MR2296419 32Jxx
Debarre, Olivier (F-STRAS-I)
Classes de cohomologie positives dans les variétés kählériennes compactes (d'après Boucksom, Demailly, Nakayama, Păun, Peternell et al.). (French. French summary) [Positive cohomology classes in compact Kähler manifolds (after Boucksom, Demailly, Nakayama, Păun, Peternell et al.)]
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{A review for this item is in process.}

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2275023 (Review) 14B05 (13A35 14M05)
Takagi, Shunsuke [Takagi, Shunsuke[2]] (J-KYUSM)
Formulas for multiplier ideals on singular varieties. (English summary)
Amer. J. Math. **128** (2006), *no. 6,* 1345–1362.

The paper under review uses characteristic p methods including tight closure to generalize certain properties of multiplier ideals to the case of singular varieties.

Recall that for a $\mathbb{Q}$-Gorenstein normal variety X over a field of characteristic zero, an ideal sheaf $\mathfrak{a} \subset \mathcal{O}_X$, and a real number $t \geq 0$, one has the associated multiplier ideal sheaf $\mathcal{J}(\mathfrak{a}^t) \subseteq \mathcal{O}_X$. Details may be found in [R. K. Lazarsfeld, *Positivity in algebraic geometry. II*, Springer, Berlin, 2004; MR2095472 (2005k:14001b)]. One also defines “mixed” multiplier ideals $\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s)$. The multiplier ideals satisfy a number of interesting properties, including the following two:

Subadditivity: J.-P. Demailly, L. M. H. Ein and Lazarsfeld [Michigan Math. J. **48** (2000), 137–156; MR1786484 (2002a:14016)] showed the following subadditivity property of multiplier ideals on a smooth variety:

$$\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s) \subseteq \mathcal{J}(\mathfrak{a}^t)\mathcal{J}(\mathfrak{b}^s).$$

Summation: M. Mustaţă [Trans. Amer. Math. Soc. **354** (2002), no. 1, 205–217 (electronic); MR1859032 (2002k:14006)] showed the following summation formula for multiplier ideals on a

smooth variety:

$$\mathcal{J}((\mathfrak{a}+\mathfrak{b})^t)=\sum_{\lambda+\mu=t}\mathcal{J}(\mathfrak{a}^\lambda)\mathcal{J}(\mathfrak{b}^\mu).$$

The paper under review generalizes these formulas to the case of $\mathbb{Q}$-Gorenstein normal X over a field K of characteristic zero. The generalizations involve the Jacobian ideal of X over K, $\mathfrak{J}(X/K)$:

$$\mathfrak{J}(X/K)\mathcal{J}(\mathfrak{a}^t\mathfrak{b}^s)\subseteq\mathcal{J}(\mathfrak{a}^t)\mathcal{J}(\mathfrak{b}^s),$$
$$\mathcal{J}((\mathfrak{a}+\mathfrak{b})^t)=\sum_{\lambda+\mu=t}\mathcal{J}(\mathfrak{a}^\lambda\mathfrak{b}^\mu).$$

The method relies on the theory of tight closure introduced by M. Hochster and C. L. Huneke [J. Amer. Math. Soc. **3** (1990), no. 1, 31–116; MR1017784 (91g:13010)]. A generalization, called $\mathfrak{a}^t$-tight closure, was introduced by N. Hara and K. Yoshida [Trans. Amer. Math. Soc. **355** (2003), no. 8, 3143–3174 (electronic); MR1974679 (2004i:13003)]. The test ideals $\tau(\mathfrak{a})$ introduced by Hochster and Huneke were generalized to $\mathfrak{a}^t$-test ideals $\tau(\mathfrak{a}^t)$ by Hara and Yoshida, who showed that the $\mathfrak{a}^t$-test ideals correspond to the multiplier ideal $\mathcal{J}(\mathfrak{a}^t)$ via reduction to characteristic $p\gg 0$.

In the paper under review, analogs of the subadditivity and summation formulas are proved for the test ideals $\tau(\mathfrak{a}^t)$, allowing the author to prove the promised generalizations for multiplier ideals on singular varieties. Similar formulas are proved for asymptotic multiplier ideals [L. M. H. Ein, R. K. Lazarsfeld and K. E. Smith, Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001)] using asymptotic versions of the test ideals.

Finally, the subadditivity formula for asymptotic multiplier ideals is applied to answer a question of Hochster and Huneke [Invent. Math. **147** (2002), no. 2, 349–369; MR1881923 (2002m:13002)] on the growth of symbolic powers of an ideal.

Reviewed by *Zach Teitler*

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MR2272098 (2007g:32022) 32U25 (32U05 32U35)
Rashkovskii, Alexander [Rashkovskiĭ, A. Yu.]
Relative types and extremal problems for plurisubharmonic functions.
Int. Math. Res. Not. **2006,** *Art. ID* 76283, 26 *pp.*

The paper under review is concerned with singularities of plurisubharmonic (psh) functions. A psh function u defined in a neighborhood of a point $\zeta \in \mathbf{C}^n$ is said to have a singularity at ζ if $u(\zeta) = -\infty$.

There are many ways of measuring the "strength" of the singularity. The most basic invariant is the Lelong number. It can be defined in two different ways: as a growth order of u at the origin, or as a Monge-Ampère mass (or intersection number). More precisely, if we set $\varphi(x) = \log|x - \zeta|$, then the first characterization of the Lelong number is as the $\liminf$ when $x \to \zeta$ of $u(x)/\varphi(x)$.

The second characterization is as the mass of the measure $dd^c u \wedge dd^c \varphi$ at ζ.

J.-P. Demailly [Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)] defined a generalized Lelong number $\nu(\varphi, u)$ with respect to an arbitrary psh weight φ (i.e. a psh function with an isolated singularity at ζ) using the second formula above. He observed that these generalized Lelong numbers share many of the same properties as the usual Lelong number, especially when the weight is locally maximal outside the origin.

Here the author instead generalizes the first characterization of the Lelong number, thus defining the relative type $\sigma(\varphi, u)$ which in general differs from the corresponding generalized Lelong number $\nu(\varphi, u)$. They do, however, agree when $\varphi = \max_{1\le k\le n} a_k^{-1} \log|x_k - \zeta_k|$ for some constants $a_k > 0$, and in this case one obtains the directional Lelong numbers defined by C. O. Kiselman [Ann. Polon. Math. **60** (1994), no. 2, 173–197; MR1301603 (95i:32024)].

One can define a tropical structure on the cone of psh functions, with the tropical addition given by $\max\{u, v\}$ and the tropical multiplication by $u + v$. Endow the set $[0, +\infty]$ with an analogous tropical structure, but where tropical addition is given by $\min\{s, t\}$. For a given psh weight φ, the generalized Lelong number $\nu(\varphi, \cdot)$ is then tropically multiplicative, but not tropically additive in general. On the other hand, the relative type $\sigma(\varphi, \cdot)$ is tropically additive, but not tropically multiplicative in general.

A main result in the paper (Theorem 4.3) characterizes relative weights as functionals on psh functions near ζ that are upper semicontinuous, positively homogeneous and tropically additive.

If φ is chosen such that the relative weight $\sigma(\varphi, \cdot) \equiv \nu(\varphi, \cdot)$, then the resulting functional is both tropically additive and multiplicative. It can then be checked that the assignment $f \mapsto \sigma(\varphi, \log|f|)$ defines a valuation on the ring of holomorphic germs at ζ [see C. Favre and M. Jonsson, Invent. Math. **162** (2005), no. 2, 271–311; MR2199007 (2006k:32064)].

As the author points out, "Tropical additivity of the relative types make them a perfect tool for dealing with upper envelopes of families of plurisubharmonic functions with prescribed singularities." The paper contains several applications of relative weights to pluricomplex Green functions and a Siu-type theorem on the analyticity of superlevel sets of relative weights.

Reviewed by *Mattias Jonsson*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2257847 (2007h:14021) 14F10 (32Q45)
Rousseau, Erwan (3-QU)
Équations différentielles sur les hypersurfaces de $\mathbb{P}^4$. (French. English, French summaries)
[Differential equations on hypersurfaces in $\mathbb{P}^4$]
J. Math. Pures Appl. (9) **86** (2006), *no. 4,* 322–341.

The paper under review deals with entire holomorphic curves into hypersurfaces in complex projective spaces.

After the studies of J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] and Y. T. Siu [in *The legacy of Niels Henrik Abel*, 543–566, Springer, Berlin, 2004; MR2077584 (2005h:32061)], the author gives theorems on existence of global sections of holomorphic jet bundles. Let X be a smooth hypersurface of degree $d \geq 2$ in $\mathbf{P}^4$ and T_X its holomorphic cotangent bundle. Denote by $E_{k,m}T^*_X$ the vector bundle of jet differentials on X of order k and of degree m. Let A be an ample line bundle over X. Then he first shows that $H^0(X, E_{2,m}T^*_X) = 0$, and if $d \geq 97$, then $H^0(X, E_{3,m}T^*_X \otimes A^{-1}) \neq 0$ for sufficiently large m.

This yields that every entire holomorphic curve $f\colon \mathbf{C} \to X$ must satisfy an algebraic differential equation of third order, which is the main result in the present paper. The logarithmic version of this result is also obtained, that is, every entire curve in the complement of a smooth hypersurface in $\mathbf{P}^3$ of degree $d \geq 92$ must satisfy an algebraic differential equation of third order.

REVISED (June, 2007)

Current version of review. Go to earlier version.

Reviewed by *Yoshihiro Aihara*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2254806 (2007e:14066) 14J70 (14J25 32Q45)
Bogomolov, Fedor (1-NY-X); **De Oliveira, Bruno** (1-MIAM)
Hyperbolicity of nodal hypersurfaces. (English summary)
J. Reine Angew. Math. **596** (2006), 89–101.

S. Kobayashi [*Hyperbolic manifolds and holomorphic mappings*, Dekker, New York, 1970; MR0277770 (43 #3503)] conjectured that a generic surface X of $\mathbb{P}^3$ of degree $d \geq 5$ is Kobayashi hyperbolic, i.e., there is no nonconstant holomorphic map from $\mathbb{C}$ into X. J.-P. Demailly and J. El Goul [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)] proved the conjecture for very generic surfaces of degree $d \geq 21$. In the paper under review, the authors deal with algebraic quasi-hyperbolicity, i.e., the property of having only finitely many rational and elliptic curves. Their main result is that a nodal hypersurface X of $\mathbb{P}^3$ of degree d with a sufficiently large number l of nodes, $l > \frac{8}{3}(d^2 - \frac{5}{2}d)$, is algebraically quasi-hyperbolic. Such surfaces exist for degrees $d \geq 6$ [Y. Miyaoka, Math. Ann. **268** (1984), no. 2, 159–171; MR0744605 (85j:14060)].

The strategy of the proof is to study symmetric differentials on the minimal resolution Y of X, i.e., global sections of $S^m\Omega^1_Y$. In a previous work by one of the authors [F. A. Bogomolov, Dokl. Akad. Nauk SSSR **236** (1977), no. 5, 1041–1044; MR0457450 (56 #15655)], it was shown that the existence of sufficiently many symmetric differentials on a surface of general type implies its algebraic quasi-hyperbolicity. It is a well-known result of F. Sakai [in *Algebraic geometry (Proc. Summer Meeting, Univ. Copenhagen, Copenhagen, 1978)*, 545–563, Lecture Notes in Math., 732, Springer, Berlin, 1979; MR0555717 (82b:32043)] that smooth surfaces in $\mathbb{P}^3$ have no symmetric differentials. So here, the main observation of the authors is to show the contribution of the singularities to the existence of symmetric differentials. This is done using Riemann-Roch computations generalized to orbifolds.

Reviewed by *Erwan Rousseau*

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1287, English translation: Math. USSR Izv. **13** (1) (1979), 499–544. MR0522939 (80j:14014)
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MR2229475 (2007b:14034) 14E30 (14J40)

Peternell, Thomas (D-BAYR-IM)

Kodaira dimension of subvarieties. II. (English summary)

Internat. J. Math. **17** (2006), *no. 5,* 619–631.

In this paper the author proves that, given a subvariety A of a smooth projective variety X with various special properties, X is uniruled, continuing an investigation which was initiated in Part I [T. Peternell, M. H. Schneider and A. J. Sommese, Internat. J. Math. **10** (1999), no. 8, 1065–1079; MR1739364 (2001e:14016)].

To give the flavour of the type of results contained in this paper, suppose that the normal bundle of A is ample. Then it was proved in [op. cit.] that either A is of general type or the Kodaira dimension of X is $-\infty$. Using a beautiful result of S. Boucksom, J.-P. Demailly, M. Paun and Peternell ["The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension", preprint, arxiv.org/abs/math/0405285] which says that if K_X is not pseudo-effective, then X is uniruled, the author is able to conclude that if A is not of general type, then in fact X is uniruled, and he is able to weaken the hypothesis on the positivity of the normal bundle of A.

Reviewed by *James McKernan*

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14. Th. Peternell, M. Schneider and A. J. Sommese, Kodaira dimension of subvarieties, *Int. J. Math.* **10** (1999) 1065–1079. MR1739364 (2001e:14016)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2228686 (2007e:14067) 14J70 (32Q45)
Debarre, Olivier (F-STRAS-I); **Pacienza, Gianluca** (F-STRAS-I); **Păun, Mihai** (F-NANC-IE)
Non-deformability of entire curves in projective hypersurfaces of high degree. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **56** (2006), *no. 1*, 247–253.

The Kobayashi conjecture asserts that the general hypersurface in $\mathbb{P}^n$ of degree $d \geq 2n-1$ is Kobayashi hyperbolic. Brody's criterion of hyperbolicity states that X is Kobayashi hyperbolic if and only if there are no entire curves on X, i.e. any holomorphic map $f\colon \mathbb{C} \to X$ is constant. For $n=2$, J.-P. Demailly and J. El Goul showed that the very general hypersurface in $\mathbb{P}^3$ of degree $d \geq 21$ is hyperbolic [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)]. Previously M. McQuillan had done this for the case $d \geq 36$ [Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)]. More generally, Y. T. Siu showed that the general hypersurface of degree d in $\mathbb{P}^n$ for any n is hyperbolic if d is sufficiently large [in *The legacy of Niels Henrik Abel*, 543–566, Springer, Berlin, 2004; MR2077584 (2005h:32061)].

This article is short and well written. The authors prove that there is no entire curve on a smooth hypersurface X of degree $d \geq 2n$ that deforms with X along an open subset of the parameter

space S of the universal family $\mathcal{X} \to S$ of hypersurfaces of $\mathbb{P}^n$ of degree d. This result says that the Kobayashi conjecture can only fail for $d \geq 2n$ if the general hypersurface X has an entire curve which is not preserved by any local universal deformation of X. As the authors mention, their result follows from the Kobayashi conjecture, hence it is only new for the cases not covered by the results stated in the previous paragraph. To prove their main theorem the authors, as Siu did in the paper previously mentioned, bring to the transcendental case the variational approach initiated by C. H. Clemens to prove algebraic hyperbolicity [Ann. Sci. École Norm. Sup. (4) **19** (1986), no. 4, 629–636; MR0875091 (88c:14037); see also L. M. H. Ein, Invent. Math. **94** (1988), no. 1, 163–169; MR0958594 (89i:14002); C. Voisin, J. Differential Geom. **44** (1996), no. 1, 200–213; MR1420353 (97j:14047)]. The authors use and prove Siu's result about global generation of $T_{\mathcal{X}} \otimes p^* \mathcal{O}_{\mathbb{P}^n}(1)$ to obtain a sequence of functions that proves their main theorem by contradiction using the standard negative curvature arguments.

Reviewed by *Bruno N. de Oliveira*

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J. Diff. Geom. (1998), no. 49, p. 601–611. MR1669712 (99m:14078)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2226021 (2007c:32031) 32Q45 (13A50)
Rousseau, Erwan (3-QU)
Étude des jets de Demailly-Semple en dimension 3. (French. English, French summaries) [Study of Demailly-Semple jets in dimension 3]
Ann. Inst. Fourier (Grenoble) **56** (2006), *no. 2,* 397–421.

The paper under review deals with the characterization of Demailly-Semple jets, which is closely related to the hyperbolicity of algebraic varieties. After the study of J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)], the author gives an algebraic characterization of Demailly-Semple jets in dimension three by making use of the invariant theory of nonreductive groups. Let X be a complex manifold of dimension three and T^*_X its cotangent bundle. Denote by $E_{k,m}T^*_X$ the vector bundle of jet differentials on X of order k and of degree m. Put $A_k = \bigoplus_m (E_{k,m}T^*_X)_x$ for $x \in X$. The author describes the structure of A_3 and gives a characterization for $\mathrm{Gr}^\bullet E_{3,m}T^*_X$.

Namely, he proves that $\mathrm{Gr}^\bullet E_{3,m}T^*_X$ can be written as a direct sum of the spaces of Schur polynomials on $S^{a_j}T^*_X$ for some $a_j \in \mathbf{Z}^+$, which is the main result in this paper.

Some results of Riemann-Roch type are also obtained in the case in which X is a hypersurface in projective 4-space.

Reviewed by *Yoshihiro Aihara*

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MR2212883 (2007b:32039) 32Q20 (14J45 14J70)

Arezzo, Claudio (I-PARM); **Ghigi, Alessandro** (I-PAVI); **Pirola, Gian Pietro** (I-PAVI)

Symmetries, quotients and Kähler-Einstein metrics. (English summary)

J. Reine Angew. Math. **591** (2006), 177–200.

It is well known that it is extremely difficult to check whether a Fano manifold has a Kähler-Einstein metric or not. For instance, even the case of del Pezzo surfaces is far from being obvious, and in full generality it is not known for hypersurfaces of the projective space. G. Tian proved that Fermat hypersurfaces are Kähler-Einstein, showing the properness of a certain energy functional F_ω whose Euler-Lagrange equation is the Monge-Ampère equation. In this paper, the behaviour of F_ω under a Galois covering is studied, and some algebraic conditions on the covering maps are found to ensure the properness of this functional. This allows the authors to extend widely the work of Tian to other classes of Fano manifolds:

(1) hypersurfaces of the form
$$\{x_0^d+\cdots+x_{k-1}^d+f(x_k,\dots,x_{n+1})=0\}\subset\mathbb{P}^{n+1}$$
where f is a homogeneous polynomial of degree d, and $k>n+2-d$;

(2) n-dimensional intersections of hypersurfaces of the same form as above, all of the same degree d, and $k>n+2-d$;

(3) arbitrary intersections of two (hyper)quadrics;

(4) double covers of $\mathbb{P}^n$ ramified along a smooth hypersurface of degree $2d$ with $\frac{n+1}{2}<d\le n$;

(5) double covers of the n-dimensional quadric $Q_n\subset\mathbb{P}^{n+1}$ with smooth branching locus cut out by a hypersurface of degree $2d$ with $\frac{n}{2}<d<n$.

