–103; MR1064578 (91h:58122); J.-M. Bismut, J. Algebraic Geom. **6** (1997), no. 1, 19–149; MR1486991 (2000a:58084)] for the case of ordinary singularities.) The main theorem is applied in this paper to study the asymptotic behavior of analytic torsion in the case of smoothing of IHS, whose principal term turns out to be determined by the total Milnor number of the singular fiber and determinant of period integrals. (See [M. S. Farber, J. Differential Geom. **41** (1995), no. 3, 528–572; MR1338482 (96e:58165)] for a related result.) Concerning the proof, the author first proves the main theorem in the case where (X, E) is globalizable, i.e. X can be embedded in a projective algebraic manifold of the same dimension, and E extends as a coherent sheaf. For this part a main tool is a theorem of Bismut and G. Lebeau [Inst. Hautes Études Sci. Publ. Math. No. 74 (1991), ii+298 pp. (1992); MR1188532 (94a:58205)] and the method of proof is similar to that of Bismut [op. cit.]. After this is done, using an approximation result by J.-P. Demailly, L. Lempert and B. Shiffman [Duke Math. J. **76** (1994), no. 2, 333–363; MR1302317 (95i:32022)] the author proceeds with the approximation by algebraic families and completes the proof of the main theorem. All of the proofs in this paper are essentially analytical in nature; concerning an

algebraic approach to similar problems, see [Y. L. L. Tong and I. H. Tsai, Comm. Math. Phys. **171** (1995), no. 3, 589–606; MR1346173 (96j:58177)].

Reviewed by *I-Hsun Tsai*

MR1637927 (99k:32049) 32L10 (32C17)

Takayama, Shigeharu (J-OSAKEGS)

Adjoint linear series on weakly 1-complete Kähler manifolds. I. Global projective embedding.

Math. Ann. **311** (1998), *no. 3,* 501–531.

The paper under review successfully applies the recent developments of effective adjunction theory for ample line bundles over weakly 1-complete Kähler manifolds. It has been known for a while (T. Ohsawa [Proc. Japan Acad. Ser. A Math. Sci. **55** (1979), no. 5, 193–194; MR0533546 (80e:32017)], answering a question of Nakano) that a line bundle L may be positive over such a manifold, but not ample, in the sense that global holomorphic sections of a high tensor power generate 1-jets at every point (and hence define a global one-to-one immersion into some projective space). The original idea, based on recent work related to the Fujiki conjecture, is to show that some adjoint bundles $K_X \otimes L^{\otimes m}$ will be ample for large m. This idea is shown to work extremely well on the mentioned problem. The main theorem, whose proof is rather involved but very cleanly exposed, yields an explicit bound for the separation of points by sections of $K_X \otimes L^{\otimes m}$ over "pseudo-balls" (or level sets of an exhaustive psh function). Its proof follows the scheme developed by Angehrn and Siu or Tsuji in the compact case: the Riemann-Roch formula is replaced by Demailly's holomorphic Morse inequalities (applied on relatively compact level sets) in order to construct singular metrics with controlled singularity at some points, thanks to Shokurov's concentration method; the Nadel vanishing theorem is then applied to that situation. Among the fine results derived from these main technical results, we can single out: "Let X be an n-dimensional weakly 1-complete manifold with a positive line bundle L. Then the line bundle $(K_X \otimes L^{\otimes m})^{\otimes n+2}$ is very ample for $m > n(n+1)/2$." Also, "If the anticanonical bundle $K_X^{\otimes -1}$ is positive, X is Stein iff X has no compact complex subspaces of positive dimension."

Reviewed by *Thierry Bouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1632996 (99e:32013) 32C30

Meo, Michel (F-ANGR)

Inégalités d'auto-intersection pour les courants positifs fermés définis dans les variétés projectives. (French) [Self-intersection inequalities for closed positive currents defined in projective varieties]

Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **26** (1998), *no. 1,* 161–184.

Let T be a closed positive current of bidimension (p,p) defined on a complex variety X of dimension n. For all $c>0$, the sets $E_c=\{x\in X:\ \nu_T(x)\geq c\}$ are analytic subsets of dimension at most p. For X compact and equipped with a Kähler metric ω, J.-P. Demailly [J. Algebraic Geom. **1** (1992), no. 3, 361–409; MR1158622 (93e:32015)] gave an upper bound on the degree with respect to ω of q-dimensional irreducible components in E_c in terms of the cohomology class

of T when T is of bidimension $(n-1,n-1)$; namely, he proved that

$$\sum_{k\geq 1}(\nu_q,k-b_{n-1})\cdots(\nu_q,k-b_q)\{Z_q,k\}\{\omega\}^q \leq$$
$$(\{T\}+b_{n-1}\{u\})\cdots(\{T\}+b_q\{u\})\{\omega\}^q,$$

with u a semipositive cohomology class in X such that $c_1(\mathcal{O}_{TX}(1))+\pi_X^*\{u\}$ is semipositive, $b_q = \inf\{c>0;\ \dim E_c \leq q\}$, $b_{-1}=\max_{x\in X}\nu(T,x)$ and $(Z_{q,k})_{k\geq 1}$ is the at most countable family of q-dimensional irreducible components in E_c for $c\in(b_q,b_{q-1}]$ and $\nu_{q,k}=\min_{x\in Z_{q,k}}\nu(T,x)$.

