Report on the paper

Rationally connected manifolds and semipositivity of the Ricci curvature

by F. Campáná, J.-P. Demailly and T. Peternell.

This is a very nice article, whose main goal is to establish a structure theorem (Theorem 1.4) for compact Kähler manifolds with semi-positive anti-canonical bundle. The authors show that, up to a finite cover, such manifolds may be written, both holomorphically and isometrically, as Cartesian products of Ricci flat manifolds and rationally connected smooth varieties. I believe the article to be complete and correct, and recommend that it be accepted for publication, as it is well-written and pertinent, and will surely by useful in a number of ways.

Let me say a little about the proof. The first step in proving the decomposition is to establish a generalized holonomy principle (Theorem 1.5), which itself is deduced from a variant of the Bochner technique of computing the Laplacian of the pointwise L^2-norm of a section. Instead of L^2 norms, the authors compute the Laplacian of a quotient of metrics.

Once the holonomy principle is available, the authors rely on a criterion (Criterion 1.1) for rational connectedness in terms of either curvature of bundles of forms or tensors, or (non)existence of sections of such bundles. (The latter criterion is quite close to the assumption of a conjecture attributed to Mumford, and in fact, the authors obtain Mumford's conjecture, in the case where $-K_X$ is semipositive, as a consequence of their Structure Theorem 1.4.) The criterion is a fairly straight-forward consequence of a now-well known result of Boucksom-Demailly-Păun-Peternell.

Finally, there is an interesting and well-written appendix by the second author establishing a variant of the holonomy principle and the rational connectedness criterion in the setting a flag varieties.

Let me conclude with a couple of minor typos.

(1) In section 4, in the second line of the first paragraph, "tangent vector vector" should be "tangent bundle".
(2) In section 4, at the end of the proof of 1.4(a), "can be pseudoeffective" should be "can't (or cannot) be pseudoeffective".

I also have some minor suggestions or a grammatical or lexical nature, which the authors are welcome to ignore. (I might ignore them myself if I were one of the authors, but I feel badly not giving any constructive criticism.)

(1) In the first page, the authors refer to points which are "aligned" and "located on a conic". The latter is not so bad, but the former is non-standard and could be ambiguous (though not to experts). Its dictionary definition means more about being of the same mindset, or having the same purpose, than being on the same line. I would recommend "are aligned $\rightarrow$ are collinear", and less so, "located on a conic $\rightarrow$ lie on a conic".
(2) The authors are inconsistent about the use of the prefix "non". In a number of places, it appears separate from the word it modifies, for example, at the top of page 2 they have "non existence". In some other places, it appears connected to the word: for example, at the bottom of page 1, the word "nonnegative" appears. The correct way to do it is either the second way, or with a hyphen: non-existence.

Finally, I think it would be better to have all the references at the end of the article, instead of breaking things up into two articles, even if the appendix is quite independent of the rest of the paper. This suggestion makes it easier to find the references, since the appendix is about as long as the whole paper, and flipping to the middle of the article is a little annoying. (Very little.)