In order to get the result, the authors prove the following interesting fact. Let $\pi\colon M\to N$ be a ramified Galois covering of degree d with structure group G, ω_N a Kähler-Einstein metric on N and $\omega\in 2\pi c_1(M)$ a G-invariant Kähler metric on M. If we denote $R(\pi)$ the ramification divisor of π and assume that in homology $R(\pi)=-\beta K_M$ for a certain $\beta\in\mathbb{Q}_+$, then there is a constant C such that for all G-invariant smooth potential φ with $\omega+\sqrt{-1}\partial\overline{\partial}\varphi>0$,
$$F^0_\omega(\varphi)\ge\frac{1}{1+\beta}\log\left(\frac{1}{V}\int_M e^{-(1+\beta)\varphi}\pi^*\omega_N^n\right)-C.$$
Here M has complex dimension n and $V=\int_M\omega^n$. Note that for a potential φ such that $\frac{1}{V}\int_M e^{h(\omega)-\varphi}\omega^n=1$, where $\mathrm{Ric}(\omega)-\omega=\sqrt{-1}\partial\overline{\partial}h(\omega)$, one has $F^0_\omega(\varphi)=F_\omega(\varphi)$. We refer to the survey [G. Tian, *Canonical metrics in Kähler geometry*, Birkhäuser, Basel, 2000; MR1787650 (2001j:32024)] for the details about the functionals F_ω and F^0_ω and the equivalence between the existence of a Kähler-Einstein metric and the properness of F_ω.

With a view to applying their result to the examples above, the authors need to control the integral $\int_M e^{-(1+\beta)\varphi}\omega^n$ with the term $\int_M e^{-(1+\beta)\varphi}\pi^*\omega^n_N$ in order to get the required properness. Interestingly, this leads one to the consideration for $\eta=\frac{\pi^*\omega^n_N}{\omega^n}$ of the real number $c=\sup\{r\ge 0\colon \frac{1}{\eta^r}\in L^1(M,\omega^n)\}$, which is just the infimum over M of the complex singularity exponent of the ideal sheaf induced by the divisor $R(\pi)$ [J.-P. Demailly and J. Kollár, Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]. Although the singularities of the ramification divisor are quite mild, it is difficult to compute the complex singularity

exponent in full generality. Nevertheless, if the reduced divisor associated to $R(\pi)$ is smooth at $p \in M$, then there is a holomorphic function f defined on a neighborhood of p such that $R(\pi) = \{f^m = 0\}$ with $m \leq \deg(\pi) - 1$ and $Df(p) \neq 0$. This gives the required lower bound for the complex singularity exponent at the point p under a natural assumption on the covering. Then, the manifolds quoted previously enter into this framework.

Reviewed by *Julien Keller*

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MR2211329 (2007f:32027) 32L10 (32L20 47B35)
Berman, Robert (S-CHAL)
Super Toeplitz operators on line bundles. (English summary)
J. Geom. Anal. **16** (2006), *no. 1,* 1–22.

Let L be a Hermitian line bundle over a compact complex manifold X. Denote by $X(q)$ the subset of X where the curvature form of L is nondegenerate and has exactly q negative eigenvalues, by $H^0(X,L)$ the space of all holomorphic sections of L, and by $\{\psi_j\}$ any orthonormal basis of $H^0(X,L)$. The Bergman kernel of $H^0(X,L)$ is then the holomorphic section of $L\otimes\overline{L}$ defined by $K(x,y)=\sum_j \psi_j(x)\otimes\overline{\psi_j(y)}$, and the author calls

$$B(x):=\|K(x,x)\|=\sup\{\|f(x)\|^2:\ f\in H^0(X,L),\ \|f\|\leq 1\}$$

the Bergman function. Let $B_k(x)$ be similarly defined for L^k in the place of L. Using Demailly's holomorphic Morse inequalities, the author shows that if $X(1)=\varnothing$ then

$$k^{-n}B_k(x)\to\pi^{-n}\,\mathbf{1}_{X(0)}(x)\,\|\det\frac{i}{2}\partial\overline{\partial}\varphi(x)\|.$$

This is then elaborated on in two directions. First, applications are given to the asymptotic sets of sampling in $H^0(X,L^k)$ as $k\to\infty$, generalizing, in particular, the result of Boutet de Monvel and Guillemin on the counting function for the eigenvalues of a Toeplitz operator T_f with real-valued symbol f. Second, an analogous theory is developed if $H^0(X,L^k)$ is replaced by the spaces $\mathcal{H}^q(X,L^k)$ of harmonic $(0,q)$-forms on X with values in L^k. It turns out that this admits a very convenient formulation in the language of supermanifolds and superintegrals and, again, has implications concerning the asymptotic distribution of eigenvalues, as $k\to\infty$, of certain "super-Toeplitz" operators T_f whose symbols f are differential forms on X. The super formalism also allows a compact notation.

Reviewed by *Miroslav Engliš*

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MR2209220 (2006k:32046) 32Q15 (32Q05 53C25 53C55)
Fang, Fuquan (PRC-CAP)
Kähler manifolds with numerically effective Ricci class and maximal first Betti number are tori. (English, French summaries)
C. R. Math. Acad. Sci. Paris **342** (2006), *no. 6,* 411–416.

In [Compositio Math. **89** (1993), no. 2, 217–240; MR1255695 (95b:32044)], J.-P. Demailly, T. Peternell and M. H. Schneider generalized the notion of a numerically effective (nef) holomorphic line bundle over an algebraic variety to any compact complex manifold. A Kähler manifold with nef anticanonical bundle, or equivalently, numerically effective Ricci class, is called a nef Kähler manifold in the paper under review. For a nef Kähler manifold M, Demailly, Peternell and Schneider conjectured that the Albanese map $\alpha\colon M \to \mathrm{Alb}(M)$ is surjective. When M is a projective manifold of arbitrary dimension, Q. Zhang proved this conjecture in [J. Reine Angew. Math. **478** (1996), 57–60; MR1409052 (97m:14039)] by using relative deformation theory and mod p reductions (which were originally used by S. Mori to settle Hartshorne's conjecture). Using the analytic techniques of differential geometry, M. Paun proved the conjecture under the assumption that the Ricci class of M is integrable in [Comm. Anal. Geom. **9** (2001), no. 1, 35–60; MR1807951 (2001m:32050)]. F. Campana, Peternell and Zhang confirmed the conjecture when the dimension of M is not greater than four in [Proc. Amer. Math. Soc. **131** (2003), no. 2, 549–553 (electronic); MR1933346 (2004e:32020)].

Let M be a nef Kähler manifold of dimension n. In this well-written and very readable paper the author proves: (1) If the first Betti number $b_1(M) = 2n$, then M is biholomorphic to a complex torus of dimension n. In particular, if $b_1(M) = 2n$, then the Albanese map $\alpha\colon M \to \mathrm{Alb}(M)$ is surjective. (2) Let G be the fundamental group of M and $G' = [G, G]$ be the commutator subgroup of G. If the first Betti number $b_1(M) = 2n - 2$, and $G'/[G', G]$ has rank at least two, then the Albanese map $\alpha\colon M \to T_{\mathbb{C}}^{n-1}$ is surjective.

The author claims that the proof of the second result follows along the same lines as the first one. The following is the main idea of the proof of the first main result. By the Aubin-Calabi-Yau theorem, Demailly, Peternell and Schneider proved that a Kähler manifold is nef if and only if there exists a sequence of Kähler metrics $\{\omega_k\}$ such that for any $k > 0$, $\{\omega_k\}$ belongs to a fixed Kähler class $[\omega]$, and the Ricci curvature of ω_k is bounded from below by $-\frac{1}{k}$. Let $\widetilde{M}_k$ be the Riemannian covering space of M_k (the manifold M with metric ω_k). Let $\overline{M}_k = \widetilde{M}_k/G'$. Using the equivariant Gromov-Hausdorff convergence and Gromov compactness theorem, and a splitting theorem of Cheeger-Colding for limit spaces, the author proves that there is a finite index torsion-free subgroup Γ_k of $\Gamma = G/G'$ such that $(\overline{M}_k, \Gamma_k)$ converges to $(\mathbb{R}^{2n}, \mathbb{Z}^{2n})$. From this the author concludes that $\overline{M}_k/\Gamma_k$ is homeomorphic to a torus, and hence so is M. From the Poincaré-Lelong equation it follows that the Albanese map has no zeros and is actually a biholomorphism.

Reviewed by *Qi Lin Yang*

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MR2199007 (2006k:32064) 32U05 (13A18 32U25)
Favre, Charles (F-PARIS7-GDM); **Jonsson, Mattias** (1-MI)
Valuative analysis of planar plurisubharmonic functions. (English summary)
Invent. Math. **162** (2005), *no. 2,* 271–311.

This is the first of a series of papers in which the authors give applications of the theory of the valuative tree, a notion introduced in their book [*The valuative tree*, Lecture Notes in Math., 1853, Springer, Berlin, 2004; MR2097722 (2006a:13008)].

The valuations considered here are valuations acting on germs of holomorphic functions at $0 \in \mathbf{C}^2$, that is, functions $\nu\colon \mathcal{O}(\mathbf{C}^2, 0) \mapsto [0, \infty]$ such that:

(i) $\nu(\psi\psi') = \nu(\psi) + \nu(\psi')$;

(ii) $\nu(\psi + \psi') \geq \min(\nu(\psi), \nu(\psi'))$;

(iii) $\nu(0) = \infty$, $\nu(1) = 0$, $\min(\nu(x), \nu(y)) = 1$.

In their aforementioned work, the authors proved that the space $\mathcal{V}$ of such valuations has the structure of an $\mathbf{R}$-tree, hence is eligible to be treated by the methods of analysis and measure theory.

The most classical example of valuation is the multiplicity of a germ. The Lelong number can be seen as an extension of this multiplicity, acting on the space of germs of plurisubharmonic (psh) functions.

Using the formalism of the valuative tree, the authors first show that (quasimonomial) valuations can be evaluated on psh functions, giving rise to generalized Lelong numbers in the sense of J.-P. Demailly [Acta Math. **159** (1987), no. 3-4, 153–169; MR0908144 (89b:32019)] (see also [C.-O. Kiselman, in *Séminaire d'Analyse Complexe et Géométrie 1985–1987*, 61–70, Fac. Sci. Tunis/Fac. Sci. Tech. Monastir; per bibl.]).

This allows them to describe singularities of psh functions quite accurately as follows. If u is a psh function with singularity at the origin, $\nu \mapsto \nu(u)$ is a function on the space of valuations, with special convexity properties that allow one to apply the methods of analysis on the valuative tree and obtain a “tree measure” associated to it.

The general idea of the paper is that this tree measure contains a lot of information about u. The authors give some applications of this principle.

The first application is a process of attenuation of singularities of positive closed currents. Given a germ of positive closed current at $0 \in \mathbf{C}^2$, there exists a composition of blowups π such that π^*T decomposes as a current supported on the exceptional divisor plus a current with arbitrary small Lelong numbers. This extends results of Mimouni and Guedj.

The second application is an exact formula for the mass of $dd^c u \wedge dd^c v\{0\}$, where u and v are germs of singular psh functions, under some regularity assumptions on u and v. The value of the mass depends on the tree measures of u and v. As a consequence of this formula, it is proved that every generalized Lelong number is an average of valuations.

An important third application to the theory to multiplier ideal sheaves associated to psh functions appears in a separate paper [C. Favre and M. Jonsson, J. Amer. Math. Soc. **18** (2005), no. 3, 655–684 (electronic);; MR2138140 (2007b:14004)].

Reviewed by *Romain Dujardin*

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MR2191703 (2006k:31004) 31C10
Cegrell, Urban (S-UMEA-IM); **Wiklund, Jonas** (S-UMEA-IM)
A Monge-Ampère norm for delta-plurisubharmonic functions. (English summary)
Math. Scand. **97** (2005), *no. 2,* 201–216.

Let $\Omega \subset \mathbb{C}^n$ be a bounded hyperconvex domain and $\mathrm{PSH}(\Omega) \subset L^1_{\mathrm{loc}}(\Omega)$ be the cone of plurisubharmonic functions on Ω. It is well known from the pioneering work of Bedford and Taylor that the complex Monge-Ampère operator $(dd^c u)^n$ is well defined on the class of bounded plurisubharmonic functions u on Ω, but not for all plurisubharmonic functions on Ω. Later on, Cegrell introduced interesting classes of (singular) plurisubharmonic functions on Ω for which the complex Monge-Ampère operator is well defined as a Radon measure on Ω and which play an important role in the solution of the Dirichlet problem for the complex Monge-Ampère equation [see U. Cegrell, Acta Math. **180** (1998), no. 2, 187–217; MR1638768 (99h:32016)].

In the paper under review, the authors study one of these classes, namely, the class $\mathcal{F}(\Omega)$ of functions $\varphi \in \mathrm{PSH}(\Omega)$ for which there exists a decreasing sequence (φ_j) of plurisubharmonic functions on Ω with boundary values 0 which converges to φ on Ω and satisfies $\sup_j \int_\Omega (dd^c\varphi_j)^n < +\infty$. It follows from Cegrell's work that for $\varphi \in \mathcal{F}(\Omega)$ the Monge-Ampère measure $(dd^c\varphi)^n$ is well defined on Ω and is of finite mass on Ω.

The set $\mathcal{F}(\Omega)$ is a cone in the linear space $L^1_{\mathrm{loc}}(\Omega)$. It is then natural to consider the linear subspace generated by this cone. This is the set $\delta\mathcal{F}(\Omega)$ of functions $u \in L^1_{\mathrm{loc}}(\Omega)$ such that there exist $u_1, u_2 \in \mathcal{F}(\Omega)$ satisfying $u = u_1 - u_2$.

The authors then define a norm on $\delta\mathcal{F}(W)$ by

$$\|u\| := \inf\left\{\int_\Omega (dd^c(u_1+u_2))^n;\ u_1, u_2 \in \mathcal{F}(\Omega), u = u_1 - u_2\right\}.$$

The first main result of the paper states that $\|\cdot\|$ is a norm on the linear space $\delta\mathcal{F}(W)$ and this space is a Banach space. Moreover the authors characterize its dual as the linear space generated by the dual cone $\mathcal{F}'(\Omega)$.

Finally the authors prove that Demailly's generalized Lelong numbers functional is continuous on the Banach space $\delta\mathcal{F}(W)$.

Reviewed by *Ahmed Zeriahi*

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MR2192217 (2006k:32042) 32L20 (14F17)
Laytimi, F. (F-LILL); **Nahm, W.** (IRL-DIAS-P)
On a vanishing problem of Demailly.
Int. Math. Res. Not. **2005,** *no. 47,* 2877–2889.

Let X be a smooth projective complex algebraic variety of dimension n. In the first section of the paper the authors discuss several vanishing results for the cohomology of certain vector bundles on X, and explain that one of them (Theorem 1.4) implies all the other ones. (The proof of Theorem 1.4 is given in the second section.) For instance, two of the corollaries of the main result of the paper are the following generalizations of vanishing theorems of Demailly and Griffiths, respectively. In both of these results, we let E be a holomorphic vector bundle of rank e on X, we let L be a line bundle on X, we let p, q be integers with $0 \le p, q \le n$, and we put $r = \min(n-p, n-q)$.

(1) Let $k \ge 1$ be an integer, let $m = \min(e-1, k)$, and assume that $S^{k+(r+m)e}E \otimes L$ is ample. Then
$$H^{p,q}(X, E^{\otimes k} \otimes (\det E)^{m+r} \otimes L) = 0 \qquad \text{for } \; p+q-n > 0.$$

(2) Let $\alpha \ge 0$ be an integer, and assume that $S^{\alpha+r+re}E \otimes L$ is ample. Then
$$H^{p,q}(X, S^{\alpha}E \otimes (\det E)^{r+1} \otimes L) = 0 \qquad \text{for } \; p+q-n > 0.$$

However, the original motivation for the work was trying to answer the following question of Demailly (which is alluded to in the title of the paper). We keep the notation X, n, E, e, L, p, q and r introduced above. Let $a = (a_1, \dots, a_l)$ be a nonincreasing sequence of positive integers of length $l \le e$; its weight is defined to be $|a| = \sum_i a_i$, and a can then be thought of as a partition of the integer $|a|$. This partition determines a Schur functor on the category of vector bundles on X; the value of this functor on E will be denoted by S_aE. (Schur functors arise from the representation theory of the general linear group, and they generalize the symmetric power and exterior power functors.)

If E is ample and L is nef or vice versa, Demailly posed the problem of determining the smallest exponent $j_0 = j_0(n, p, q, a)$ such that $H^{p,q}(X, S_aE \otimes (\det E)^j \otimes L) = 0$ for $j \ge j_0$. Demailly also suggested that $j_0 = r + l$ is sufficient. The authors confirm Demailly's prediction with the

following result (Theorem 1.2 in the paper) which turns out to be equivalent to a special case (Theorem 1.3) of the main theorem of the paper (Theorem 1.4). Namely, let $\widetilde{a}=(\widetilde{a}_1,\widetilde{a}_2,\dots)$ denote the transpose of the partition a, let $m\geq 0$ be an integer, and assume that $S^{|a|+(m+r)e}E\otimes L$ is ample. Then

$$H^{p,q}(X,S_aE\otimes(\det E)^{m+r}\otimes L)=0$$

for $p+q-n>\sum_{\widetilde{a}_i>m}(e-\widetilde{a}_i)$. As the authors remark, the ampleness condition in this result is satisfied if E is ample and L is nef, or vice versa. If we take $m=l$ in the theorem, then the condition on the right-hand side of the last formula becomes $p+q-n>0$, and we arrive at a solution of Demailly's problem, with the exponent being $j_0=r+l$, as he suggested. In the third and final section of the paper the authors show that this result is optimal in a certain range of the parameters n, p, q and a, which, unfortunately, is not symmetric with respect to p and q.

Reviewed by *Dmitriy S. Boyarchenko*

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MR2190241 (2007c:53056) 53C25 (14J25)
Kollár, János (1-PRIN)
Einstein metrics on five-dimensional Seifert bundles. (English summary)
J. Geom. Anal. **15** (2005), *no. 3,* 445–476.

Summary: "The aim of this article is to study Seifert bundle structures on simply connected 5-manifolds. We classify all such 5-manifolds which admit a positive Seifert bundle structure, and in a few cases all Seifert bundle structures are classified. These results are then used to construct positive Ricci curvature Einstein metrics on these manifolds.

"The proof has 4 main steps: first, the study of the Leray spectral sequence of the Seifert bundle, based on work of P. Orlik and P. Wagreich [Invent. Math. **28** (1975), 137–159; MR0361150 (50 #13596)]; second, the study of log del Pezzo surfaces; third, the construction of Kähler-Einstein metrics on del Pezzo orbifolds using the algebraic existence criterion of J.-P. Demailly and J. Kollár [Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]; fourth, the lifting of the Kähler-Einstein metric on the base of a Seifert bundle to an Einstein metric on the total space using the Kobayashi-Boyer-Galicki method."