In the case $X=\mathbf{P}^n$, one can take $u=0$, and the above inequality becomes

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq \delta(T)^{n-q}.$$

The author's goal in this paper is to prove an analogous inequality for a closed positive current T in $\mathbf{P}^n$:

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq \delta(T)^{p+1-q}.$$

The idea is to construct a closed positive current of bidegree $(1,1)$ in $\mathbf{P}^n$ which has the same degree as T and the same Lelong number at every point. This construction is similar to that of Lelong and Skoda, which consists of constructing a potential associated with the current T.

When T is defined on a projective variety X and ω is a Kähler metric on X defining an entire cohomology class, the author proves the inequality

$$\sum_{k\geq 1}(\nu_{q,k}-b_{n-1})\cdots(\nu_{q,k}-b_q)\delta(Z_{q,k})\leq C\delta(T)^{p+1-q}.$$

He uses Matsusaka's embedding theorem to prove the existence of the constant $C>0$.

Reviewed by *Mongi Blel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1622747 (99e:32047) 32J17
Campana, Frédéric (F-NANC); **Demailly, Jean-Pierre** (F-GREN-F);
Peternell, Thomas (D-BAYR-IM)

The algebraic dimension of compact complex threefolds with vanishing second Betti number. (English summary)

Compositio Math. **112** (1998), *no. 1,* 77–91.

A compact complex threefold with vanishing second Betti number cannot be algebraic or Kähler. Then the natural question is: What possibilities are there for the algebraic dimension of such manifolds? (Algebraic dimension is the transcendence degree of the field of meromorphic functions over **C**.)

The main result of this article is that if the algebraic dimension is positive, then the topological Euler characteristic is 0 and then either $b_1 = 0$ and $b_3 = 2$ or $b_1 = 1$ and $b_3 = 0$. An interesting corollary is that S^6 does not admit a complex structure with a non-constant meromorphic function. The authors deduce the main result as a straightforward consequence of a vanishing theorem for vector bundles twisted by generic elements of Pic^0. Examples of threefolds with positive algebraic dimension and vanishing second Betti number and topological Euler characteristics are also given showing that the result is optimal.

The authors also investigate more deeply threefolds with vanishing second Betti number and whose algebraic dimension is 1.

This is another interesting article from these distinguished authors presenting ideas of great interest to algebraic and complex analytic geometers alike.

Reviewed by *Sándor J. Kovács*

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MR1613763 (99d:32037) 32L07 (53C55)
Mihai, Paun [Păun, Mihai] (F-GREN-FM)
Sur les variétés kählériennes compactes à classe de Ricci numériquement effective. (French)
[On compact Kähler manifolds with numerically effective Ricci class]
Bull. Sci. Math. **122** (1998), *no. 2,* 83–92.

In this paper compact Kähler manifolds with numerically effective (nef) anticanonical bundle are investigated; in particular the structures of the Albanese map and of the fundamental group are studied. The main result is the following: Let X be a compact Kähler manifold of complex dimension n with nef anticanonical bundle. Then: (i) $h^1(X, \mathcal{O}_X) \leq n$; (ii) if Γ is a finitely generated subgroup of the fundamental group of X, then there exists a normal subgroup Γ_1 of Γ of finite index generated by at most $4^{2n}+1$ elements. The proof makes use of the Aubin-Calabi-Yau theorem and of some results of Gromov and Demailly-Peterell-Schumacher. This paper continues the results obtained in papers by F. Campana and T. Peternell [Math. Ann. **289** (1991), no. 1, 169–187; MR1087244 (91m:14061)] and J.-P. Demailly, Peternell and M. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037); Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)].

Reviewed by *Antonella Nannicini*

MR1632491 (99f:32021) 32F05 (32F07)
Lelong, Pierre
Remarks on pointwise multiplicities in complex spaces.
Dedicated to Professor Vyacheslav Pavlovich Zahariuta.
Linear Topol. Spaces Complex Anal. **3** (1997), 112–119.