Reviewed by *Massimiliano Pontecorvo*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2188444 (2006j:14063) 14K12 (14F10 14M10)

Debarre, Olivier (F-STRAS-I)

Varieties with ample cotangent bundle. (English summary)

Compos. Math. **141** (2005), *no. 6,* 1445–1459.

The goal of the paper under review is to construct examples of projective algebraic varieties X with ample cotangent bundle. Such varieties are always of general type. Moreover, when defined over $\mathbb{C}$ they do not admit nontrivial holomorphic maps $\mathbb{C} \to X$ [J.-P. Demailly, in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)], and when defined over a number field K they are conjectured to have only finitely many K-rational points [A. Moriwaki, Math. Res. Lett. **2** (1995), no. 1, 113–118; MR1312981 (96b:14021)]. Although such varieties are expected to be abundant, there are few known concrete examples. This situation is now somewhat remedied.

The main part of the paper is Section 2, in which the author proves the following three theorems: (i) Let $L_1, \ldots, L_c$ be very ample line bundles on a simple abelian variety of dimension n, where $c \geq n/2$. Let $H_1 \in |L_1^{e_1}|, \ldots, H_C \in |L_C^{e_c}|$ be general divisors where $e_2, \ldots, e_c$ are all $> n$; then the complete intersection $H_1 \cap \cdots \cap H_c$ has ample cotangent bundle. (ii) If we change the condition on the e_i's to be large enough and divisible enough, then the same property holds for any abelian variety. (iii) If the dimension n is 4 and $c = 2$, it suffices to take $e_1, e_2 \geq 5$ (this result has been subsequently improved by the author and E. Izadi ["Ampleness of intersections of translates of theta divisors in an abelian fourfold", preprint, arxiv.org/abs/math/0506374], who proved that for any 4-dimensional Jacobian the intersection of two general translates of the theta divisor has ample cotangent bundle). The author proves (i) by showing that the fibers of the natural map $\mathbb{P}(\Omega_X) \to \mathbb{P}(\Omega_{A,0}) \times X \to \mathbb{P}(\Omega_{A,0})$ are 0-dimensional. This is done by showing that if ∂ is a nontrivial constant vector field on A, then there is an inequality $\dim(H_1 \cap \partial H_1 \cap \cdots \cap H_c \cap \partial H_c) \leq \max(n - 2c, 0)$. This inequality is proved by induction on c and is the most technical part of the paper. The proofs of (ii) and (iii) are similar to the proof of (i).

Section 3 of the paper is mostly conjectural—the author conjectures that similar theorems to his theorems on abelian varieties would hold in $\mathbb{P}^n$. In Section 4 he reproduces an unpublished result of F. Bogomolov: Let $X_1, \ldots, X_n$ be smooth projective varieties with big cotangent bundles, all of dimension $\geq d > 0$, and let V be a general linear section of $X_1 \times \cdots \times X_m$ such that $\dim(V) \leq$

$(d(m+1)+1)/2(d+1)$. Then the cotangent bundle of V is ample.

Reviewed by *David Lehavi*

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MR2178969 (2006j:53058) 53C25 (32Q20 57R60)
Boyer, Charles P. (1-NM); **Galicki, Krzysztof** (1-NM); **Kollár, János** (1-PRIN)
Einstein metrics on spheres.
Ann. of Math. (2) **162** (2005), *no. 1,* 557–580.

This paper provides existence theorems for large families of Einstein metrics on the sphere S^5, on all 28 oriented diffeomorphism classes on S^7 as well as on the standard and Kervaire spheres S^{4m+1}. This is a substantial advance on previous work and the techniques used here motivate the authors' conjecture that all odd-dimensional homotopy spheres which bound parallelizable manifolds admit Sasakian-Einstein metrics.

The study of special Riemannian metrics on exotic spheres has quite a long history. D. Gromoll and W. Meyer [Ann. of Math. (2) **100** (1974), 401–406; MR0375151 (51 #11347)] wrote explicitly a metric of non-negative sectional curvature on one of the Milnor 7-spheres. Much later, K. Grove and W. Ziller [Ann. of Math. (2) **152** (2000), no. 1, 331–367; MR1792298 (2001i:53047)] proved that all exotic 7-spheres which are S^3-bundles over S^4 admit such a metric. It is however not known whether exotic spheres admit metrics of positive sectional curvature. Concerning metrics with positive Ricci curvature, they are known to exist on all spheres which bound parallelizable manifolds by a result of D. Wraith [J. Differential Geom. **45** (1997), no. 3, 638–649; MR1472892 (98i:53058)]. In dimension 7, all homotopy spheres have this property, and part of the ideas in the paper under review come from a re-proving of Wraith's theorem by Boyer, Galicki and M. Nakamaye [Topology **42** (2003), no. 5, 981–1002; MR1978045 (2004c:53055)] in the framework of the classical Brieskorn presentation. Dimension 7 was also deeply analyzed by M. Kreck and S. Stolz [Ann. of Math. (2) **127** (1988), no. 2, 373–388; MR0932303 (89c:57042)], who proved the existence of 7-manifolds with the maximal number of 28 smooth structures each of which admits an Einstein metric with positive scalar curvature.

The present paper attacks the problem of existence of Einstein metrics on standard and exotic spheres by combining the original Brieskorn point of view through links L_f of isolated hypersurface singularities with their Sasakian geometry and with a continuity method to construct Kähler-Einstein metrics on their associated transversal structure.

We now sketch some of the ideas entering in the proof.

First, links L_f are defined by choosing a weight vector $\overrightarrow{w}=(w_0,\dots,w_n)\in\mathbb{Z}_+^{n+1}$ and a weighted homogeneous polynomial f of weighted degree $w(f)$: $f(\lambda^{w_0}z_0,\dots,\lambda^{w_n}z_n)=\lambda^{w(f)}f(z_0,\dots,z_n)$. The definition is $L_f=\{z\in\mathbb{C}^{n+1}:\ f(z)=0\}\cap S^{2n+1}$, and when 0 is the only isolated singularity

of f, the weighted S^1-fibration $S^{2n+1}_{\overrightarrow{w}} \to P^n_{\mathbb{C}}(\overrightarrow{w})$ gives rise to an induced fibration $L_f \to Z_f$, relating the Sasakian geometry of the smooth link L_f to its transversal Kähler geometry, encoded in the orbifold Z_f. As a first result, the authors prove that the orbifold Z_f is Fano if and only if $w(f) - \sum w_i < 0$.

A remarkable class of links L_f is given by Brieskorn-Pham links $L(\overrightarrow{a})$, $\overrightarrow{a} = (a_0, \dots, a_n)$, corresponding to polynomials $f = \sum z_i^{a_i}$. Then $w(f) = \operatorname{lcm}(a_0, \dots, a_n)$, the weights are $w_i = \frac{w(f)}{a_i}$, and the Fano condition reads $1 < \sum \frac{1}{a_i}$. It was proved in E. Brieskorn's famous 1966 paper [Invent. Math. **2** (1966), 1–14; MR0206972 (34 #6788)] that $L(\overrightarrow{a})$ is homeomorphic to a sphere provided some conditions hold on an associated graph $G(\overrightarrow{a})$. More generally, perturbed Brieskorn-Pham links $L(\overrightarrow{a}, p)$, corresponding to $f = \sum z_i^{a_i} + p(z_0, \dots, z_n)$, satisfy the same Fano condition for the corresponding Kähler orbifolds $Z(\overrightarrow{a}, p)$. The strategy is now to apply a continuity method to $Z(\overrightarrow{a}, p)$ to ensure on them the existence of a Kähler-Einstein metric. A Sasakian-Einstein metric on $L(\overrightarrow{a}, p)$ will then be obtained from the classical Kobayashi circle bundle construction, suitably adapted to orbifolds.

A second key ingredient in the proof is the continuity method developed through the work of T. Aubin, Y. T. Siu, G. Tian and A. M. Nadel. The aim is to show the existence of a Kähler-Einstein metric of positive sign through the Monge-Ampére equation by starting from a Yau solution for the value $t = 0$ of the parameter and by getting conditions that ensure that the value $t = 1$ can be reached by continuity. The relevant theorem in the authors' proof is in the orbifold category and is due to J.-P. Demailly and Kollár [Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 4, 525–556; MR1852009 (2002e:32032)]. For Fano Kähler hypersurfaces $Z_f \subset \mathrm{P}^n_{\mathbb{C}}(\overrightarrow{w})$ associated to the links L_f, the Demailly and Kollár theorem is in fact used to show that a Kähler-Einstein metric of positive sign can be obtained on Z_f provided the following condition holds: there is a $\gamma > \frac{n}{n+1}$ such that for every weighted homogeneous polynomial g of weighted degree $s(\sum w_i - w(f))$, not identically zero on $Y_f = \{\{f = 0\} \subset \mathbb{C}^{n+1}\}$, the function $|g|^{-\frac{\gamma}{s}}$ is locally L^2 on $Y_f - \{0\}$. Then, by working out this condition for Brieskorn-Pham links $L(\overrightarrow{a}, p)$, the authors prove that a Kähler-Einstein metric on the orbifolds $Z(\overrightarrow{a}, p)$ exists if $1 < \sum \frac{1}{a_i} < 1 + \frac{n}{n+1} \min\{\frac{1}{a_i}, \frac{1}{b_i b_j}\}$, where $b_i = \gcd(C_i, a_i)$, $C_i = \operatorname{lcm}(a_0, \dots, \widehat{a}_i, \dots, a_n)$. Moreover, given two vectors $\overrightarrow{a}$, $\overrightarrow{a}'$ satisfying these conditions, the links $L(\overrightarrow{a})$ and $L(\overrightarrow{a}')$ are isometric if and only if $\overrightarrow{a}$ is a permutation of $\overrightarrow{a}'$.

An enumeration of all sequences $(a_0, \dots, a_n)$ satisfying the above condition can thus be accomplished in some cases, also with the help of computer programs. Moreover, possible perturbing polynomials $p(z_0, \dots, z_n)$ can be considered as well, and some significant cases are examined in the last section of the paper. This leads the authors to obtain 68 inequivalent families of Sasakian-Einstein metrics on S^5, from 231 to 452 families on each of the 28 oriented diffeomorphism classes on S^7, and a doubly exponential number of inequivalent families on standard and Kervaire S^{4m+1}. All these results of course suggest the examination also of homotopy spheres S^{4m+3} and the companion paper by Boyer et al. [Experiment. Math. **14** (2005), no. 1, 59–64; MR2146519 (2006a:53042)] gives a computer assisted proof for S^{11} and S^{15} of the authors' conjecture stated at the beginning of this review.

This extensive range of ideas and techniques has already originated some remarkable developments. Among them, the construction of Einstein metrics on 5-dimensional Seifert bundles, described by Kollár [J. Geom. Anal. **15** (2005), no. 3, 445–476;; MR2190241 (2007c:53056)], and

of further new Einstein metrics on odd-dimensional spheres by A. Ghigi and Kollár ["Kaehler-Einstein metrics on orbifolds and Einstein metrics on spheres" preprint, arxiv.org/abs/math.DG/0507289], are certainly deserving of mention. Very likely, many other developments will follow from the present rich paper.

Reviewed by *Paolo Piccinni*

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MR1489351 (99h:57060)

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2172891 (2006d:32054) 32W20 (32U15)
Kołodziej, Sławomir
The complex Monge-Ampère equation and pluripotential theory. (English summary)
Mem. Amer. Math. Soc. **178** (2005), *no. 840*, *x*+64 *pp*.

This is a survey and update of results in pluripotential theory; mostly due to the author and primarily on existence theorems for the complex Monge-Ampère operator. Chapter 1 gives background material on positive currents and plurisubharmonic (psh) functions. The complex Monge-Ampère operator $(dd^c u)^n$ is defined for locally bounded psh functions in $\mathbf{C}^n$; convergence theorems and comparison theorems are proved for such functions. The relative extremal function and relative capacity cap are introduced and utilized to prove quasicontinuity of psh functions. Josefson's theorem, that locally pluripolar sets are globally pluripolar, and the Bedford-Taylor result, that negligible sets are pluripolar, are also proved. Much of the material is from the Bedford-Taylor papers [E. Bedford and B. A. Taylor, Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351); Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024)], but proofs of some of the convergence theorems and the negligible sets are pluripolar result have been slightly simplified. Chapter 2 introduces the Lelong classes $\mathcal{L}$ and $\mathcal{L}^+$ and the Siciak-Zaharjuta extremal function of a bounded set and proves the capacity comparison theorem of H. J. Alexander and Taylor [Math. Z. **186** (1984), no. 3, 407–417; MR0744831 (85k:32034)]. Chapter 3 solves the Dirichlet problem for the complex Monge-Ampère operator on a strictly pseudoconvex domain Ω: given $\varphi\in C(\partial\Omega)$ and $f\in C(\overline{\Omega})$ with $f\ge 0$, find $u\in \mathrm{PSH}(\Omega)\cap C(\overline{\Omega})$ with $(dd^c u)^n = f\,dV$ in Ω and $\lim_{z'\to z} u(z') = \varphi(z)$ for all $z\in\partial\Omega$. The procedure of Bedford and Taylor in [op. cit., 1976] is followed with some minor modifications of J.-P. Demailly [in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)].

The final three chapters cover more recent results—all within the past ten years—on the Dirichlet problem and the complex Monge-Ampère operator. In Chapter 4, the author generalizes the class of admissible data f for solvability of the Dirichlet problem. First, define a class of measures μ that satisfy a type of local domination by relative capacity: for an "admissible" function $h\colon \mathbf{R}_+\to (1,\infty)$ and a positive constant A,

$$\mathcal{F}(A,h) := \{\mu\colon\ \mu(K)\le F(\mathrm{cap}(K,\Omega)),\ F(x)=\frac{Ax}{h(x^{-1/n})},\ K \text{ compact}\}.$$

A priori estimates for the sup-norm of solutions of the Dirichlet problem with μ in place of $f\,dV$ are proved which imply the existence of continuous solutions of the Dirichlet problem for a wide class of absolutely continuous measures $f\,dV$ (Theorem 4.6). This class includes all $f\in L^p_{\mathrm{loc}}(\Omega)$ for $p>1$. If one relaxes the assumption of continuity of the solution and asks for a bounded solution of the Dirichlet problem, Theorem 4.7 shows that if such a Dirichlet problem admits a subsolution then it admits a solution. These results are from [S. Kołodziej, Ann. Polon. Math. **65** (1996), no. 1,

11–21; MR1414748 (98a:32015); Indiana Univ. Math. J. **44** (1995), no. 3, 765–782; MR1375348 (96m:32013); Acta Math. **180** (1998), no. 1, 69–117; MR1618325 (99h:32017); Math. Z. **240** (2002), no. 4, 835–847; MR1922732 (2003f:32043)]. The complex Monge-Ampère operator can be defined in a reasonable way for certain unbounded psh functions; this is the content of Chapter 5. The so-called energy classes $\mathcal{E}_p$ and $\mathcal{F}_p$ of Cegrell are introduced and a characterization of the finite measures μ in a hyperconvex domain which are Monge-Ampère measures $(dd^c u)^n$ of a function u in $\mathcal{F}_p$ is given (Theorem 5.5). Much of this chapter follows [U. Cegrell, Acta Math. **180** (1998), no. 2, 187–217; MR1638768 (99h:32016)]. In addition, if $\mu \in \mathcal{F}(A,h)$, one obtains a continuous solution [S. Kołodziej, in *Complex geometric analysis in Pohang (1997)*, 241–243, Contemp. Math., 222, Amer. Math. Soc., Providence, RI, 1999; MR1653056 (99i:32019)]. Finally, Chapter 6 presents work of the author on the complex Monge-Ampère equation on a compact Kähler manifold M [S. Kołodziej, op. cit., 1998; Indiana Univ. Math. J. **52** (2003), no. 3, 667–686; MR1986892 (2004i:32062)]. Given a fundamental form ω normalized so that $\int_M \omega^n = 1$, a continuous function φ on M is called ω-plurisubharmonic ($\varphi \in \mathrm{PSH}(\omega)$) if $\omega_\varphi := \omega + dd^c\varphi \geq 0$. Given a nonnegative function f on M normalized so that $\int_M f\omega^n = 1$, the problem is to solve the Monge-Ampère equation $\omega_\varphi^n = f\omega^n$ for φ. Defining

$$\mathcal{F}(A,h) := \{f \in L^1(M)\colon f \geq 0,\ \int_E f\omega^n \leq F(\mathrm{cap}_\omega(E)),\ E \text{ Borel}\}$$

where cap_ω is the appropriate notion of capacity, the author shows (Theorem 6.7) that if h is admissible and $1 \in \mathcal{F}(A,h)$, then for any $f \in \mathcal{F}(A,h)$ there exists a continuous solution φ of $\omega_\varphi^n = f\omega^n$. If one normailizes φ so that $\max_M \varphi = 0$, the solution is unique; this follows from a stability estimate in the final section.

Reviewed by *Norman Levenberg*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2166340 (2006k:32070) 32U40 (32C30 32H50 37A25 37B40 37F10)
Guedj, Vincent (F-TOUL3-LM)
Courants extrémaux et dynamique complexe. (French. English, French summaries)
[Extremal currents and complex dynamics]
Ann. Sci. École Norm. Sup. (4) **38** (2005), *no. 3,* 407–426.

The goal of the paper is to construct extremal positive closed currents of any degree in $\mathbb{P}^k$ (i.e., extremal elements in the closed convex cone of positive closed currents of bidimension (p,p), $0 \leq p \leq k$) which are not currents of integration along irreducible analytic subsets. Such currents, of dynamical character, were constructed before by various authors [J.-P. Demailly, Invent. Math. **69** (1982), no. 3, 347–374; MR0679762 (84f:32007); E. Bedford and J. Smillie, Math. Ann. **294** (1992), no. 3, 395–420; MR1188127 (93k:32062); J. E. Fornæss and N. Sibony, Duke Math. J. **65** (1992), no. 2, 345–380; MR1150591 (93d:32040)], but all examples were of bidimension $(k-1,k-1)$. The author considers a polynomial endomorphism $f\colon \mathbb{C}^k \to \mathbb{C}^k$ and its meromorphic extension to $\mathbb{P}^k$. Throughout the paper, he makes the assumption (H1): $X_f \cap I_f = \varnothing$, where $X_f := f((t=0)) \setminus I_f$, $(t=0)$ is the hyperplane at infinity and I_f is the indeterminacy set for f. Under this assumption, there exists a positive closed current T_+ in $\mathbb{P}^k$ such that

$$T_+ = \omega + dd^c g_+, \;\; f^* T_+ = dT_+,$$

where $d > 1$ is the first dynamical degree of f, ω is the Fubini-Study form in $\mathbb{P}^k$ and g_+ is continuous in $\mathbb{P}^k \setminus I_f$. Then $\dim X_f = r-1, \; \dim I_f = k-r-1$ for some integer $1 \leq r \leq k-1$ and it is possible to define $T_+^j, \; 1 \leq j \leq r+1$. T_+^r is one of the currents whose extremal properties

are established in the paper. The key point in the author's proof is the existence of potentials that allow him to control sign. In Section 1, under the assumption (H2) that I_f is f^{-1}-attracting, the author observes that T_+^r admits almost positive potentials, i.e., $T_+^r = \omega^r + dd^c T_{\rm can}$, where $T_{\rm can}$ is a positive current in $\mathbb{P}^k \setminus W_{I_f}$, W_{I_f} is a suitable neighborhood of I_f and $\lambda^{-j}(f^j)^* T_{\rm can} \to 0$ in $\mathbb{C}^k$ with $\lambda = d^r$. Proposition 1.1 proves the existence of an almost everywhere negative potential (with control on mass) for any positive closed current S of bidegree (s,s) in $\mathbb{P}^k$ satisfying $0 \le S \le \sigma$ in $\mathbb{P}^k$, where σ is a given positive closed current of bidegree (s,s) in $\mathbb{P}^k$, $1 \le s \le k$.

Section 2 deals with extremality properties. Theorem 2.1 says the following: Let $f\colon \mathbb{C}^k \to \mathbb{C}^k$ be a polynomial automorphism satisfying conditions (H1) and (H2). Then T_+^r is extremal in the cone $\mathcal{T}^r(\mathbb{P}^k)$ of positive closed (r,r)-currents in $\mathbb{P}^k$. Also, it is extremal in $\mathcal{T}^r(\mathbb{C}^k)$. Theorem 2.3 is its counterpart for a polynomial endomorphism f of $\mathbb{C}^k$ satisfying (H1),(H2), with the conclusion that T_+^r is extremal in the sub-cone of $\mathcal{T}^r(\mathbb{P}^k)$ consisting of currents S satisfying $f^*S = \lambda S$. Both proofs are based on uniform convergence of sequences of pullbacks under iterates of f of some quite general currents to constant multiples of T_+^r. Later, the author makes an assumption (H3) about the degrees of f:

$$\lambda = d^r > \lambda_{r+1}(f) := \lim_{j\to\infty} (\delta_{r+1}(f^j))^{1/j},$$

where $\delta_{r+1}(f)$ is the $(r+1)$-st algebraic degree of f. For f satisfying (H1) and (H3) it is possible to construct a positive closed current $T_{k-r}^- = \theta + dd^c T_\infty^-$ such that $T_\infty^- \ge 0$, $f_* T_{k-r}^- = \lambda T_{k-r}^-$ and $\|T_{k-r}^-\| = 1$, where θ is a suitable positive closed smooth $(k-r,k-r)$-form. The existence of the positive potential T_∞^- implies that T_{k-r}^- is not a current of integration over an analytic set of dimension r. To deal with singularities of $f_*\theta$ when f is not invertible, the author assumes (H4): $\lim_{z\in C_f, |z|\to\infty} f(z) \in X_f$, where C_f is the critical set of f. By Theorem 2.6, for f satisfying conditions (H1), (H3) and (H4), the current T_{k-r}^- is extremal in the cone $\mathcal{T}_{f_*}^{k-r}(\mathbb{P}^k) = \{S \in \mathcal{T}^{k-r}(\mathbb{P}^k)\colon f_*S = \lambda S\}$. If f is an automorphism, T_{k-r}^- is also extremal in $\mathcal{T}^{k-r}(\mathbb{P}^k)$. Proposition 2.5 states that quasi-plurisubharmonic functions in $\mathbb{P}^k$ are integrable with respect to $T_{k-r}^- \wedge \omega^r$. In particular, T_{k-r}^- does not charge pluripolar sets. In Remark 2.7 it is observed that T_{k-r}^- can be extremal in $\mathcal{T}^{k-r}(\mathbb{P}^k)$ even though f is not invertible, as proved by the author in [Amer. J. Math. **124** (2002), no. 1, 75–106; MR1879000 (2003b:32021)] for $k=2, r=1$.

Section 3 is devoted to a canonical invariant measure for f. It is $\mu_f = T_+^r \wedge T_{k-r}^-$, which is a well-defined measure, since T_+^r and T_{k-r}^- have complementary bidegrees and T_+ has bounded potentials on the support of T_{k-r}^-. Theorem 3.1 states that μ_f is an f-invariant probability measure with compact support in $\mathbb{C}^k$ such that $\mathcal{L}(\mathbb{C}^k) \subset L^1(\mu_f)$, where $\mathcal{L}$ denotes the Lelong class of plurisubharmonic functions in $\mathbb{C}^k$ with logarithmic growth. In particular, μ_f does not charge pluripolar sets in $\mathbb{C}^k$. Theorem 3.2 says that μ_f is ergodic and of maximal entropy

$$h_{\mu_f}(f) = h_{\rm top}(f) = \log \lambda.$$

The inequality

$$h_{\mu_f}(f) \le h_{\rm top}(f)$$

between metric and topological entropy is the Misiurewicz's variational principle, and the inequal-

ity
$$h_{\mathrm{top}}(f) \le \max_{1\le j\le k} \log \lambda_j(f)$$
is due to M. L. Gromov [Enseign. Math. (2) **49** (2003), no. 3-4, 217–235; MR2026895 (2005h:37097)]. When f satisfies (H1) and (H3), λ is the largest dynamical degree. The equalities were conjectured by S. Friedland in [Ann. of Math. (2) **133** (1991), no. 2, 359–368; MR1097242 (92c:58115)].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2158978 (2006g:32007) 32A26 (32W05 53C60)
Qiu, Chunhui (PRC-XIAM-SM); **Zhong, Tongde** (PRC-XIAM-SM)
The Koppelman-Leray formula on complex Finsler manifolds. (English summary)
Sci. China Ser. A **48** (2005), *no. 6,* 847–863.

J.-P. Demailly and C. Laurent-Thiébaut constructed integral representations of Cauchy-Leray-Koppelman type for forms of arbitrary bidegree on complex manifolds [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], and later B. Berndtsson obtained more precise results [Ann. Sci. École Norm. Sup. (4) **24** (1991), no. 3, 319–337; MR1100993 (92c:32012)]. The kernel of Demailly and Laurent-Thiébaut is essentially the leading term of the kernel of Berndtsson.

The authors adapted the method of Demailly and Laurent-Thiébaut to the setting of complex Finsler manifolds in a previous article [Sci. China Ser. A **47** (2004), no. 2, 284–296; MR2068946 (2005e:32005)]. The present article, which uses the kernel of Berndtsson in place of the kernel of Demailly and Laurent-Thiébaut, is completely parallel, even reproducing much of the earlier article verbatim. The authors have added two proofs that they omitted in the earlier article as well as three examples.

Reviewed by *Harold P. Boas*

MR2157164 (2006j:32016) 32L10 (32A25 32L20)
Berman, Robert (S-CHAL)
Holomorphic Morse inequalities on manifolds with boundary. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **55** (2005), *no. 4,* 1055–1103.

The main result of this paper is a generalization of Demailly's weak holomorphic Morse inequalities to the case of compact n-dimensional complex manifolds with boundary.

To be more precise, let X be a compact complex manifold with boundary, let ρ be a defining function for the boundary and let $\mathcal{L} = i\partial\overline{\partial}\rho$ restricted to $T^{1,0}(\partial X)$ be the Levi form of the boundary. Further, assume that L is a holomorphic line bundle over X with fiber metric φ and let $\Theta = i\partial\overline{\partial}\varphi$ be its curvature form. Now let $X(q)$ be the subset of X where Θ has exactly q negative eigenvalues, i.e. the set where $\mathrm{index}(\Theta) = q$, and let

$$T(q)_{\rho,x} := \{t > 0\colon \mathrm{index}(\Theta + t\mathcal{L}) = q \text{ along } T^{1,0}_x(\partial X)\}.$$

The author shows that if the Levi form is nondegenerate on the boundary then, up to terms of order $o(k^n)$, the dimension of $H^{0,q}(X, L^k)$ can be estimated by

$$k^n(-1)^q\left(\frac{1}{2\pi}\right)^n\left(\int_{X(q)}\frac{\Theta^n}{n!} + \int_{\partial X}\int_{T(q)_{\rho,x}}\frac{(\Theta + t\mathcal{L})^{n-1}}{(n-1)!}\wedge\partial\rho\wedge dt\right).$$

The proof of this theorem uses in an essential way the estimates of some Bergman functions, which can be derived by explicitly computing some model cases. The author also gives some examples that illustrate the sharpness of the obtained result.

Some of the other results obtained in this paper are the strong holomorphic Morse inequalities, applications to the volume of semi-positive line bundles and some relations to hole filling and contact geometry.

Reviewed by *Bert Fischer*

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MR2142242 (2006d:32033) 32Q15 (32G20 32J27)
Demailly, Jean-Pierre (F-GREN-F); **Eckl, Thomas** (D-KOLN);
Peternell, Thomas (D-BAYR-IM)
Line bundles on complex tori and a conjecture of Kodaira.
Comment. Math. Helv. **80** (2005), *no. 2,* 229–242.

A compact Kähler manifold is called almost algebraic if it can be approximated by smooth projective varieties. K. Kodaira proved in [Ann. of Math. (2) **78** (1963), 1–40; MR0184257 (32 #1730)] that every Kähler surface is almost algebraic. The statement that this should be true also in higher dimensions is known as the Kodaira conjecture. Recently, C. Voisin ["On the homotopy types of Kähler manifolds and the birational Kodaira problem", preprint, arxiv.org/abs/math/0410040] and K. Oguiso ["Automorphisms of hyperkähler manifolds in the view of topological entropy", preprint, arxiv.org/abs/math/0407476] constructed counterexamples by constructing rigid non-algebraic Kähler threefolds. The present paper, which was completed before the counterexamples appeared, gives some observations concerning the Kodaira conjecture. A certain blow-up of a $\mathbb{P}^3_1$-bundle over a 3-dimensional complex torus with Picard number ≥ 3 is shown to be rigid. It turns out, however, that these complex tori are algebraic. Some interesting generalizations are also considered.

Reviewed by *H. Lange*

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MR2140209 (2006a:30047) 30F60 (32G15)
Haïssinsky, Peter (F-PROV-LAT)
Déformation localisée de surfaces de Riemann. (French. English summary) [Localized deformations of Riemann surfaces]
Publ. Mat. **49** (2005), *no. 1,* 249–255.

Let Y be a Riemann surface with compact boundary embedded into a hyperbolic Riemann surface of finite type X. The following results are proved:

(1) The space of deformations $\mathcal{D}$ of the complex structure of Y in X is an open subset of the Teichmüller space $T(X)$ of X.

(2) The space $\mathcal{D}$ has compact closure in $T(X)$ if and only if Y is simply connected or isomorphic to a punctured disk.

(3) $\mathcal{D} = T(X)$ if and only if the components of $X \setminus Y$ are all disks or punctured disks.

Point (1) was already known by results of J.-P. Demailly and C. T. McMullen. Points (2) and (3) are original. The proof follows from the standard parametrization of Teichmüller space given by Fenchel-Nielsen coordinates and quasi-conformal Teichmüller theory.

Reviewed by *Marco Boggi*

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MR2128092 (2006i:32024) 32L15 (32J99 32L10)
Marinescu, George [Marinescu, Gheorghe] (D-HUMB-IM)
A criterion for Moishezon spaces with isolated singularities. (English summary)
Ann. Mat. Pura Appl. (4) **184** (2005), *no. 1,* 1–16.

Summary: "We give a criterion for a compact complex space with isolated singularities to be Moishezon in the spirit of Siu-Demailly's solution to the Grauert-Riemenschneider conjecture [see Y. T. Siu, J. Differential Geom. **19** (1984), no. 2, 431–452; MR0755233 (86c:32029); J.-P. Demailly, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)]. It refines a previous work by A. M. Nadel and H. Tsuji [J. Differential Geom. **28** (1988), no. 3, 503–512; MR0965227 (89m:32047)], and another one by S. Takayama [Tohoku Math. J. (2) **46** (1994), no. 2, 281–291; MR1272883 (95d:32032)], in a more specific situation."

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J. Geom. Anal. **3**, 37–61 (1993) MR1197016 (93m:32014)
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MR2113990 (2005m:32038) 32L05 (32J27)
Biswas, Indranil (6-TIFR-SM); **Subramanian, Swaminathan** (6-TIFR-SM)
Numerically flat principal bundles. (English summary)
Tohoku Math. J. (2) **57** (2005), *no. 1,* 53–63.

The notion of "numerical effectiveness" of vector bundles over smooth projective varieties is very well known. It was extended by J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)] to the case of vector bundles over compact Kähler manifolds. A vector bundle E is said to be numerically flat if it is numerically effective and its dual E^* is also numerically effective.

In the paper under review the authors extend the notion of numerical flatness to the case of principal bundles and give some characterizations of numerically flat principal bundles.

Let G be a semisimple algebraic group, M a compact Kähler manifold and E_G a holomorphic principal G-bundle over M. The principal G-bundle E_G is said to be numerically flat if for all pairs (P,χ), where P is a parabolic subgroup of G and χ is an anti-dominant character of P with respect to some Borel subgroup contained in P, the associated line bundle $E_\chi = E_G \times_\chi \mathbb{C}$ over E_G/P is a numerically effective line bundle. The main results of the paper under review are the following:

(1) If E_G is numerically flat and V is a finite dimensional complex G-module, then the associated vector bundle $E_G \times_G V$ is numerically flat. Conversely, let $\rho\colon G \to \mathrm{SL}(V)$ be a representation of G, where V is as above. If $\ker(\rho)$ is finite and if $E_G \times_G V$ is numerically flat then E_G is numerically flat. In particular a principal G-bundle E_G is numerically flat if and only if the adjoint vector bundle $\mathrm{ad}(E_G)$ is numerically flat.

(2) A numerically flat principal G-bundle E_G over a compact Kähler manifold M is semistable (with respect to the Kähler metric as a polarization) and all its (rational) characteristic classes of degree at least one vanish. The converse is true if we assume that M is projective. Hence a characterization of numerical flatness of holomorphic principal bundles in terms of semistability in the case M is projective.

In the last part of the paper, the authors give another characterization of numerical flatness of holomorphic principal bundles in terms of a reduction of the structure group and the existence of some "special connections". More precisely, they prove that:

(3) A principal G-bundle E_G over a compact Kähler manifold M is numerically flat if and only if there exist a parabolic subgroup P of G and a reduction E_P of the structure group of E_G to P, such that the principal P-bundle E_P admits a flat holomorphic connection ∇ with the property that the monodromy of the (flat) connection on $E_{L(P)}$ induced by ∇ is contained in a maximal compact subgroup of $L(P)$, where $L(P)$ is a Levi factor of the parabolic subgroup P and $E_{L(P)}$ is the principal $L(P)$-bundle obtained by extending the structure group of E_P to $L(P)$ via the morphism $P \longrightarrow L(P)$.

Reviewed by *Boudjemâa Anchouche*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2099777 (2005i:14052) 14J45 (14C05 53D10)
Kebekus, Stefan (D-KOLN)
Lines on complex contact manifolds. II. (English summary)
Compos. Math. **141** (2005), *no. 1,* 227–252.

Let X be a complex projective manifold of dimension $2n+1$. The manifold X is called a contact manifold if there exist a line bundle L and a twisted holomorphic form $\theta \in H^0(X, \Omega_X \otimes L)$ such that $\theta \wedge (d\theta)^{\wedge n}$ is nowhere zero. Following [S. Kebekus et al., Invent. Math. **142** (2000), no. 1, 1–15; MR1784795 (2002a:14047)] and [J.-P. Demailly, in *Complex geometry (Göttingen, 2000)*, 93–98, Springer, Berlin, 2002; MR1922099 (2003f:32029)] it is known that either X is the projectivisation of the (co)tangent bundle of an $(n+1)$-fold (with the standard contact structure),

or $b_2(X) = 1$ and X is Fano. In the latter case X is conjectured to be a rational homogeneous manifold and the closed orbit of the projectivisation of the adjoint representation of a simple algebraic group. It is expected that the conjecture should be proved by understanding rational curves on X. The paper under review concerns rational curves on X which are of degree 1 with respect to the line bundle L; they are called contact lines, or just lines. By [S. Kebekus, in *Complex geometry (Göttingen, 2000)*, 147–155, Springer, Berlin, 2002; MR1922103 (2003j:14065)] X is either the projective space $\mathbf{P}^{2n+1}$ or it is covered by lines. The main theorem of the paper under review is as follows: Let us choose an irreducible component H of the space parameterizing lines and for $x \in X$ let H_x denote the subvariety in H parameterizing lines passing through x. Then for a general x the variety H_x is irreducible and smooth, and the locus of curves in X parametrized by H_x forms a cone over H_x. In particular, all lines through x are smooth, they meet only in x and they do not share a common tangent direction at x. This implies, for example, that the variety of tangents to lines at x is a smooth Legendrian subvariety of the contact distribution $\mathbf{P}(\ker(\theta)_x) \subset \mathbf{P}(T_xX)$. In an appendix the author discusses some properties of jet bundles which are interesting for their own sake.

{For Part I see [S. Kebekus, J. Reine Angew. Math. **539** (2001), 167–177; MR1863858 (2002h:14069)].}

Reviewed by *Jarosław A. Wiśniewski*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2144160 32-06 (14-06)

Komplexe Analysis. [Complex analysis]

Abstracts from the workshop held August 22–28, 2004.

Organized by Jean-Pierre Demailly, Klaus Hulek and Thomas Peternell.

Oberwolfach Reports. Vol. 1, no. 3.

Oberwolfach Rep. **1** (2004), *no. 3,* 2171–2215.

{This item will not be reviewed individually. For details of the collection in which this item appears see MR2022954 (2006c:00027) .}

{For the entire collection see MR2022954 (2006c:00027)}

MR2132649 (2007h:14003) 14B05 (13H99)
Blickle, Manuel (D-DUES2); **Lazarsfeld, Robert** (1-MI)
An informal introduction to multiplier ideals. (English summary)
Trends in commutative algebra, 87–114, *Math. Sci. Res. Inst. Publ.*, 51, *Cambridge Univ. Press*, *Cambridge*, 2004.

From the introduction: "Given a smooth complex variety X and an ideal (or ideal sheaf) $\mathfrak{a}$ on X, one can attach to $\mathfrak{a}$ a collection of multiplier ideals $\mathcal{J}(\mathfrak{a}^c)$ depending on a rational weighting parameter $c > 0$. These ideals, and the vanishing theorems they satisfy, have found many applications in recent years. In the global setting they have been used to study pluricanonical and other linear series on a projective variety [J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007); U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036); Y. T. Siu, Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035); L. M. H. Ein and R. K. Lazarsfeld, J. Amer. Math. Soc. **10** (1997), no. 1, 243–258; MR1396893 (97d:14063); Invent. Math. **137** (1999), no. 2, 427–448; MR1705839 (2000j:14028); J.-P. Demailly, Astérisque No. 266 (2000), Exp. No. 852, 3, 59–90; MR1772670 (2001m:32042)]. More recently they have led to the discovery of some surprising uniform results in local algebra [L. M. H. Ein, R. K. Lazarsfeld and K. E. Smith, Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001); Amer. J. Math. **125** (2003), no. 2, 409–440; MR1963690 (2003m:13004); Duke Math. J. **123** (2004), no. 3, 469–506; MR2068967 (2005k:14004)]. The purpose of these lectures is to give an easy-going and gentle introduction to the algebraically-oriented local side of the theory.