Let F be a holomorphic function in a neighborhood of the closed unit polydisk $D = \{x \in \mathbf{C}^n\colon \sup_k |x_k| \leq 1\}$ with $F(0) = 0$. The theme of the paper is a discussion of generalizations of

the classical notion of the multiplicity m of the zero of F at the origin; i.e., the order of the zero of the restriction of F to a generic complex line; the non-generic lines form a small exceptional set. For any n-tuple $a=(a_1,\cdots,a_n)$ with $a_k>0$, the index $I(F,0,a)$ of F at the origin 0 is defined as follows: for $0\le w\le 1$, replace x_k by $w^{a_k}x_k$ in the power series for $F(x)$ at 0 to get $F(w,x):=\sum_{J\ge I} w^J P_J(x)$, where $P_J(x)$ are polynomials. Then

$$I=I(F,0,a)=\lim_{w\to 0}\frac{\log|F(w,x)|}{\log w}$$

is the degree in w of $w\to F(w,x)$ for generic x, the exceptional set being the algebraic set $P_I(x)=0$. This recovers the classical notion of multiplicity of F at 0 if $a_1=\cdots=a_n=1$. This notion is extended to nonpositive plurisubharmonic (psh) functions f which are $-\infty$ at 0; the index, denoted $s(f,0,a)$, is defined by considering

$$h(w,x):=\frac{f(w^{a_1}x_1,\cdots,w^{a_n}x_n)}{\log 1/w};$$

then $K(x):=\limsup_{w\to 0} h(w,x)$ has the property that its upper semicontinuous regularization $K^*(x)$ is constant; this constant value is defined to be $-s(f,0,a)$. Here, the exceptional set $A:=\{x\in \mathbf{C}^n\colon\ K(x)<-s(f,0,a)\}$ is pluripolar. The index $s(f,0,a)$ reduces to the Lelong number ν of the current $(1/2\pi)dd^cf$ at the origin when $a_1=\cdots=a_n=1$. In particular, if

$$f(x):=\frac{1}{2}\log\sum_j |F_j(x)|^2$$

where F_j are holomorphic in a neighborhood of D, we have $s(f,0,a)=\inf_j I(F_j,0,a)$. J.-P. Demailly's notion of a generalized Lelong number $\nu(T,0,\varphi)$ of a closed positive current for a psh weight φ (with $\exp\varphi$ continuous) is recalled [Mém. Soc. Math. France (N.S.) No. 19 (1985), 124 pp.; MR0813252 (87g:32030); Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)]; for certain special weights φ, one gets nice formulas for $\nu(dd^cf,0,\varphi)$ for f psh. Finally, the author studies polynomial mappings $P=(P_1,\cdots,P_n)\colon \mathbf{C}^n\to\mathbf{C}^n,\ \deg P_k=d_k$, near points x of their zero sets W_P which have regular multiplicity $\nu(x)$, i.e.,

$$\nu(x):=\lim_{y\to x}\frac{\log|P(x)|}{\log|x-y|}\ge 1.$$

Let $\mu_P:=(2\pi)^{-n}(dd^c\log|P|)^n$ be the Monge-Ampère measure of $\log|P|$. Let W_P be compact and of dimension zero. It is shown that if $\{\zeta_s\}$ are the isolated zeros of P with regular multiplicity, then

$$\mu_P=\sum_s[\nu(\zeta_s)]^n\le d_1\cdots d_n.$$

{For the entire collection see MR1632477 (99a:00052)}

Reviewed by *Norman Levenberg*

MR1622653 (99c:32006) 32C30 (32F07)

Coman, Dan (R-CLUJMI)

Integration by parts for currents and applications to the relative capacity and Lelong numbers.

Mathematica **39(62)** (1997), *no. 1,* 45–57.

Let Ω be a bounded domain in $\mathbf{C}^n$, let u and v be locally bounded plurisubharmonic functions in Ω, and let T be a closed positive current of bidimension (1,1). When is the integration by parts formula

$$\int_\Omega udd^cv\wedge T=\int_\Omega vdd^cu\wedge T \tag{1}$$

or the inequality

$$\int_\Omega udd^cv\wedge T\le\int_\Omega vdd^cu\wedge T \tag{2}$$

valid? The author first shows that if u and v are negative and agree outside a compact subset of Ω, then equality (1) holds; he then relaxes the hypotheses a bit to give weaker conditions sufficient to guarantee (1) or (2); e.g., (1) holds if Ω is hyperconvex, u and v are negative exhaustion functions for Ω, and both $\int_\Omega dd^cu\wedge T$ and $\int_\Omega dd^cv\wedge T$ are finite. These results are used to obtain upper and lower bounds for the relative capacity of a regular compact sublevel set $K=\{z\in\Omega\colon\ \psi(z)\le-\gamma<0\}$ of a hyperconvex domain $\Omega=\{z\in\Omega'\colon\ \psi(z)<0\}\subset\subset\Omega'$ where $\psi\in C^2(\Omega)$ and $\psi=0$ on $\partial\Omega$ and also to generalize slightly a formula of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)] regarding Lelong numbers with weights.

Reviewed by *Norman Levenberg*

MR1614576 (2000a:32042) 32L05 (14F05 32J27)
Mourougane, Christophe (F-GREN-F)
Images directes de fibrés en droites adjoints. (French. English summary) [Direct images of adjoint line bundles]
Publ. Res. Inst. Math. Sci. **33** (1997), *no. 6,* 893–916.

Recall that some results for the direct image of the canonical fiber bundle over a projective manifold had been obtained by Y. Kawamata [Compositio Math. **43** (1981), no. 2, 253–276; MR0622451 (83j:14029)] and J. Kollár [Ann. of Math. (2) **123** (1986), no. 1, 11–42; MR0825838 (87c:14038)]. A study of the numerically effective properties of fiber bundles was given by J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)]. In the paper under review the more general situation of holomorphic line bundles L over compact complex manifolds M is considered and especially its direct images φ_*L under a smooth holomorphic mapping $\varphi\colon M \to N$. The author's attention is focussed on the properties of ampleness and positivity and on the question of their implications for the direct image φ_*L or at least for that of the adjoint line bundle. The methods used include a variety of techniques, algebraic and analytic in spirit, which help the author to separate some original algebraic version based on some algebraic treatment of positivity, without curvature tensors, etc. Applications are given. Sometimes the reviewer found the author's exposition hard to understand.

Reviewed by *S. Dimiev*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1492596 (99a:32033) 32H20 (32J10 32L05)

Demailly, Jean-Pierre (F-GREN-F)

Variétés projectives hyperboliques et équations différentielles algébriques. (French) [Hyperbolic projective varieties and algebraic differential equations]

Journée en l'Honneur de Henri Cartan, 3–17, *SMF Journ. Annu.*, 1997, *Soc. Math. France, Paris*, 1997.

This is a very well-written survey of some of the more recent developments in the theory of holomorphic curves in algebraic varieties, a holomorphic curve in an algebraic variety being a holomorphic mapping from the complex plane to the variety. The author's survey concentrates in particular on the recent work of Y. T. Siu and S.-K. Yeung [Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072 (98h:32044)]. An extensive bibliography is also provided. Although probably best suited for those readers already familiar with the language of complex differential geometry, and in particular the language used when working with Hermitian vector bundles and meromorphic connections, the survey is for the most part a very accessible introduction to some of the latest developments in the field and assumes little prior knowledge of Nevanlinna theory, algebraic geometry, or the other techniques commonplace in the study of holomorphic curves.

The reviewer's translation of the author's first paragraph reads as follows: "The goal of this text is to offer an introduction, which is as elementary as possible, to an important result concerning the geometry of the images of holomorphic curves in complex algebraic varieties. This result finds its origin in the fundamental work of A. Bloch [J. Math. Pures Appl. (9) **5** (1926), 19–66; JFM 52.0373.05] and in the thesis of H. Cartan [Ann. Sci. École Norm. Sup. **45** (1928), 255–346; JFM 54.0357.06]. The proof that we give here is a very recent contribution by Siu and Yeung [op. cit.]. It proceeds in a relatively simple manner with help from classical estimates in Nevanlinna theory, like the lemma on the logarithmic derivative, and by making use of differential operators such as Wronskians, all ideas whose germs were already sown in Henri Cartan's thesis [op. cit.]."

More specifically, the author explains techniques for showing that a holomorphic curve in an algebraic variety is algebraically degenerate, meaning that its image is contained in a proper algebraic subvariety. A fundamental conjecture along these lines is the conjecture of Green and Griffiths stating that a holomorphic curve in a variety of general type must be algebraically degenerate. The survey is centered around the following fundamental vanishing theorem. If $f\colon \mathbf{C}\to X$ is a holomorphic curve in a projective variety X, if L is a positive line bundle on X, and if P is an algebraic differential operator on X with values in L^{-1}, then P applied to f is zero. For hypersurfaces in projective space, this theorem can be applied to Wronskian-like differential operators coming from explicitly constructed meromorphic connections, as in the work of A. M. Nadel [Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444 (91a:32036)]. This results in specific examples of general type projective varieties in which every holomorphic curve is algebraically degenerate. This method also proves that in some of these varieties, the image of every holomorphic curve must be constant; such varieties are called hyperbolic.

{For the entire collection see MR1492594 (98h:00041)}

Reviewed by *William A. Cherry*

MR1492594 (98h:00041) 00B30
Hirzebruch, Friedrich; **Demailly, Jean-Pierre** (F-GREN-F); **Lannes, Jean**
★**Journée en l'Honneur de Henri Cartan. (French) [Conference in Honor of Henri Cartan]**
SMF Journée Annuelle [SMF Annual Conference], 1997.
Société Mathématique de France, *Paris*, 1997. iv+27 *pp*.