"Multiplier ideals can be approached (and historically emerged) from three different viewpoints. In commutative algebra they were introduced and studied by J. Lipman [Bull. Soc. Math. Belg. Sér. A **45** (1993), no. 1-2, 223–244; MR1316244 (97a:13030)] (under the name 'adjoint ideals', which now means something else), in connection with the Briançon-Skoda theorem. On the analytic side of the field, A. M. Nadel [Ann. of Math. (2) **132** (1990), no. 3, 549–596; MR1078269 (92d:32038)] attached a multiplier ideal to any plurisubharmonic function, and proved a Kodaira-type vanishing theorem for them. (In fact, the 'multiplier' in the name refers to their analytic construction; see Section 2.4.) This machine was developed and applied with great success by Demailly, Siu and others. Algebro-geometrically, the foundations were laid in passing by Esnault and Viehweg in connection with their work involving the Kawamata-Viehweg vanishing theorem. More systematic developments of the geometric theory were subsequently undertaken by Ein, Kawamata and Lazarsfeld. We take the geometric approach here.

"The present notes follow closely a short course on multiplier ideals given by Lazarsfeld at the Introductory Workshop for the Commutative Algebra Program at the MSRI in September 2002. The three main lectures were supplemented with a presentation by Blicke on multiplier ideals associated to monomial ideals (which appears here in Section 3). We have tried to preserve in this write-up the informal tone of these talks: thus we emphasize simplicity over generality in statements of results, and we present very few proofs. Our primary hope is to give the reader a feeling for what multiplier ideals are and how they are used. For a detailed development of the theory from an algebro-geometric perspective we refer to the forthcoming Part III of [R. K. Lazarsfeld, *Positivity in algebraic geometry. I*, Springer, Berlin, 2004; MR2095471 (2005k:14001a); *II*, MR2095472

(2005k:14001b)]. The analytic picture is covered in J.-P. Demailly's lectures [in *School on Vanishing Theorems and Effective Results in Algebraic Geometry (Trieste, 2000)*, 1–148, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2001; MR1919457 (2003f:32020)]."

{For the entire collection see MR2132646 (2005j:13002)}

MR2119243 (2006c:32045) 32U40 (32C30 32H04 32Q15 37B40)
Dinh, Tien-Cuong (F-PARIS11); **Sibony, Nessim** (F-PARIS11)
Regularization of currents and entropy. (English, French summaries)
Ann. Sci. École Norm. Sup. (4) **37** (2004), *no. 6,* 959–971.

In the paper, the authors prove several results about the regularization of (p, p)-currents on a compact Kähler manifold (X, ω) of dimension k. The main theorem is Theorem 1.1: Let (X, ω) be a compact Kähler manifold of dimension k. Then, for every positive closed (p, p)-current T on X there exist smooth positive closed (p, p)-forms T_n^+ and T_n^- such that $T_n^+ - T_n^-$ converges weakly to the current T. Moreover, $\|T^\pm\| \leq c_X \|T\|$, where $c_X > 0$ is a constant independent of T. This is a generalization of a result by J.-P. Demailly [in *Several complex variables (Berkeley, CA, 1995–1996)*, 233–271, Cambridge Univ. Press, Cambridge, 1999; MR1748605 (2002e:32046)] on the regularization of positive closed $(1, 1)$-currents on X. In Section 1, they derive other results as corollaries from Theorem 1.1. Corollary 1.2 extends their previous result on the regularization of positive closed (p, p)-currents on a projective manifold [T.-C. Dinh and N. Sibony, "Une borne supérieure pour l'entropie topologique d'une application rationnelle", Ann. of Math. (2), to appear]. Corollary 1.3 defines the pullback of the current T by a surjective holomorphic map $\Pi: X' \mapsto X$, where (X', ω') is another compact Kähler manifold of dimension $k' \geq k$. Theorem 1.4 says that if f is a dominating meromorphic self-map of X, then $h(f) \leq \mathrm{lov}(f) = \max_{1 \leq p \leq k} \log d_p$, where $h(f)$ is the topological entropy of f, d_p is the dynamical degree of order p of f and $\mathrm{lov}(f)$ measures the growth of the volume of the graphs of $(f, \ldots, f^{n-1})$ (over the subset of X on which all iterates of f are defined). However, it should be noted that for a compact Kähler manifold X of dimension k and $f: X \mapsto X$ holomorphic, S. Friedland proved a stronger result, namely that $h(f) = \mathrm{lov}(f) = \max_{1 \leq p \leq k} d_p$ (see [J. Fourier Anal. Appl. **1995**, Special Issue; MR1364875 (96f:00039)] for the whole collection). His proof does not rely on Lemma 3 in his previous publication [S. Friedland, Ann. of Math. (2) **133** (1991), no. 2, 359–368; MR1097242 (92c:58115)], which the authors of the paper under review point out to be wrong. The equality $h(f) = \mathrm{lov}(f)$ for f meromorphic still remains a conjecture. In Section 2, the authors prove an auxiliary Lemma 2.1, which gives properties of a linear operator P defined on the set $\mathcal{M}$ of Radon measures on $\mathbb{R}^m$ as integration against a kernel K with compact support on $B \times B$, smooth in $B \times B \setminus \Delta$ and bounded pointwise by a constant multiple of the fundamental kernel

of classical potential theory. Section 3 contains the proof of Theorem 1.1. First they give a weak regularization of the current of integration $[\Delta]$, i.e., they construct positive closed (k,k)-forms $K_n^{\pm}$ with coefficients in L_1 and smooth out Δ such that $K_n^+ - K_n^- \to [\Delta]$ weakly and $\|K_n^{\pm}\| \leq c_1$, where $c_1 > 0$. Then they define $T_n^{\pm}$ by wedging $K_n^{\pm}$ with T, show that $T_n^+ - T_n^- \to T$ and $\|T_n^{\pm}\| \leq c\|T\|$, and obtain the smoothness of $T_n^{\pm}$ by repeated use of Lemma 2.1. In Section 4, they prove Theorem 4.1, which is an analog of Theorem 1.1 for positive dd^c-closed (p,p)-currents on X, with $T_n^{\pm}$ smooth, positive and dd^c-closed. Later, they introduce a special class $\mathrm{DSH}^p(X)$ of (p,p)-currents equipped with a suitable norm. Theorem 4.4 is a regularization result for currents in this class. Proposition 4.6 is a variant of Theorem 1.1 for a continuous form T. Section 5 is devoted to the study of the intersection $S \wedge T$ of a positive closed $(1,1)$-current S with a positive pluriharmonic (p,p)-current T, $1 \leq p \leq k-1$. S can be written as $S = \alpha + dd^c u$, where u, called the potential of S, is a quasi-psh function. Theorem 5.1 says that if u is continuous, then $S \wedge T$ is well defined and is a positive dd^c-closed current, which depends continuously on S and T in a suitable sense. Theorem 5.3 is a similar result for T in the class $\mathrm{DSH}^p(X)$; $S \wedge T$ is then in $\mathrm{DSH}^{p+1}(X)$. Proposition 5.5 is an analog of Theorem 5.1 for T of bidegree $(1,1)$ and S with bounded potential.

Reviewed by *Małgorzata Stawiska*

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preprint, 2003, math.DS/0306095. cf. MR 2004e:37063
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2112584 (2006b:14011) 14C25 (32J27)
Demailly, Jean-Pierre (F-GREN-IF)
On the geometry of positive cones of projective and Kähler varieties. (English summary)
The Fano Conference, 395–422, *Univ. Torino*, *Turin*, 2004.

Some of the more fundamental problems and results in complex geometry revolve around such questions as when a complex manifold would be projective or Kähler, or how much of its geometry could be determined by divisors and curves. All projective manifolds are Kähler, and a famous theorem of Kodaira proves that a Kähler manifold (X, ω) is projective precisely when the class

$[\omega] \in H^{1,1}(X) \subset H^2(X,\mathbb{R})$ moreover represents a class in $H^2(X,\mathbb{Z})$. Kodaira had also conjectured that a compact complex surface admits a Kähler metric if and only if the first Betti number is even. A closely related question concerns the ampleness of holomorphic line bundles L on X. When X is projective, the Nakai-Moishezon criterion establishes that ampleness of L is equivalent to having a strictly positive integral for the p-th exterior power of the Chern class of L over any algebraic subset of dimension p for $1 \le p \le n = \dim(X)$. Mori's theory of complex three-manifolds brought new techniques to bear on the projective context via the geometry of cones of divisors and curves lying within their respective cohomology groups. For example, a conjecture of Fano asserts that a projective X is "uniruled" by rational curves precisely when the Chern class of the canonical line bundle lies outside the closure of the cone of effective divisors. The article under review is a survey of relatively recent achievements of the author and his collaborators, S. Boucksom, M. Paun and T. Peternell, in further unifying and extending the theory surrounding these questions. Central to their programme are the powerful techniques associated with positive currents of type $(1,1)$ on compact Kähler manifolds, and the interplay between the open convex cone of Kähler forms and the enveloping closed convex cone of positive $(1,1)$-currents (the "pseudo-effective" cone). While some basic familiarity with Kähler geometry and the theory of currents is assumed, the author's exposition is designed to be informative to the non-specialist. Among the results surveyed, some highlights are a generalization of the Nakai-Moishezon criterion and its application to the characterisation of Kähler currents on compact complex manifolds, as well as a theory of Poincaré duality between cones of positive currents of type $(1,1)$ and $(n-1,n-1)$, which leads in particular to a proof of Fano's conjecture.

{For the entire collection see MR2112562 (2005g:14003)}

Reviewed by *Adam Gregory Harris*

MR2112562 (2005g:14003) 14-06

★**The Fano Conference.**

Proceedings of the conference to commemorate the 50th anniversary of the death of Gino Fano (1871–1952) held in Torino, September 29–October 5, 2002.

Edited by Alberto Collino, Alberto Conte and Marina Marchisio.

Università di Torino, Dipartimento di Matematica, Turin, 2004. *xiv*+804 *pp*. €99.90.
ISBN 88-900876-1-7

Contents: Robert Fano, In loving memory of my father Gino Fano (1–4) MR2112563; Herbert C. Clemens [C. Herbert Clemens], A version of Abel's theorem for surfaces (5–23); MR2112564 (2006d:14010); Vasilij A. Iskovskikh [Vasiliĭ Alekseevich Iskovskikh], On the Noether-Fano inequalities (25–35); MR2112565 (2005m:14017); Shigefumi Mori and Shigeru Mukai, Extremal

rays and Fano 3-folds (37–50); MR2112566 (2005k:14085); Jacob P. Murre, Fano varieties and algebraic cycles (51–68); MR2112567 (2005i:14053); Valery A. Alexeev and Michel Brion, Boundedness of spherical Fano varieties (69–80); MR2112568 (2005k:14081); Vincenzo Ancona and Marco Maggesi, On the quantum cohomology of Fano bundles over projective spaces (81–98); MR2112569 (2005m:14103); Marco Andreatta, The projective space and other simple varieties (99–108); MR2112570 (2005j:14019); Victor V. Batyrev, Toric degenerations of Fano varieties and constructing mirror manifolds (109–122); MR2112571 (2006b:14070); Ingrid C. Bauer and Fabrizio Catanese, Some new surfaces with $p_g = q = 0$ (123–142); MR2112572 (2005k:14079).

Arend Bayer and Yuri I. Manin [Yuriĭ Ivanovich Manin], (Semi)simple exercises in quantum cohomology (143–173); MR2112573 (2006a:14090); Arnaud Beauville, Fano threefolds and $K3$ surfaces (175–184); MR2112574 (2006b:14071); Mauro C. Beltrametti, Maria L. Fania and Andrew J. Sommese, Mukai varieties as hyperplane sections (185–208); MR2112575 (2005i:14008); Tzachi Ben-Itzhak and Mina Teicher, Topological algorithm for conjugation of half-twists in the braid group with application to Hurwitz equivalence (209–221); MR2112576 (2006e:20064); Fedor Bogomolov and Yuri Tschinkel [Yu. Tschinkel], Special elliptic fibrations (223–234); MR2112577 (2005k:14017); Gavin Brown [Gavin Brown^2], Alessio Corti and Francesco Zucconi, Birational geometry of 3-fold Mori fibre spaces (235–275); MR2112578 (2005k:14031); Alberto Calabri, Ciro Ciliberto, Flaminio Flamini and Rick Miranda, On the geometric genus of reducible surfaces and degenerations of surfaces to unions of planes (277–312); MR2112579 (2005j:14010); Frédéric Campana, L'anneau ordonné de la géométrie algébrique [Ordered rings in algebraic geometry] (313–338); MR2112580 (2006k:14017); Frédéric Campana, Orbifoldes à première classe de Chern nulle [Orbifolds of zero first Chern class] (339–351); MR2112581 (2005i:32019); Lucia Caporaso, Moduli theory and arithmetic of algebraic varieties (353–378); MR2112582 (2005m:14015).

Olivier Debarre, Seshadri constants of abelian varieties (379–394); MR2112583 (2005k:14093); Jean-Pierre Demailly, On the geometry of positive cones of projective and Kähler varieties (395–422); MR2112584 (2006b:14011); Igor V. Dolgachev, Abstract configurations in algebraic geometry (423–462); MR2112585 (2005k:14091); Antonella Grassi, Open-closed string dualities in geometry and modified open Gromov-Witten invariants (463–478); MR2112586 (2005k:14118); Joe Harris [Joseph Daniel Harris] and Klaus Hulek, A remark on the Schottky locus in genus 4 (479–483); MR2112587 (2005h:14077); Yasuyuki Kachi and Shashikant B. Mulay, Local-to-global correspondence in algebraic geometry (485–514); MR2112588 (2006a:14018); János Kollár and Frank-Olaf Schreyer, Real Fano 3-folds of type V_{22} (515–531); MR2112589 (2005k:14084); Viktor S. Kulikov, Generic coverings of the plane and braid monodromy invariants (533–558); MR2112590 (2005j:14050); Yoichi Miyaoka, Numerical characterisations of hyperquadrics (559–562); MR2112591 (2005h:14103); Shigeru Mukai, Plane quartics and Fano threefolds of genus twelve (563–572); MR2112592 (2006d:14043).

Viacheslav V. Nikulin [Vyacheslav V. Nikulin], On algebraic varieties with finite polyhedral Mori cone (573–589); MR2112593 (2006c:14019); Christian Okonek and Andrei Teleman, Gauge theoretical Gromov-Witten invariants and virtual fundamental classes (591–623); MR2112594 (2005i:14072); Thomas Peternell and Andrew J. Sommese, Ample vector bundles and branched coverings. II (625–645); MR2112595 (2005k:14088); Yuri G. Prokhorov, A remark on Fano

threefolds with canonical Gorenstein singularities (647–657); MR2112596 (2005k:14086); Aleksandr V. Pukhlikov, Birationally rigid Fano varieties (659–681); MR2112597 (2005j:14017); Kristian Ranestad, Non-abelian Brill-Noether loci and the Lagrangian Grassmannian $\mathrm{LG}(3,6)$ (683–692); MR2112598 (2005j:14031); Nicholas I. Shepherd-Barron, Stably rational irrational varieties (693–700); MR2112599 (2006b:14021); Andrei N. Tyurin, Fano versus Calabi-Yau (701–734); MR2112600 (2005h:14098); Alessandro Verra, The Prym map has degree two on plane sextics (735–759); MR2112601 (2005k:14057); Claire Voisin, Intrinsic pseudo-volume forms and K-correspondences (761–792); MR2112602 (2006b:14020); Pelham M. H. Wilson, Metric limits of Calabi-Yau manifolds (793–804); MR2112603 (2005h:32057).

{Most of the papers are being reviewed individually.}

MR2113021 (2005i:32020) 32J27 (32Q15)

Demailly, Jean-Pierre (F-GREN); **Paun, Mihai** (F-STRAS)

Numerical characterization of the Kähler cone of a compact Kähler manifold. (English summary)

Ann. of Math. (2) **159** (2004), *no. 3,* 1247–1274.

This article gives a beautiful solution of a long-standing basic problem in Kähler geometry and, as such, can be viewed as a classic. It is likely to have a lasting impact on the field.

The problem was to generalize the classical Nakai-Moishezon criterion of ampleness to a numerical characterization of the Kähler cone of a compact Kähler manifold.

Let us first recall the statement of the Nakai-Moishezon theorem. Let k be a field and X be a projective scheme over k. Let L be a Cartier divisor on X. Then L is ample iff for every positive dimensional reduced closed subscheme $Z\subset X$, $L^{\dim Z}\cdot Z>0$.

If $k=\mathbf{C}$ and X is smooth, we can reformulate this using Kodaira's theorem that ample divisors L on X are characterized by the existence of a smooth Hermitian metric of positive curvature or, in an equivalent fashion, by the fact that the first Chern class $c_1(L)$, as an element of the vector space $H^{1,1}(X)$ of degree 2 de Rham real cohomology classes represented by closed $(1,1)$-forms, has a Kähler representative. The open convex cone in $H^{1,1}(X)$ consisting of classes with a Kähler representative is called the Kähler cone and will be denoted by $\mathcal{K}(X)$.

Thus, we get the following statement: Let X be a complex projective manifold. Let $\mathrm{NS}(X)\subset H^{1,1}(X)$ be the subset of $H^{1,1}(X)$ consisting of classes with integral periods. A class $\omega\in\mathrm{NS}(X)$ lies in $\mathcal{K}(X)$ iff $\int_Z\omega^{\dim Z}>0$ for every positive-dimensional closed analytic subset Z of X.

We will denote by $\mathcal{P}(X)$ the set of classes ω cut out by the conditions that $\int_Z\omega^{\dim Z}>0$ for every positive-dimensional closed analytic subset Z of X.

It was widely believed that a similar result holds for general real $(1,1)$ classes, namely that

$\mathcal{K}(X) = \mathcal{P}(X)$.

The article under review confirms this conjecture in the more general case of compact Kähler manifolds. Here the statement should be modified to the effect that $\mathcal{K}(X)$ is a connected component of $\mathcal{P}(X)$.

The proof consists in a reduction to the nef case and a subtle application of Yau's fundamental work on the solution of the inhomogeneous complex Monge-Ampère equation in which the volume form acquires a singularity.

Reviewed by *Philippe P. Eyssidieux*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2111016 (2006d:32027) 32J25 (32C30)
Eckl, Thomas (D-BAYR-IM)
Numerically trivial foliations. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **54** (2004), *no. 4,* 887–938.

In the present paper, the author continues his investigation of numerically trivial fibrations on projective complex manifolds from [T. Eckl, J. Algebraic Geom. **13** (2004), no. 4, 617–639; MR2072764 (2005f:14014)] by studying the localized notion of foliations with numerically trivial leaves.

Tsuji's criterion for numerical triviality of a pair (L, h) involving the curvature current of the singular metric h on the pseudo-effective line bundle L is taken as the definition for the numerical triviality of an arbitrary closed positive $(1, 1)$-current. The main result is the existence of a maximal foliation with numerically trivial leaves with respect to such a current. Y. T. Siu's decomposition theorem [Invent. Math. **27** (1974), 53–156; MR0352516 (50 #5003)] plays an important role in the proof. On a projective complex manifold, Tsuji's numerically trivial fibration with respect to a singular metric having the given current as curvature current turns out to be maximal among those fibrations whose fibers are contained in the leaves. Using an appropriate metric, the Iitaka fibration can also be characterized in this way (i.e., it coincides with Tsuji's fibration).

Furthermore, based on ideas of Demailly and Boucksom regarding moving intersection numbers, the author describes the nef fibration of a nef line bundle as the maximal fibration contained in the appropriately defined numerically trivial maximal foliation with respect to the first Chern class (called the nef foliation). The Iitaka fibration in turn is shown to contain the nef foliation. As a consequence, when the nef foliation is not a fibration, the Iitaka dimension is strictly less than the numerical dimension. It is not known whether the converse to this statement holds true.

In fact, to define the numerically trivial foliation in the preceding paragraph, only a pseudo-effective class α is needed; under the assumption that the singularities of the numerically trivial foliation with respect to α are isolated points, it is shown that the codimension of the leaves is an upper bound for the numerical dimension of α. It is unknown to what extent the assumption of isolated singularities can be removed.

Finally, some explicit surface examples are discussed, and an appendix gives the basics of singular foliations.

Reviewed by *Gordon Heier*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2099189 (2005f:14103) 14N05 (32Q45)
Duval, Julien (F-TOUL3-LM)
Une sextique hyperbolique dans $\mathrm{P}^3(\mathbf{C})$. (French. English, French summaries) [A hyperbolic sextic in $\mathrm{P}^3(\mathbf{C})$]
Math. Ann. **330** (2004), *no. 3,* 473–476.

From the text (translated from the French): "A subset of $\mathrm{P}^3(\mathbf{C})$ is said to be hyperbolic if it does not contain an entire curve, i.e. a non-constant holomorphic image of $\mathbf{C}$. The Kobayashi conjecture in projective space stipulates that a generic surface of degree ≥ 5 in $\mathrm{P}^3(\mathbf{C})$ is hyperbolic. It was proved for degrees ≥ 36 by M. McQuillan [Geom. Funct. Anal. **9** (1999), no. 2, 370–392; MR1692470 (2000f:32035)] and then for degrees ≥ 21 by J.-P. Demailly and J. El Goul [Amer. J. Math. **122** (2000), no. 3, 515–546; MR1759887 (2001f:32045)]. In a parallel and more modest effort, a number of authors (see the references in [M. Zaidenberg, "Hyperbolic surfaces in $\mathbf{P}^3$: examples", preprint, arxiv.org/abs/math/0311394]) have sought to construct examples of

hyperbolic surfaces of the lowest possible degree. Until now, the best bound was degree 8. Our goal in this note is to show the existence of hyperbolic surfaces of degree 6."

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MR2099122 (2006c:32041) 32S65 (32Q25 37D20 37F10)

Cantat, Serge (F-RENNB-IM)

Difféomorphismes holomorphes Anosov. (French. English summary) [Anosov holomorphic diffeomorphisms]

Comment. Math. Helv. **79** (2004), *no. 4,* 779–797.

The theme of this article is the classification of compact complex manifolds X which admit a holomorphic Anosov diffeomorphism $f\colon X \to X$. When X is a complex surface, it has been proved by É. Ghys (see Theorem A in [Invent. Math. **119** (1995), no. 3, 585–614; MR1317651(95k:58116)]) that X is a complex torus and f is a linear automorphism.

The situation is not as simple in higher dimensions, as shown by Example 1.5 of the article under review. The transverse properties of the stable/unstable foliations $\mathcal{F}^{s/u}$ play an important role here. In the case where the stable (or unstable) leaves have dimension 1, Ghys has proved that these foliations are holomorphic foliations. Moreover, if f has a dense orbit, then (f, X) is topologically conjugate to a linear automorphism of a complex torus (see Theorem B in [op. cit.]).

The purpose of this article is to establish similar results when the stable/unstable leaves have

complex dimension ≥ 2. It is not clear in this case that $\mathcal{F}^{s/u}$ are holomorphic foliations (the transverse structure is a priori merely continuous). The main result of the paper (Theorem 1.4.a) asserts that if $\mathcal{F}^{s/u}$ are holomorphic foliations and X is projective, then (f, X) is—up to an étale cover—a linear automorphism of a complex torus.

The proofs use a lot of complex analytic and algebraic geometry, hence they are quite different in nature from those given by Ghys. They rely notably on a recent alternative of S. Boucksom, J.-P. Demailly, M. Paun and T. Peternell which says that either X is uniruled or K_X is pseudoeffective.

Reviewed by *Vincent Guedj*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2095471 (2005k:14001a) 14-02 (14C20)
Lazarsfeld, Robert (1-MI)

★**Positivity in algebraic geometry. I.**
Classical setting: line bundles and linear series.
Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 48.
Springer-Verlag, *Berlin*, 2004. *xviii*+387 *pp*. $129.00. *ISBN* 3-540-22533-1

Citations
From References: 0
From Reviews: 0

MR2095472 (2005k:14001b) 14-02 (14C20 14F05 14F17)
Lazarsfeld, Robert (1-MI)

★**Positivity in algebraic geometry. II.**
Positivity for vector bundles, and multiplier ideals.
Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 49.
Springer-Verlag, *Berlin*, 2004. *xviii*+385 *pp*. $129.00. *ISBN* 3-540-22534-X

Positivity has long been a major theme in various branches of algebraic geometry, both as an object

of study and as a technical tool. A first book attempting to treat positivity, under the heading of ampleness, appeared already three and a half decades ago. That is R. Hartshorne's book [*Ample subvarieties of algebraic varieties*, Springer, Berlin, 1970; MR0282977 (44 #211)], where the main goal was to extend the notion of ample divisor to arbitrary subvarieties of a given variety. Since then, the subject has seen major developments in a large number of different directions: intersection theory, singularities, topology of algebraic varieties, vanishing theorems and their applications to linear series, and higher dimensional geometry, to give a certainly incomplete list. However, these developments have mostly remained scattered in the literature and some, maybe precisely for this reason, have not been worked out in a systematic fashion. In the book under review the author succeeds wonderfully in putting together under the same heading most of the areas of classical and modern complex algebraic geometry dedicated to, or influenced by, the study of positivity.

The book is divided into three parts, each with a separate introduction. In addition, each chapter contains introductory remarks and concluding notes which emphasize the history of the topic, sources of inspiration, and further references. I will present in what follows the rough contents of each part, chapter by chapter.

The first part is equivalent to the content of Volume I, and is devoted to a fundamental topic, namely that of line bundles and linear series. The main notion of positivity here is that of ampleness. Its importance has become widely recognized after the foundational papers of J.-P. Serre [Ann. of Math. (2) **61** (1955), 197–278; MR0068874 (16,953c)] on the algebraic viewpoint, and K. Kodaira [Proc. Nat. Acad. Sci. U. S. A. **39** (1953), 1268–1273; MR0066693 (16,618b)] on the analytic one. The author presents in Chapter 1 the basic theory of ampleness, with an accent on modern concepts like $\mathbf{Q}$- and $\mathbf{R}$-divisors, nefness, and cones of divisors in the Néron-Severi space of numerical equivalence classes. He continues in Chapter 2 by presenting the theory of linear series which may not be ample. The following is in my view the most remarkable and novel feature of this part of the book: a systematic study of the asymptotic theory of non-ample linear series, with a special emphasis on the role of big divisors (the birational analogues of ample divisors). Chapter 3 takes up a more geometric and topological approach to positivity. Its main focus is on the Lefschetz and Bertini theorems, together with subsequent generalizations by Barth, Fulton and Hansen and others, and we get a glimpse of the interesting geometry associated with subvarieties of small codimension in projective spaces. Chapter 4 contains a treatment of vanishing theorems, including the classical Kodaira and Nakano theorems for ample line bundles, and the very useful generalization by Kawamata and Viehweg to the case of big and nef divisors (but not the $\mathbf{Q}$-divisors case, which is treated separately later, in §9.1). With respect to this part, the author mentions that the exposition is somewhat more elementary than the standard presentations. The main generic vanishing theorem of M. L. Green and Lazarsfeld [Invent. Math. **90** (1987), no. 2, 389–407; MR0910207 (89b:32025)] is also presented. Finally, Chapter 5 deals with the topic of local positivity. This is a more recent development, based on Demailly's notion of Seshadri constant, which measures how much of the positivity of a given line bundle is concentrated at a given point of the variety. The most important result presented here is a lower bound for the Seshadri constant given by L. M. H. Ein, O. Küchle and Lazarsfeld [J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)]. It should be said, however, that the picture

is not definitive, and a substantially better bound is proposed, partly in view of Fujita's famous conjecture on the freeness of adjoint bundles.

A note about a couple of things I particularly enjoyed reading in the first part: first, it is nice to see the notion of Castelnuovo-Mumford regularity take a quite prominent role in a text on positivity. The idea roughly speaking is to quantify directly how much one has to twist a sheaf by a positive line bundle in order to achieve vanishing. This makes later arguments particularly concise, for example in the context of multiplier ideals. Second, the machinery of Cutkosky of producing examples of line bundles on higher dimensional varieties with interesting base-locus or volume behavior (e.g. disproving the existence of Zariski decomposition on threefolds or higher) is systematized and made widely available.

The second part of the book (first part of Volume II) focuses on positivity for vector bundles of higher rank. This is an area of study started in the 1960's by people like Grauert, Griffiths and Hartshorne, with the aim of generalizing to higher rank as much of the geometry of ample divisors as possible. Important and beautiful applications of these generalizations have emerged during the subsequent decades, and they naturally belong to the general context of positivity. Chapter 6 is devoted to introducing the notions of ampleness and nefness for vector bundles, mainly through many examples associated to interesting geometric contexts. The adopted definition is that of Hartshorne [Inst. Hautes Études Sci. Publ. Math. No. 29 (1966), 63–94; MR0193092 (33 #1313)], namely a vector bundle E on a projective variety X is ample (or nef) if the Serre line bundle $\mathcal{O}_{\mathbf{P}E}(1)$ is so on the projectivized bundle $\mathbf{P}E \to X$. Some of the most basic examples to be mentioned are the following: the normal bundles of smooth subvarieties in projective space and in simple abelian varieties are ample, smooth projective varieties with ample cotangent bundle are Kobayashi hyperbolic, the duals of Picard bundles parametrizing sufficiently positive linear series of fixed numerical class are ample, push-forwards of relative canonical bundles are nef (under mild hypotheses). Chapter 7 focuses on the geometric properties of ample vector bundles, particularly on the topology associated with zero loci of their sections, or more generally to degeneracy loci. After presenting the Lefschetz type theorems of A. J. Sommese [Math. Ann. **233** (1978), no. 3, 229–256; MR0466647 (57 #6524)] and S. Bloch and D. Gieseker [Invent. Math. **12** (1971), 112–117; MR0297773 (45 #6825)], the author discusses his joint theorem with W. Fulton on the connectedness of degeneracy loci [Acta Math. **146** (1981), no. 3-4, 271–283; MR0611386 (82k:14016)], together with its beautiful applications to Brill-Noether theory and to other contexts. Vanishing theorems for vector bundles, for example the celebrated result of J. Le Potier [Math. Ann. **218** (1975), no. 1, 35–53; MR0385179 (52 #6044)], are also presented. Chapter 8 deals with numerical consequences of ampleness for vector bundles, and it is here that the idea of positivity comes to the forefront. The central result is the theorem of Fulton and Lazarsfeld [Ann. of Math. (2) **118** (1983), no. 1, 35–60; MR0707160 (85e:14021)], stating that the cone of numerically positive polynomials in the Chern classes of ample vector bundles is spanned by Schur polynomials.

An interesting original thing in this part of the book is the formalism developed by the author regarding twisting vector bundles by $\mathbf{Q}$-divisors. This allows one to keep track formally of the finer positivity properties of vector bundles, particularly when studying nefness.

The third part (second part of Volume II) makes the transition from classical to the most modern developments in the field. It takes up ideas and techniques from higher dimensional geometry,

under the form of multiplier ideals. This is the star attraction of the book, since the theory of multiplier ideals has only recently taken a well-defined shape, due in part to efforts of people like Siu, Demailly, Ein and the author. To quote Lazarsfeld: "it seems safe to predict that multiplier ideals and their variants are destined to become fundamental tools in algebraic geometry". Multiplier ideals appeared first in the complex analytic setting, where they are defined naturally in terms of integrability conditions. It was noted later that they can be defined in purely algebro-geometric terms, using log-resolutions. In fact this definition had already been worked-out in passing by H. Esnault and E. Viehweg in [*Lectures on vanishing theorems*, Birkhäuser, Basel, 1992; MR1193913 (94a:14017)]. For those less familiar with the topic, let me comment that multiplier ideals arise in this context from the wish to avoid passing to a birational modification in order to satisfy the normal crossing hypothesis required for the Kawamata-Viehweg vanishing theorem for **Q**-divisors (which by the Kawamata-Reid-Shokurov technique had become an essential tool in the study of linear series). Another, in fact very much related, approach to this circle of ideas is the notion of singularities of pairs. The author explains the connection between the two, but does not go deeper into the context of singularities of pairs. For this there are already excellent references, such as J. Kollár's survey [in *Algebraic geometry—Santa Cruz 1995*, 221–287, Proc. Sympos. Pure Math., 62, Part 1, Amer. Math. Soc., Providence, RI, 1997; MR1492525 (99m:14033)] and Kollár and S. Mori's book [*Birational geometry of algebraic varieties*, Translated from the 1998 Japanese original, Cambridge Univ. Press, Cambridge, 1998; MR1658959 (2000b:14018)].

The author begins Chapter 9 with a proof of the Kawamata-Viehweg vanishing theorem for **Q**-divisors, and continues with the basic definitions, properties and examples of multiplier ideals. Very interesting results presented at the end of the chapter are the Skoda and Briançon-Skoda theorems, which are local statements concerning multiplier ideals (and the integral closure) of powers of ideals, together with global version proved by Ein and the author [Invent. Math. **137** (1999), no. 2, 427–448; MR1705839 (2000j:14028)]. Chapter 10 is devoted to various applications of multiplier ideals to the general theory of divisors and linear series. Among the most interesting ones, where multiplier ideals seem to have provided a real breakthrough, are the Ein-Lazarsfeld theorems on the singularities of theta divisors [J. Amer. Math. Soc. **10** (1997), no. 1, 243–258; MR1396893 (97d:14063)], the approach of Y. T. Siu [Houston J. Math. **28** (2002), no. 2, 389–409; MR1898197 (2003i:32038)] and J.-P. Demailly [Invent. Math. **124** (1996), no. 1-3, 243–261; MR1369417 (97a:32035)] to Matsusaka's big theorem, and the Angehrn-Siu Fujita-type theorem on the global generation of adjoint bundles [U. Angehrn and Y. T. Siu, Invent. Math. **122** (1995), no. 2, 291–308; MR1358978 (97b:32036)]. Finally, Chapter 11 introduces probably the most modern concept in the book, asymptotic multiplier ideals, stemming from work of Siu on the deformation invariance of plurigenera. The author develops (for the first time) a coherent theory of asymptotic multiplier ideals, following the basic properties presented earlier in the ordinary case. This is then applied in a few directions. First, an application to uniform results in commutative algebra is described, following work of Ein, K. E. Smith and the author [Invent. Math. **144** (2001), no. 2, 241–252; MR1826369 (2002b:13001)]. We are then guided through a nice presentation of the context of Fujita's approximation theorem and its applications to the study of volumes of line bundles. Here the author includes a very nice recent result of S. Boucksom, Demailly, M. Paun and T. Peternell ["The pseudo-effective cone of a compact Kähler manifold and varieties of negative

Kodaira dimension", preprint, arxiv.org/abs/math/0405285], which describes the pseudoeffective cone of an irreducible variety as that dual to the cone of mobile curves (roughly speaking curves which move in families covering an open dense subset of the variety). The book concludes with what is widely regarded as the most spectacular application of multiplier ideals to date—the proof of the invariance of plurigenera for varieties of general type—due to Siu [Invent. Math. **134** (1998), no. 3, 661–673; MR1660941 (99i:32035)]. Although the original proof was analytic, the author presents here a relatively quick proof based on the algebro-geometric ideas developed in the text. (It should be said, however, that in the meantime, Siu [in *Complex geometry (Göttingen, 2000)*, 223–277, Springer, Berlin, 2002; MR1922108 (2003j:32027a)] has proved the deformation invariance of plurigenera for arbitrary varieties still based on the analytic theory of multiplier ideals, and in this case, no purely algebraic proof is currently known.)

Very much in this last part of the book is original, but as a single example expressing simply a preference of the reviewer, it is again remarkable, how Castelnuovo-Mumford regularity is used, together with vanishing theorems, in order to give purely conceptual proofs to key results on non-vanishing or global generation for twists of multiplier ideal sheaves.

Switching to overall comments, one interesting feature of the text is the fact that at times it provides new simplified proofs of, or new approaches to, well-established results. One such example is the proof of the Campana-Peternell theorem (Theorem 2.3.18), which is an analogue of the Nakai-Moishezon ampleness criterion in the setting of $\mathbf{R}$-divisors. Another is a generalization of the Nadel product lemma (Theorem 8.4.10), using the positivity of cone classes. More importantly, the book contains genuinely new results. A favorite of the reviewer is Theorem 2.2.44 on the continuity of the volume function. This has inspired a great deal of subsequent work on asymptotic invariants of line bundles.

There are numerous examples scattered throughout the text (together with various applications, they form about a third of the book). Some are very explicit, making essentially every concept introduced in the book quite easy to grasp. Others serve as a guide to further literature and encourage independent study. One recognizes here one of the features that has also made Fulton's *Intersection theory* [Second edition, Springer, Berlin, 1998; MR1644323 (99d:14003)] so successful.

For understanding the material presented in this text, the reader is assumed to have some familiarity only with standard introductory texts like Hartshorne's *Algebraic geometry* [Springer, New York, 1977; MR0463157 (57 #3116)] and P. Griffiths and J. Harris' *Principles of algebraic geometry* [Wiley-Intersci., New York, 1978; MR0507725 (80b:14001)], with only very occasional need to go beyond these sources. The author emphasizes that there is relatively little in the book about the Hodge-theoretic and complex analytic side of the story. For this he suggests the texts of C. Voisin [*Hodge theory and complex algebraic geometry. I*, Translated from the French original by Leila Schneps, Cambridge Univ. Press, Cambridge, 2002; MR1967689 (2004d:32020); *Hodge theory and complex algebraic geometry. II*, Translated from the French by Leila Schneps, Cambridge Univ. Press, Cambridge, 2003; MR1997577 (2005c:32024b)] and Demailly's *Complex analytic and algebraic geometry*, fragments of which can already be found in various places in the literature.