Contents: F. Hirzebruch, Learning complex analysis in Muenster-Paris, Zuerich and Princeton from 1945 to 1953 (1–2); Jean-Pierre Demailly, Variétés projectives hyperboliques et équations différentielles algébriques [Hyperbolic projective varieties and algebraic differential equations] (3–17); Jean Lannes, Divers aspects des opérations de Steenrod [Various aspects of the Steenrod operations] (18–27).

{Most of the papers are being reviewed individually.}

MR1492539 (99b:32037) 32H20 (14J40 32L10)
Demailly, Jean-Pierre (F-GREN-F)
Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. (English summary)
Algebraic geometry—Santa Cruz 1995, 285–360, *Proc. Sympos. Pure Math.*, 62, *Part* 2, *Amer. Math. Soc.*, *Providence, RI*, 1997.

The article under review is an expanded version of five lectures delivered at the Santa Cruz AMS Summer School on Algebraic Geometry. It proposes an important framework for solving several geometry questions related to hyperbolicity in the sense of Kobayashi. This framework was initiated by M. Green and P. Griffiths [in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557 (82h:32026)]. Aiming, among other things, to fix a gap in Green-Griffiths' proof of the pointwise version of the Ahlfors-Schwarz lemma for jet differentials, Demailly introduces the concept of "directed manifold" and an associated tower of projective bundles over X (called Semple jet bundles). The Ahlfors-Schwarz lemma is then established in this setting, and the proof of Bloch's theorem is recovered following the approach of Green and Griffiths. Several important new results are also obtained in this paper. Moreover, the author believes that the Semple bundle construction should be an efficient tool to calculate the case locus; therefore several important open problems in the theory of complex

hyperbolicity hopefully could be settled under this framework. It should be noted that, since the appearance of this article, Demailly has proved jointly with J. El Goul that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 42 is Kobayashi hyperbolic. It has been conjectured by Kobayashi that every generic surface in $\mathbf{P}^3$ of degree greater than or equal to 5 is Kobayashi hyperbolic.

This paper is a quite important contribution to the theory of complex hyperbolicity. The paper is self-contained and the exposition is excellent. It is highly recommended to the experts in this field, as well as to anyone who desires a general overview of this subject.

We will now try to outline this article. A complex directed manifold is a pair (X, V) where X is a complex manifold and V is a holomorphic subbundle of T_X; here T_X is the tangent bundle of X. To study the complex hyperbolicity of (X, V), a well-known major technique is the so-called "negative curvature method". The method is based on the following observation: by the Ahlfors-Schwarz lemma, the existence of a Hermitian metric on the line bundle $\mathcal{O}_{\mathbf{P}(V)}(-1)$ over $\mathbf{P}(V)$ (i.e. a Finsler metric on V) with negative curvature implies that (X, V) is hyperbolic. Let us recall here how to construct such a metric. Assume that V^* is "very big" in the following sense: there exist an ample line bundle L and a sufficiently large integer m such that the global sections $H^0(X, S^m V^* \otimes L^{-1})$ generate all fibers over $X \smallsetminus Y$, for some analytic subset $Y \subset X$. Let $\sigma_1, \cdots, \sigma_N$ be such global sections, and define

$$N(\xi) = \Big(\sum_{1 \le j \le N} |\sigma_j(x) \cdot \xi^m|^2 \Big)^{1/2m}, \quad \xi \in V_x^*;$$

N then gives rise to such a metric. Therefore we have the following result: Let (X, V) be a directed complex manifold. Assume that V^* is "very big". Then every entire curve $f\colon \mathbf{C} \to X$ tangent to V satisfies $f(\mathbf{C}) \subset Y$, where Y is the subset of X defined above. In particular, if V^* is ample, then (X, V) is hyperbolic.

The heart of the article consists of Chapters 4 to 7. They are devoted to extending the above result to k-jet differentials. The idea is based on the important fact, first observed by Green and Griffiths, that the Ahlfors-Schwarz lemma still works for k-jet differentials, and thus k-jet negativity also implies hyperbolicity. Unfortunately, there is a slight technical gap in Green and Griffiths' approach in the step proving the pointwise Ahlfors-Schwarz lemma for jet differentials. In his paper, Demailly fills the gap in the case of invariant jet differentials, and also extends the result to the more general situation of directed manifolds. (Note: Another solution has been provided later by Y. T. Siu and S.-K. Yeung by means of Nevanlinna's second main theorem [see Amer. J. Math. **119** (1997), no. 5, 1139–1172; MR1473072 (98h:32044)].)

To do this, Demailly introduces a canonical tower of projective bundles (also called Semple jet bundles). Given a complex directed manifold (X, V), a new complex directed manifold $(\widetilde{X}, \widetilde{V})$ is produced as follows. Let $\widetilde{X} = \mathbf{P}(V)$ be the projectivized bundle of lines of V, and let $\widetilde{V} \subset T_{\widetilde{X}}$ be the subbundle of $T_{\widetilde{X}}$ defined as follows: for every point $(x, [v]) \in \widetilde{X}$ associated with a vector $v \in V_x \smallsetminus \{0\}$,

$$\widetilde{V}_{(x,[v])} = \{\xi \in T_{\widetilde{X},(x,[v])}\colon \pi_* \xi \in \mathbf{C}\}, \quad V_x \subset T_{X,x},$$

where $\pi\colon \widetilde{X} = \mathbf{P}(V) \to X$ is the natural projection. The projectivized k-jet bundle $\mathbf{P}_k V = X_k$ (or Semple k-jet bundle) and the associated subbundle $V_k \subset T_{X_k}$ are defined inductively by $(X_0, V_0) =$

(X,V), $(X_k,V_k)=(\widetilde{X}_{k-1},\widetilde{V}_{k-1})$. Every non-constant tangent trajectory $f\colon\Delta_R\to X$ of (X,V) lifts to a well-defined and unique tangent trajectory $f_{[k]}\colon\Delta_R\to X_k$ of (X_k,V_k).

The author shows that the Ahlfors-Schwarz lemma works at each level of the tower of projective bundles. That is: If (X,V) has a k-jet metric h_k on the line bundle $\mathcal{O}_{\mathbf{P}_kV}(-1)$ (i.e. a Finsler metric on the vector bundle V_{k-1} over $\mathbf{P}_{k-1}V$), with negative jet curvature, then every entire curve $f\colon\mathbf{C}\to X$ tangent to V satisfies $f_{[k]}(\mathbf{C})\subset\Sigma_{h_k}$, where Σ_{h_k} is the singularity set of the metric h_k.

To produce such metrics h_k, one uses global sections of $H^0(\mathbf{P}_kV,\mathcal{O}_{\mathbf{P}_kV}(m)\otimes\pi_{0,k}^*L^{-1})$, where L is an ample line bundle on X. The author also shows that the direct images $(\pi_{0,k})_*\mathcal{O}_{\mathbf{P}_kV}(m)$ can be viewed as bundles of algebraic differential operators of order k and degree m, acting on germs of curves and invariant under reparametrization. This bundle is denoted by $E_{k,m}(V^*)$. Therefore $H^0(\mathbf{P}_kV,\mathcal{O}_{\mathbf{P}_kV}(m)\otimes\pi_{0,k}L^{-1})\simeq H^0(X,E_{k,m}(V^*)\otimes L^{-1})$.

The above discussion leads to the following result: Assume that there exist integers $k,m>0$ and an ample line bundle L on X such that $H^0(X,E_{k,m}(V^*)\otimes L^{-1})$ has nonzero sections $\sigma_0,\cdots,\sigma_N$. Let $Z\subset\mathbf{P}_kV$ be the base locus of these sections. Then every entire curve $f\colon\mathbf{C}\to X$ tangent to V satisfies $f_{[k]}(\mathbf{C})\subset Z$. In other words, for every global parametrization invariant polynomial differential operator P with values in L^{-1}, every entire curve f as above must satisfy the algebraic differential equation $P(f)=0$.

The dimension

$$h^0(X,E_{k,m}(V^*)\otimes L^{-1})=\dim H^0(X,E_{k,m}(V^*)\otimes L^{-1})$$

can be computed by using the Riemann-Roch theorem and a vanishing theorem due to Bogomolov. In particular, in the surface case, the Riemann-Roch theorem yields the following (see Chapter 13, Corollary 13.9): If X is an algebraic surface of general type and L an ample line bundle over X, then

$$h^0(X,E_{2,m}T^*X\otimes\mathcal{O}(-L))\geq\frac{m^4}{648}(13c_1^2-9c_2)-O(m^3).$$

In particular, every smooth surface $X\subset\mathbf{P}^3$ of degree $d\geq 15$ admits a nontrivial section, and every entire function $f\colon\mathbf{C}\to X$ must satisfy the corresponding algebraic differential equations.

However, it seems very difficult to conclude that f satisfies an algebraic equation. The author suggests in Chapter 13 that the Riemann-Roch calculations might be helpful to locate the base locus, thus to conclude the algebraic degeneracy.