The book under review is exceptionally well written. It treats a large number of concepts and topics, but always in a gentle and explicit manner. It can be used both as a textbook and as a source

for current research problems. As such, it will be of great value to both students and experts in the field. It is also excellent as a guide to further literature. In my opinion, Lazarsfeld's book will become one of the fundamental references in the field of complex algebraic geometry.

Reviewed by *Mihnea Popa*

MR2088931 (2005g:32024) 32L10 (32A25 32L20)

Berman, Robert (S-CHAL)

Bergman kernels and local holomorphic Morse inequalities. (English summary)

Math. Z. **248** (2004), *no. 2,* 325–344.

The author proves a local version of J.-P. Demailly's holomorphic Morse inequalities [Ann. Inst. Fourier (Grenoble) **35** (1985), no. 4, 189–229; MR0812325 (87d:58147)] by a clever comparison of the Bergman kernel of a complex Hermitian manifold together with a Hermitian holomorphic line bundle and the Bergman kernel of the model $\mathbb{C}^n$ with flat metric and trivial line bundle with constant metric. After scaling, the comparison relies on elliptic theory.

Reviewed by *Christophe Mourougane*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2087040 (2005f:32041) 32Q45 (32L05)

El Goul, Jawher (F-TOUL3-LM)

Demailly's 2-jet negativity of certain hyperbolic fibrations. (English summary)

Complex analysis in several variables—Memorial Conference of Kiyoshi Oka's Centennial Birthday, 85–93, *Adv. Stud. Pure Math.*, 42, *Math. Soc. Japan, Tokyo*, 2004.

The paper under review concerns a conjecture of Demailly on k-jet negativity. J.-P. Demailly [in *Algebraic geometry—Santa Cruz 1995*, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997; MR1492539 (99b:32037)] conjectured that the existence of a metric with k-negativity on a k-jet bundle should characterize Kobayashi hyperbolicity for compact complex manifolds. In this paper the author deals with the special case of this conjecture. He considers the case of 2-jet bundles of a hyperbolic (singular) fibration on hyperbolic curves with certain conditions on the singularities of special fibers and proves a weak negativity property on this bundle. It is noticed that his method only works up to the 2-jet stage.

{For the entire collection see MR2087033 (2005c:32002)}

Reviewed by *Yoshihiro Aihara*

MR2087033 (2005c:32002) 32-06 (00B30)

★**Complex analysis in several variables—Memorial Conference of Kiyoshi Oka's Centennial Birthday.**

Papers from the conference held in Kyoto, October 30–November 5, 2001 and Nara, November 6–8, 2001.

Edited by Kimio Miyajima, Mikio Furushima, Hideaki Kazama, Akio Kodama, Junjiro Noguchi, Takeo Ohsawa, Hajime Tsuji and Tetsuo Ueda.

Advanced Studies in Pure Mathematics, 42.

Mathematical Society of Japan, *Tokyo*, 2004. *x*+345 *pp*. *ISBN* 4-931469-27-2

Contents: Oka, Kiyoshi (3–5) MR2087034; Toshio Nishino, Mathematics of Professor Oka—a landscape in his mind (17–30) MR2087035; Yoshihiro Aihara, Uniqueness problem for meromorphic mappings under conditions on the preimages of divisors (31–35); MR2087036 (2005f:32029); Takao Akahori, On the middle dimension cohomology of A_l singularity (37–44); MR2087037 (2005g:32042); Takashi Aoki, Takahiro Kawai and Yoshitsugu Takei, The exact steepest descent method—a new steepest descent method based on the exact WKB analysis (45–61); MR2087038 (2005h:34237); Eric Bedford, Excursions of a complex analyst into the realm of dynamical systems (63–84); MR2087039 (2005e:37095); Jawher El Goul, Demailly's 2-jet negativity of certain hyperbolic fibrations (85–93); MR2087040 (2005f:32041); John Erik Fornæss, Short $\mathbb{C}^k$

(95–108); MR2087041 (2005h:32027); Hirotaka Fujimoto, Some constructions of hyperbolic hypersurfaces in $P^n(\mathbf{C})$ (109–114); MR2087042 (2005h:32059); Kengo Hirachi, A link between the asymptotic expansions of the Bergman kernel and the Szegö kernel (115–121); MR2087043 (2005e:32004); Andrei Iordan, On the non-existence of smooth Levi-flat hypersurfaces in $\mathbb{CP}_n$ (123–126); MR2087044 (2005f:32061); Su-Jen Kan, Recent development on Grauert domains (127–133); MR2087045 (2005m:32021).

Kang-Tae Kim, Analytic polyhedra with non-compact automorphism group (135–140); MR2087046 (2005e:32039); Shoshichi Kobayashi, Problems related to hyperbolicity of almost complex structures (141–146); MR2087047 (2005g:32035); Joseph J. Kohn, Ideals of multipliers (147–157); MR2087048 (2005e:32056); Gen Komatsu, The Bergman kernel of Hartogs domains and transformation laws for Sobolev-Bergman kernels (159–164); MR2087049 (2005f:32005); Masatake Kuranishi, An approach to the Cartan geometry. II. CR manifolds (165–187); MR2087050 (2006e:32046); László Lempert, The $\overline{\partial}$ equation in N variables, as N varies (189–202); MR2087051 (2005g:32053); Kazuko Matsumoto, Levi form of logarithmic distance to complex submanifolds and its application to developability (203–207); MR2087052 (2005h:32016); Yoichi Miyaoka, Numerical characterisations of hyperquadrics (209–235); MR2087053 (2005h:14123); Seiki Mori, Meromorphic mappings and deficiencies (237–242); MR2087054 (2005h:32040); Junjiro Noguchi, Intersection multiplicities of holomorphic and algebraic curves with divisors (243–248); MR2087055 (2005f:32030); Takeo Ohsawa, Generalization of a precise L^2 division theorem (249–261); MR2087056 (2005h:32012).

Mikael Passare, Amoebas, convexity and the volume of integer polytopes (263–268); MR2087057 (2005e:32011); R. Michael Range, On the decomposition of holomorphic functions by integrals and the local CR extension theorem (269–273); MR2087058 (2005e:32055); Oswald Riemenschneider, The monodromy covering of the versal deformation of cyclic quotient surface singularities (275–282); MR2087059 (2006b:32041); Georg Schumacher, Moduli as algebraic spaces (283–288); MR2087060 (2005g:14027); Satoru Shimizu [Satoru Shimizu[1]], Prolongation of holomorphic vector fields on a tube domain and its applications (289–299); MR2087061 (2005h:32051); Manabu Shirosaki, Hypersurfaces and uniqueness of holomorphic mappings (301–305); MR2087062 (2005f:32031); Shigeharu Takayama, Seshadri constants and a criterion for bigness of pseudo-effective line bundles (307–312); MR2087063 (2005h:14018); Hajime Tsuji, Subadjunction theorem (313–318); MR2087064 (2005j:32010); Tetsuo Ueda, Fixed points of polynomial automorphisms of C^n (319–324); MR2087065 (2005e:32033); Katsutoshi Yamanoi, On Nevanlinna theory for holomorphic curves in abelian varieties (325–331); MR2087066 (2005d:32026); Stephen S.-T. Yau, Numerical characterization for affine varieties be a cone over nonsingular projective varieties (333–338); MR2087067 (2005f:32047); Ken-Ichi Yoshikawa, Nikulin's $K3$ surfaces, adiabatic limit of equivariant analytic torsion, and the Borcherds Φ-function (339–345); MR2087068 (2005h:14093).

{Most of the papers are being reviewed individually.}

MR2086081 (2005i:14009) 14C20 (14K05)
Tutaj-Gasińska, Halszka (PL-JAGL)
Seshadri constants in half-periods of an abelian surface. (English summary)
J. Pure Appl. Algebra **194** (2004), *no. 1-2,* 183–191.

In recent years there has been a major interest in studying the local positivity of ample line bundles on algebraic varieties. Seshadri constants, introduced by J.-P. Demailly [in *Complex algebraic varieties (Bayreuth, 1990)*, 87–104, Lecture Notes in Math., 1507, Springer, Berlin, 1992; MR1178721 (93g:32044)], partially in connection with the study of T. Fujita's conjecture [in *Algebraic geometry, Sendai, 1985*, 167–178, Adv. Stud. Pure Math., 10, North-Holland, Amsterdam, 1987; MR0946238 (89d:14006)], incorporate in a natural way information about the local positivity of a line bundle.

Let X be a smooth projective variety of dimension n, $x_1,\dots,x_r \in X$ distinct points and L an ample line bundle on X. Let π be the blow-up of X in the considered points with $E_1,\dots,E_r$ the exceptional divisors. Then the real number

$$\varepsilon(L,x_1,\dots,x_r)=\sup\{\varepsilon\in\mathbf{R}|\ \pi^*L-\varepsilon\sum_{i=1}^r E_i \text{ is nef}\}$$

is the Seshadri constant of L at $x_1,\dots,x_r$ (called the multiple point Seshadri constant if $r\geq 2$). Equivalently

$$\varepsilon(L,x_1,\dots,x_r):=\inf_{C\ni x_1,\dots,x_r}\frac{L\cdot C}{\sum_{i=1}^r \operatorname{mult}_{x_i}C},$$

(where the infimum is taken over all irreducible curves passing through $x_1,\dots,x_r$).

Despite their apparently "easy" definition, Seshadri constants are very difficult to compute. In fact their exact value is known only for a few cases (even for $r=1$) and even on surfaces it is difficult to control them, as was already pointed out by Demailly. To put things into perspective, the computation of multiple point Seshadri constants for $\mathbf{P}^2$ is equivalent to the unsolved Nagata conjecture [M. Nagata, Chinese J. Math. **11** (1983), no. 1, 1–4; MR0692988 (84f:14008)]. Therefore, any contribution towards bounding or calculating them is of interest.

One has the bounds $0<\varepsilon(L,x_1,\dots,x_r)\leq\sqrt[n]{\frac{L^n}{r}}$ from [L. M. H. Ein, O. Küchle and R. K. Lazarsfeld, J. Differential Geom. **42** (1995), no. 2, 193–219; MR1366545 (96m:14007)]. It is still not known whether a Seshadri constant can be non-rational. However, some useful information is that if $\varepsilon(L,x_1,\dots,x_r)<\sqrt[n]{\frac{L^n}{r}}$, then $\varepsilon(L,x_1,\dots,x_r)$ is rational, by A. Steffens [Math. Z. **227** (1998), no. 3, 505–510; MR1612681 (99c:14009)].

Seshadri constants on abelian surfaces have been studied in [T. Bauer, Math. Ann. **312** (1998), no. 4, 607–623; MR1660259 (2000a:14054)] and in [T. Bauer, Math. Ann. **313** (1999), no. 3, 547–583; MR1678549 (2000d:14006)]. In the appendix of the latter paper it is proved that a one-point Seshadri constant on an abelian surface is always rational and an upper bound is given, which is

shown to be attained in the case of Picard number one in the first paper.

In the very nice paper under review the author obtains the following result for multiple point Seshadri constants of an ample line bundle L of type $(1,d)$ at any r of the 16 half-periods $e_1,\dots,e_{16}$ of an abelian surface $S\simeq \mathbf{C}^2/\Lambda$, i.e., $e_1,\dots,e_{16}$ are the elements of $\frac{1}{2}\Lambda$:

(a) If $\sqrt{\frac{2d}{r}}\in\mathbf{Q}$, then $\varepsilon(L,e_1,\dots,e_r)=\sqrt{\frac{2d}{r}}$.

(b) If $\sqrt{\frac{2d}{r}}\notin\mathbf{Q}$, then $\varepsilon(L,e_1,\dots,e_r)\le 2dk_0/l_0$, where (k_0,l_0) is the primitive solution of Pell's equation $2rdk^2+1=l^2$.

In particular $\varepsilon(L,e_1,\dots,e_r)$ is rational.

The method of proof consists of using results from the last two papers mentioned and from [H. Lange and C. Birkenhake, *Complex abelian varieties*, Springer, Berlin, 1992; MR1217487 (94j:14001)] to explicitly construct curves in $|mL|$, for suitable $m>0$, passing through $e_1,\dots,e_r$ with the desired multiplicities.

Reviewed by *Andreas Leopold Knutsen*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2078708 (2005c:14018) 14E30
Solá Conde, Luis Eduardo; **Wiśniewski, Jarosław A.** (PL-WASW-IM)
On manifolds whose tangent bundle is big and 1-ample.
Proc. London Math. Soc. (3) **89** (2004), *no. 2,* 273–290.

Much of the recent work in Fano geometry and in the classification of complex manifolds is related to the problem of characterizing complex-projective manifolds X whose tangent bundles T_X have certain positivity properties.

After S. Mori's spectacular solution to the Hartshorne conjecture, asserting that the projective space $\mathbb{P}_n$ is the only manifold whose tangent bundle is ample, the next important case is that of manifolds whose tangent bundle is nef—recall that T_X is called ample or nef if the line bundle $\mathcal{O}_{\mathbb{P}(T_X)}(1)$ on the projectivization $\mathbb{P}(T_X)$ is ample or nef. It has been conjectured by Campana and Peternell that T_X nef implies that X is homogeneous.

While the Campana-Peternell conjecture is still open, the paper under review studies a more special situation where the tangent bundle T_X is assumed to be almost, but not quite, ample. More precisely, the authors classify manifolds whose tangent bundle is big and 1-ample. The assumptions imply that global sections in the bundle $\mathcal{O}_{\mathbb{P}(T_X)}(m)$, for $m \gg 0$, give rise to a morphism $\mathbb{P}(T_X) \to Y$ whose fibers are at most 1-dimensional. A part of the argumentation is then based on the observation that the complement of the zero section of the cotangent bundle $T_X^\vee$ is a complex-symplectic manifold and makes use of earlier works where morphisms from symplectic manifolds were studied.

The paper contains an appendix where the authors fill two gaps in earlier papers of J. Wierzba [J. Algebraic Geom. **12** (2003), no. 3, 507–534; MR1966025 (2003m:14023)] and J.-P. Demailly, T. Peternell and M. H. Schneider [J. Algebraic Geom. **3** (1994), no. 2, 295–345; MR1257325 (95f:32037)].

Reviewed by *Stefan Kebekus*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2069123 (2005b:32058) 32Q28 (32A07 32E10 32L05)
Pflug, Peter (D-OLD-M); **Zwonek, Wlodzimierz** (PL-JAGL)
The Serre problem with Reinhardt fibers. (English, French summaries)
Ann. Inst. Fourier (Grenoble) **54** (2004), *no. 1,* 129–146.

The Serre problem asks whether a holomorphic fiber bundle with a Stein fiber F and a Stein basis is itself Stein. Although many positive answers were given in special cases, the first counterexamples were given by H. Skoda [Invent. Math. **43** (1977), no. 2, 97–107; MR0508091 (58 #22657)] and a few afterwards by Demailly. In 1985 G. Coeuré and the reviewer [Ann. of Math. (2) **122** (1985), no. 2, 329–334; MR0808221 (87c:32033)] gave a counterexample with F a bounded Reinhardt domain in $\mathbb{C}^2$. In this paper, the authors characterize the hyperbolic Reinhardt domains F in $\mathbb{C}^2$ which can be produced as fibers for a counterexample to the Serre problem. Such an F a priori can be of three types: Type 0: F is complete; Type 1: the intersection of F with exactly one axis is nonempty; Type 2: the intersection with the two axes is empty.

The main result here is that for Types 0 and 1, F cannot be a counterexample, but there are counterexamples for Type 2, and in this paper the counterexamples are described using a matrix of $\mathrm{GL}(2,\mathbb{Z})$, generalizing the description given by Coeuré and the reviewer.

The Type 0 case was proved by K. Königsberger [Math. Ann. **189** (1970), 178–184; MR0268410 (42 #3308)] and can also be seen as a special case of general results about the Serre problem.

The Type 1 case uses explicit results of Shimizu and Kruzhilin on the automorphism group of Reinhardt domains. A theorem of Stehle and an ad hoc extending lemma are also used.

The Type 2 case also uses results of Shimizu about the algebraicity of the automorphism group. In order to construct counterexamples, the method of the paper of Coeuré and the reviewer is applied.

It should also be noted that recently K. Oeljeklaus and D. Zaffran have obtained new results in this direction.

Reviewed by *Jean-Jacques Loeb*

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MR2068946 (2005e:32005) 32A25 (32W05 53C56 53C60)

Qiu, Chunhui (PRC-XIAM); **Zhong, Tongde** (PRC-XIAM)

Integral formulas for differential forms of type (p, q) on complex Finsler manifolds. (English summary)

Sci. China Ser. A **47** (2004), *no. 2,* 284–296.

In this paper, using the invariant integral kernel introduced by J.-P. Demailly and C. Laurent-Thiébaut [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 4, 579–598; MR0932799 (89g:32023)], the authors obtain Koppelman and Koppelman-Leray formulas for relatively compact domains with C^1 boundary in strongly pseudoconvex complex Finsler manifolds, and then apply them to solve the $\overline{\partial}$-equation in such domains.

Reviewed by *Marco Abate*

MR2066940 (2005g:32020) 32H50 (32J27 32Q15 37B40 37C85 37F10)
Dinh, Tien-Cuong (F-PARIS11); **Sibony, Nessim** (F-PARIS11)
Groupes commutatifs d'automorphismes d'une variété kählérienne compacte. (French. English, French summaries) [Commutative groups of automorphisms of a compact Kähler manifold]
Duke Math. J. **123** (2004), *no. 2,* 311–328.

It is quite difficult to construct automorphisms of a compact, complex manifold V and it is even harder to find automorphisms that commute. One would therefore expect abelian subgroups of $\mathrm{Aut}(V)$ to be quite special. The main result in the paper under review is a kind of structure theorem for abelian subgroups of $\mathrm{Aut}(V)$ in the case in which V is a compact Kähler manifold.

Recall that the topological entropy of an endomorphism f of V is a number which measures the complexity of f. The dynamically most interesting endomorphisms are those with positive entropy.

Main Theorem: let $\mathcal{G}'$ be an abelian subgroup of $\mathrm{Aut}(V)$ and let U be the set of elements of $\mathcal{G}'$ of zero entropy. Then U is a group and $\mathcal{G}'$ is isomorphic to the direct product $U \times \mathcal{G}$, where $\mathcal{G}$ is a subgroup of $\mathcal{G}'$ such that all elements of $\mathcal{G} \setminus \{\mathrm{id}\}$ have positive entropy. Moreover, $\mathcal{G}$ is a free abelian subgroup of index at most $\dim V - 1$. This estimate is sharp and, in the case of equality, U is finite.