Another important part of this article is Chapter 2 and Chapter 9, where Demailly shows that Kobayashi hyperbolicity is related to other properties of a more algebraic nature. A projective directed manifold (X,V) is called algebraically hyperbolic if there exists $\varepsilon>0$ such that every algebraic curve $C\subset X$ tangent to V satisfies $2\,g(\overline{C})-2\geq\varepsilon\deg_\omega(C)$ ($\overline{C}$ is the normalization of C). The main result of Chapter 2 is that if (X,V) is hyperbolic, then (X,V) is algebraically hyperbolic. Chapter 9 extends this result to k-jet metrics and shows that the negativity of k-jet curvature implies strong restrictions of an algebraic nature on curve genera and their singularity indices.

Chapter 11 recalls the "meromorphic connection" method introduced by Siu [Y. T. Siu, Duke Math. J. **55** (1987), no. 1, 213–251; MR0883671 (89a:32030); A. M. Nadel, Duke Math. J. **58** (1989), no. 3, 749–771; MR1016444 (91a:32036)]. Using this method, the author reports on a

joint work with J. El Goul, where examples of hyperbolic surfaces in $\mathbf{P}^3$ are produced for any degree ≥ 11.

{For the entire collection see MR1492532 (98h:14003)}

Reviewed by *Min Ru*

MR1492532 (98h:14003) 14-06

★**Algebraic geometry—Santa Cruz 1995.**

Proceedings of the AMS Summer Research Institute held at the University of California, Santa Cruz, CA, July 9–29, 1995.

Edited by János Kollár, Robert Lazarsfeld and David R. Morrison.

Proceedings of Symposia in Pure Mathematics, 62, Part 2.

American Mathematical Society, *Providence*, *RI*, 1997. xviii+449 *pp*. $89.00; $159.00 the two-volume set. *ISBN* 0-8218-0895-8; 0-8218-0493-6

{For Part 1 see the preceding review [MR1492516 (98h:14002)].}

Contents: Ron Y. Donagi, Seiberg-Witten integrable systems (3–43); W. Fulton and R. Pandharipande, Notes on stable maps and quantum cohomology (45–96); Richard Hain and Eduard Looijenga, Mapping class groups and moduli spaces of curves (97–142); Jun Li [Jun Li[1]] and Gang Tian, Algebraic and symplectic geometry of Gromov-Witten invariants (143–170); L. Katzarkov, On the Shafarevich maps (173–216); Carlos Simpson, The Hodge filtration on nonabelian cohomology (217–281); Jean-Pierre Demailly, Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials (285–360); Claude LeBrun, Twistors for tourists: a pocket guide for algebraic geometers (361–385); David A. Cox, Recent developments in toric geometry (389–436); Bernd Sturmfels, Equations defining toric varieties (437–449).

{The papers are being reviewed individually.}

MR1481120 (98i:32022) 32F07 (32F05 35J65)

Zeriahi, Ahmed (F-TOUL3-LM)

Pluricomplex Green functions and the Dirichlet problem for the complex Monge-Ampère operator.

Michigan Math. J. **44** (1997), *no. 3,* 579–596.

In this paper the notion of the pluricomplex Green function (first studied by Lempert, Klimek and Demailly) is generalized in the following way. Fix a function φ which is plurisubharmonic (psh) in a hyperconvex domain D such that e^φ is continuous and the singular set $S_\varphi := \{\varphi = -\infty\}$ is compact and contains a dense subset of points A_φ where the Lelong number of φ is positive. To such φ one can associate a generalized Green function $G_D(\cdot,\varphi)$ which is the supremum over the family of negative psh functions u in D which satisfy the inequality for Lelong numbers $\nu(u,a) \geq \nu(\varphi,a)$.

The author proves the basic properties of $G = G_D(\cdot,\varphi)$. It is continuous on $D \setminus S_\varphi$ and satisfies the complex Monge-Ampère equation $(dd^cG)^n = 0$ in this set. Moreover, $G = -\infty$ on A_φ. The function is also the unique solution of the Dirichlet problem for the Monge-Ampère equation where one preassigns boundary values, Lelong numbers $\nu(\varphi,a)$ on A_φ, and the measure $(2\pi)^n \sum \nu(\varphi,a)^n \delta_a$.

It is not clear if $G = -\infty$ on the whole set S_φ. The author poses a question concerning that problem. In particular, it would be interesting to know if given S_φ one can find $u \in \mathrm{PSH}(D)$ with $S_u = S_\varphi$ and $(dd^cu)^n = 0$ on $D \setminus S_\varphi$.

Reviewed by *Sławomir Kołodziej*

MR1474805 (98i:32051) 32L30 (14J99 32J15)

Brunella, Marco (F-DJON-T)

Feuilletages holomorphes sur les surfaces complexes compactes. (French. English, French summaries) [Holomorphic foliations on compact complex surfaces]

Ann. Sci. École Norm. Sup. (4) **30** (1997), *no. 5,* 569–594.