The proof proceeds by studying the action of $\mathcal{G}'$ on the Dolbeault cohomology group $H^{1,1}(V, \mathbf{R})$. To this end, the authors exploit the structure of the Kähler cone $\mathcal{H} \subset H^{1,1}(V, \mathbf{R})$, in particular a version of the Hodge-Riemann theorem due to M. L. Gromov [in *Advances in differential geometry and topology*, 1–38, World Sci. Publishing, Teaneck, NJ, 1990; MR1095529 (92d:52018)] and a recent result by J.-P. Demailly and M. Paun [Ann. of Math. (2) **159** (2004), no. 3, 1247–1274; MR2113021]. They also use the characterization by Gromov [Astérisque No. 145-146 (1987), 5, 225–240; MR0880035 (89f:58082); Enseign. Math. (2) **49** (2003), no. 3-4, 217–235; MR2026895] and Y. Yomdin [Israel J. Math. **57** (1987), no. 3, 285–300; MR0889979 (90g:58008)] that $f \in \mathrm{Aut}(V)$ has positive entropy if and only if the spectral radius of the induced action of f on $H^{1,1}(V, \mathbf{R})$ is strictly larger than one.

By the Perron-Frobenius theorem, each $f \in \mathrm{Aut}(V)$ of positive entropy admits a class in $\overline{\mathcal{H}}$ which is an eigenvector for f^*. Roughly speaking, then, if there were too many (commuting, without relations) such f, we would find a large invariant subspace of $H^{1,1}$ generated by elements of $\overline{\mathcal{H}}$. Using the Hodge-Riemann theorem, the authors show that this is impossible.

Overall, the paper is well-written and contains a nice mix of ideas from dynamics and analytic geometry.

Reviewed by *Mattias Jonsson*

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MR2058513 (2005d:13016) 13B22 (14J17)

Takagi, Shunsuke [Takagi, Shunsuke[2]] (J-TOKYOGM);
Watanabe, Kei-ichi [Watanabe, Keiichi[1]] (J-NIHOH)

When does the subadditivity theorem for multiplier ideals hold? (English summary)

Trans. Amer. Math. Soc. **356** (2004), *no. 10,* 3951–3961 (*electronic*).

An important property of multiplier ideals is sub-additivity, which in its simplest incarnation predicts that the multiplier ideal of a product of two ideal sheaves $\mathfrak{a}$ and $\mathfrak{b}$ on a smooth complex variety is contained in the product of the multiplier ideal of $\mathfrak{a}$ and the multiplier ideal of $\mathfrak{b}$. (Subadditivity was proved by J.-P. Demailly, L. M. H. Ein and R. K. Lazarsfeld in [Michigan Math. J. **48** (2000), 137–156; MR1786484 (2002a:14016)].) The subadditivity theorem is at the heart of many of the applications of mutliplier ideals to commutative algebra and algebraic

geometry. It is natural to hope that some form of subadditivity might hold in settings more general than smooth varieties.

The paper under review shows that the subadditivity theorem, as stated in the simple form above, holds when the ambient variety is a surface with log terminal singularities. It also proves that if the general form of subadditivity (in which coefficients are allowed) holds for some surface, then that surface must be smooth. That is, if X is a two dimensional normal $\mathbf{Q}$-Gorenstein surface, and $\mathcal{J}(\mathbf{a}^c\mathbf{b}^d) \subset \mathcal{J}(\mathbf{a}^c)\mathcal{J}(\mathbf{b}^d)$ for all ideals $\mathbf{a}, \mathbf{b}$ and all positive rational numbers c, d, then the surface X is in fact smooth. The paper also provides simple examples of higher dimensional varieties with very nice singularities (including the toric case) in which even the simplest form of subadditivity fails.

Reviewed by *Karen E. Smith*

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MR2051683 (2005d:32056) 32U25 (32H50 32U40 37F10)
Coman, Dan (1-SRCS); **Guedj, Vincent** (F-TOUL3-LM)
Invariant currents and dynamical Lelong numbers. (English summary)
J. Geom. Anal. **14** (2004), *no. 2,* 199–213.

Let $f = (P_1, \ldots, P_k)\colon \mathbf{C}^k \to \mathbf{C}^k$ be a polynomial automorphism with $\lambda := \max \deg P_j \geq 2$. The meromorphic extension, which we still denote by f, to $\mathbf{P}^k = \mathbf{C}^k \cup (t=0)$ is not well-defined on the indeterminacy locus I^+. In this article, the authors consider those f for which $X^+ := f((t=0) \setminus I^+)$ reduces to a single point lying outside of I^+. Such an f is algebraically stable and it follows that the sequence of currents $\lambda^{-n}(f^n)^*\omega$, where ω is the Fubini-Study form on $\mathbf{P}^k$, converges to a positive closed current T_+ of bidegree $(1,1)$ with $f^*T_+ = \lambda T_+$ [cf. N. Sibony, in *Dynamique et géométrie complexes (Lyon, 1997)*, ix–x, xi–xii, 97–185, Soc. Math. France, Paris, 1999; MR1760844 (2001e:32026); V. Guedj and N. Sibony, Ark. Mat. **40** (2002), no. 2, 207–243; MR1948064 (2004b:32029)]. Define $\lambda_2(f) := \lim_{n\to\infty}[\delta_2(f^n)]^{1/n}$ where $\delta_2(f^n)$ is the degree of $f^{-n}(L)$ for L a generic linear subspace of codimension 2. Under the additional assumption that $\lambda > \lambda_2(f)$—this includes, e.g., complex Hénon maps in $\mathbf{C}^2$—a positive closed current σ_- of bidegree $(k-1, k-1)$ of unit mass with $(f^{-1})^*\sigma_- = \lambda\sigma_-$ can be constructed [V. Guedj and N. Sibony, op. cit. (Theorem 3.1)]. This construction is carried out again in Section 2.1 where it is shown that σ_- puts no mass on the hyperplane at infinity (Theorem 2.2). In the case where I^+ is f^{-1}-attracting, this was shown in [V. Guedj and N. Sibony, op. cit.]. It follows that for a positive closed current S of bidegree $(1,1)$ and unit mass on $\mathbf{P}^k$, $S \wedge \sigma_-$ is well-defined as a probability measure. We define the generalized Lelong number of S with respect to σ_- as $\nu(S, \sigma_-) := S \wedge \sigma_-(\{X^+\})$ (Definition 2.3). In particular, let f be a regular automorphism, i.e., $I^+ \cap I^- = \varnothing$. In this case f^{-1} is algebraically stable and the invariant Green current T_- for f^{-1} is well-defined. Here the number $\nu(S, \sigma_-)$ reduces to the Demailly number of S with respect to the weight T_- [J.-P. Demailly, in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)]. Moreover, in this setting, $\nu(S, \sigma_-)$ is positive if and only if the standard Lelong number $\nu(S, X^+)$ at the point X^+ is positive.

The main result of this article, Theorem 1.1, concerns polynomial automorphisms f with $X^+ \cap I^+ = \varnothing$, $\lambda > \lambda_2(f)$, and such that I^+ is an attracting set for f^{-1}. It states that if S is a positive closed current of bidegree $(1,1)$ and unit mass on $\mathbf{P}^k$, then

$$\frac{1}{\lambda^n}(f^n)^*S \to c_S[t=0] + (1-c_s)T_+$$

as currents on $\mathbf{P}^k$, where $c_S = \nu(S, \sigma_-)$. Moreover, as in the regular automorphism setting, $\nu(S, \sigma_-) > 0$ if and only if $\nu(S, X^+) > 0$. In Section 2.3, an interesting invariant probability measure $\mu_f = T^+ \wedge \sigma_-$ is introduced; in particular, plurisubharmonic functions of logarithmic growth are shown to be integrable with respect to μ_f (Remark 2.13). The proof of Theorem 1.1 is given in Section 3. In the fourth and final section, the authors utilize the classification of J. E. Fornæss and H. Wu [Publ. Mat. **42** (1998), no. 1, 195–210; MR1628170 (99e:14015)] to check their hypotheses on families of quadratic polynomial automorphisms of $\mathbf{C}^3$.

Reviewed by *Norman Levenberg*

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MR2050205 (2005i:32018) 32J18 (32C30)

Boucksom, Sébastien (F-GREN-F)

Divisorial Zariski decompositions on compact complex manifolds. (English, French summaries)

Ann. Sci. École Norm. Sup. (4) **37** (2004), *no. 1,* 45–76.

It was originally shown by Zariski that any effective $\mathbb{Q}$-divisor D on a projective surface X can be decomposed uniquely into a sum $D = P + N$, where P is a numerically effective $\mathbb{Q}$-divisor and $N = \sum a_j D_j$ is an effective $\mathbb{Q}$-divisor such that the Gram matrix $(D_i \cdot D_j)$ is negative definite. In terms of the intersection form, P is moreover orthogonal to N. The spaces of global holomorphic sections corresponding to $H^0(kP)$ and $H^0(kD)$ are then isomorphic, so that the ring $R(X, D) = \bigoplus_{k\geq 0} H^0(X, \mathcal{O}(kD))$ is equivalent to $R(X, P)$ via an isomorphism that respects the natural decomposition. In order to generalise this result to an arbitrary compact complex manifold, the author of the present paper employs Demailly's theory of regularization of almost positive $(1,1)$-currents to obtain a Zariski-type decomposition of a pseudo-effective class $\alpha \in H^{1,1}_{\partial\overline{\partial}}(X, \mathbb{R})$. Recall that in the world of complex manifolds beyond Kähler geometry it is convenient to work with the cohomology of d-closed smooth $(1,1)$-forms modulo $\partial\overline{\partial}$-exact ones, such that the classes are easily seen to represent a more general affine space of closed $(1,1)$-currents. Moreover, when X is compact the $\partial\overline{\partial}$-operator has closed range, so that $H^{1,1}_{\partial\overline{\partial}}(X, \mathbb{C})$ is finite-dimensional. The author's generalisation of the Zariski decomposition is therefore in the spirit of the Siu decomposition for closed positive currents. A real $(1,1)$-current T is said to be "almost positive" if there exists a smooth real $(1,1)$-form γ such that $T \geq \gamma$. Given the definition of the Lelong number of T at each $x \in X$, one defines $\nu(T, D)$ to be the infimum of the Lelong numbers over all points of a given prime divisor D, and arrives at the Siu decomposition of T accordingly. A cohomology class α, as above, is said to be "pseudoeffective" iff it contains a positive current. It is "nef" iff for each $\varepsilon > 0,\ \alpha$ contains a smooth form $\vartheta_\varepsilon \geq -\varepsilon\omega$, where ω denotes the Hermitian form associated with a Gauduchon metric on X. It is "big" iff it contains a Kähler current, i.e., a closed $(1,1)$-current T such that $T \geq -\varepsilon\omega$ for sufficiently small ε. Let $\alpha[\gamma]$ denote the set of closed almost positive currents such that $T \geq \gamma$ for a smooth real $(1,1)$-form γ. It follows that any family of elements of $\alpha[\gamma]$ has an infimum with respect to a given preorder relation. In particular, $T_{\min,\gamma}$ will denote the infimum within $\alpha[\gamma]$ itself. The author introduces the notion of "minimal multiplicity" for a pseudoeffective class α as

$$\nu(\alpha, x) = \sup_{\varepsilon>0} \nu(T_{\min,-\varepsilon\omega}, x),$$

and hence $\nu(\alpha, D)$ as the infimum of this multiplicity along a given prime divisor D. α is then nef iff $\nu(\alpha, x)$ vanishes everywhere, while the "non-nef locus" corresponds to the set of points of X where the multiplicity is positive. The main result of the paper is to derive a decomposition of the form $\alpha = N + Z$, where $N = \sum \nu(\alpha, D)D$ is the summation over all prime divisors of X, and Z is a real pseudoeffective $(1,1)$-class. The author proves that there are only finitely many prime divisors D for which $\nu(\alpha, D) > 0$ (the non-nef locus of α), while the non-nef locus of Z, though not empty in general, contains no prime divisors (hence it is said to belong to the "modified nef-cone" of the Néron-Severi space). This decomposition is naturally induced by the Siu decomposition of a closed positive current with the minimality condition above inside α when it is big. When X is a surface, the distinction between modified-nef and nef classes disappears, and when X is moreover projective one recovers the original Zariski decomposition if α is the class of an effective $\mathbb{Q}$-divisor D. Extensions of the surface theory to compact hyper-Kähler manifolds are also explored via the Beauville-Bogomolov form on $H^{1,1}(X, \mathbb{R})$.

Reviewed by *Adam Gregory Harris*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2045101 (2004m:32059) 32S65 (32Q30 32U40 37F75)
Brunella, Marco (F-DJON-IM)
On the regularity of the leafwise Poincaré metric. (English summary)
Bull. Sci. Math. **128** (2004), *no. 3,* 189–195.

This article is the continuation of the author's previous work [Invent. Math. **152** (2003), no. 1, 119–148; MR1965362 (2003m:32028)]. Let X, $\mathcal{F}$ and Ω be as in the review of the above-mentioned article. Moreover, assume that the singularities of $\mathcal{F}$ are reduced in Seidenberg's sense (one can obtain this after a finite number of blow-ups in X). On one hand, according to [Y. T. Siu, Invent. Math. **27** (1974), 53–156; MR0352516 (50 #5003)] we have $\Omega = \Omega_{\text{alg}} + \Omega_{\text{res}}$, where Ω_{alg} is a finite sum of integration currents over algebraic cycles and Ω_{res} is a closed positive current with vanishing Lelong number outside a finite set of X. On the other hand, we have Lebesgue's decomposition $\Omega = \Omega_{\text{sing}} + \Omega_{\text{ac}}$ of Ω into singular and absolutely continuous parts [see, for instance, J.-P. Demailly, J. Differential Geom. **37** (1993), no. 2, 323–374; MR1205448 (94d:14007)]. In general for a closed positive current we have $\Omega_{\text{alg}} \leq \Omega_{\text{sing}}$ and the main result of this article is the equality $\Omega_{\text{alg}} = \Omega_{\text{sing}}$. According to [M. McQuillan, in *European Congress of Mathematics, Vol. II (Barcelona, 2000)*, 47–53, Progr. Math., 202, Birkhäuser, Basel, 2001; MR1905350 (2003j:14048)], for a foliation $\mathcal{F}$ of general type, i.e. $\operatorname{kod}(\mathcal{F}) = 2$, we have $\Omega_{\text{alg}} = 0$ and so in this case Ω is absolutely continuous. The local analysis of Ω is used in the proof of the main result.

Reviewed by *Hossein Movasati*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2038086 (2004k:32039) 32Q15 (32C30)

Popovici, Dan [Popovici, Dan[2]] (F-GREN-IF)

Estimation effective de la perte de positivité dans la régularisation des courants. (French. English, French summaries) [Effective estimate of positivity loss in current regularizations]

C. R. Math. Acad. Sci. Paris **338** (2004), *no. 1,* 59–64.

Summary: "Let (X,ω) be a compact complex Hermitian manifold, and let $T \geq \gamma$ be a d-closed $(1,1)$ almost positive current on X. A variant of Demailly's regularization-of-currents theorem states that T is the weak limit of a sequence of $(1,1)$-currents T_m with analytic singularities of coefficient $1/m$, lying in the same cohomology class as T, whose Lelong numbers converge to those of T, and with a loss of positivity decaying to zero. We prove that if the $(1,1)$-form γ is assumed to be closed and C^∞, the regularizing currents T_m can be chosen such that $T_m \geq \gamma - \frac{C}{m}$ for a constant $C > 0$ independent of m."

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3. D. Popovici, Quelques applications des méthodes effectives en géométrie analytique, Thèse Univ. de Grenoble, en préparation.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2036329 (2004m:32074) 32W20
Błocki, Zbigniew (PL-JAGL)
On the definition of the Monge-Ampère operator in $\mathbb{C}^2$. (English summary)
Math. Ann. **328** (2004), *no. 3,* 415–423.

Let u be plurisubharmonic (psh) on an open subset Ω in $\mathbf{C}^N$; we write $u \in \mathrm{PSH}(\Omega)$. If $u \in C^2(\Omega)$, the complex Monge-Ampère operator $(dd^c(\cdot))^N$ applied to u,

$$(dd^c u)^N = dd^c u \wedge \cdots \wedge dd^c u = 4^N N! \det\,[\frac{\partial^2 u}{\partial z_j \partial \bar z_k}] d\lambda,$$

is a nonnegative function on Ω times the volume form $d\lambda$ on $\mathbf{C}^N$. If u is locally bounded, or, more generally, if the set of points where u is not locally bounded is relatively compact in Ω, then $(dd^c u)^N$ is well-defined [E. Bedford and B. A. Taylor, Invent. Math. **37** (1976), no. 1, 1–44; MR0445006 (56 #3351); Acta Math. **149** (1982), no. 1-2, 1–40; MR0674165 (84d:32024); J.-P. Demailly, in *Complex analysis and geometry*, 115–193, Plenum, New York, 1993; MR1211880 (94k:32009)] as a positive Radon measure and has the desirable property that it is continuous under decreasing sequences; i.e., if $u_j \downarrow u$, then $(dd^c u_j)^N \to (dd^c u)^N$ in the weak-* topology. Define $\mathcal{D}(\Omega)$ to be the class of psh functions u on Ω such that there exists a positive Radon measure μ on Ω with the property that if $\Omega' \subset \Omega$ is open and $\{u_j\}$ is any sequence of smooth, psh functions on Ω' which decrease to u in Ω', then $(dd^c u_j)^N \to \mu|_{\Omega'}$ weak-*. The main result of this paper, Theorem 1.1, is that for $N = 2$,

$$\mathcal{D}(\Omega) = \mathrm{PSH}(\Omega) \cap W^{1,2}_{\mathrm{loc}}(\Omega).$$

As observed by Bedford and Taylor in [op. cit., 1976], an integration by parts shows that $(dd^c u)^2$ is well-defined for $u \in \mathrm{PSH}(\Omega) \cap W^{1,2}_{\mathrm{loc}}(\Omega)$, $\Omega \subset \mathbf{C}^2$. The author verifies that this class coincides with $\mathcal{D}(\Omega)$, and in the final section of the paper he shows that if Ω is a bounded hyperconvex

domain in $\mathbf{C}^2$ (there exists a negative psh function φ in Ω with $\lim_{z\to\partial\Omega}\varphi(z)=0$), then the negative psh functions in $\mathcal{D}(\Omega)=\mathrm{PSH}(\Omega)\cap W^{1,2}_{\mathrm{loc}}(\Omega)$ coincide with the class $\mathcal{E}(\Omega)$ defined by U. Cegrell [in *Actes des Rencontres d'Analyse Complexe (Poitiers-Futuroscope, 1999)*, 39–42, Atlantique, Poitiers, 2002; MR1944194 (2003j:32047)].

In a forthcoming paper, the author gives a characterization of the class $\mathcal{D}(\Omega)$ when Ω is an open subset of $\mathbf{C}^N$ for any $N\geq 2$, and he shows that for a negative psh function u in a hyperconvex domain Ω, $u\in\mathcal{E}(\Omega)$ if and only if $u\in\mathcal{D}(\Omega)$.

Reviewed by *Norman Levenberg*

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