FEATURED REVIEW.

In this remarkable, carefully written and deep work, the classification of (non-singular) holomorphic foliations on compact complex surfaces is achieved. The first two sections of the paper recall and exploit the basic and essential material needed, especially P. Baum and R. Bott's formulae in the two-dimensional case [see J. Differential Geometry **7** (1972), 279–342; MR0377923 (51 #14092)], as well as behaviour under blowing-up. For instance, a nice and simple application of the contents of Section 1 is the following: Suppose X is a compact complex surface and $\mathcal{F}$

and $\mathcal{G}$ are non-singular holomorphic foliations on X which are transverse. Let $S \subset X$ be a curve which is neither $\mathcal{F}$-invariant nor $\mathcal{G}$-invariant. It follows that $S \cdot S \geq \chi(S)$ and hence, that X does not contain holomorphic spheres with negative self-intersection; in particular, X is minimal.

The remainder of the work can be divided into two parts, one dealing with surfaces of non-general type, the other with surfaces of general type.

In the first part, Sections 3, 4 and 5, foliations on surfaces of non-general type are studied. The Enriques-Kodaira classification is used throughout [see W. P. Barth, C. A. M. Peters and A. J. H. M. Van de Ven, *Compact complex surfaces*, Springer, Berlin, 1984; MR0749574 (86c:32026)]. General arguments are developed in Sections 3 and 4, especially exploiting the existence of fibrations, over "most" algebraic surfaces X, with generic fibre a rational or an elliptic curve. These fibrations are cleverly used as "reference fibrations", with which a foliation $\mathcal{F}$ in X is compared, that is, either $\mathcal{F}$ coincides with the fibration, is transverse to it, or else is a "turbulent" foliation. The concept of turbulent foliation is due to Reeb and was generalized, in the complex context, by É. Ghys [Ann. Fac. Sci. Toulouse Math. (6) **5** (1996), no. 3, 493–519; MR1440947 (98d:32037)]. Inspired by the definition of Ghys, the author defines a turbulent foliation on a compact complex surface X as a foliation $\mathcal{F}$ with the following property: there is a regular elliptic fibration of X such that: (i) a finite number of fibres are $\mathcal{F}$-invariant and (ii) all other fibres are transverse to $\mathcal{F}$. The results in the case of algebraic surfaces can be summarized as: If X is a compact complex algebraic surface and $\mathcal{F}$ is a non-singular holomorphic foliation on X, then we have one of the following (non-exclusive) possibilities: (1) $\mathcal{F}$ is a fibration; (2) $\mathcal{F}$ is transverse to a fibration; therefore, it is a suspension of an automorphism group of an algebraic curve; (3) $\mathcal{F}$ is a linear foliation on a complex torus; (4) $\mathcal{F}$ is a transversely hyperbolic foliation with dense leaves whose universal cover is a fibration over the disc, with the disc as typical fibre; (5) $\mathcal{F}$ is a turbulent foliation over an elliptic surface.

Section 5 of the paper displays the structure of foliations over surfaces of non-general type, and the results are: Let $\mathcal{F}$ be a non-singular holomorphic foliation on a compact complex surface X of Kodaira dimension < 2. Then $\mathcal{F}$ is one of the following objects (observe that such surfaces satisfy $c_1^2(X) = 2c_2(X)$): (1) an elliptic or rational fibration; (2) a foliation transverse to an elliptic or rational fibration; (3) a linear foliation on a complex torus; (4) certain (precisely described) foliations on Hopf and Inoue surfaces; (5) a turbulent foliation.

In the second part, Sections 6 and 7, the case of foliations on surfaces of general type is treated by considering dynamical properties of the foliation; essentially, a transverse invariant metric is constructed, and the arguments here are quite involved and make use of a result of J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)]. The main result of Sections 6 and 7 is: If $\mathcal{F}$ is a non-singular foliation on a surface of general type, then $\mathcal{F}$ is either a fibration or a transversely hyperbolic foliation with dense leaves whose universal cover is a fibration over the disc, with the disc as typical fibre.

Let us point out some amazing consequences of the results above: (i) The only non-minimal surface which admits a non-singular foliation is $\mathbf{P}^2_{\mathbf{C}}$ blown up at a point, and this foliation is the blow-up of the radial foliation centered at this point. (ii) If a surface X admits a regular foliation, then its signature is non-negative, that is, $\frac{1}{3}(c_1^2(X) - 2c_2(X)) \geq 0$. In particular, the only complete

intersection which admits a non-singular foliation is $\overline{\mathbf{C}} \times \overline{\mathbf{C}}$.

In conclusion, Brunella has synthesized and clearly presented a key and very important result. This is a very substantial contribution to the area.

Reviewed by *M. G. Soares*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.