STRONG K-STABILITY AND ASYMPTOTIC CHOW-STABILITY

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Abstract. For a polarized algebraic manifold (X, L), let T be an algebraic torus in the group $\mathrm{Aut}(X)$ of all holomorphic automorphisms of X. Then strong relative K-stability (cf. [6]) will be shown to imply asymptotic relative Chow-stability. In particular, by taking T to be trivial, we see that asymptotic Chow-stability follows from strong K-stability.

1. Introduction

In this paper, we consider a *polarized algebraic manifold* (X, L), i.e., a nonsingular irreducible projective variety X, defined over $\mathbb{C}$, with a very ample line bundle L on X. Let T be an algebraic torus in $\mathrm{Aut}(X)$. Then the main purpose of this paper is to show the following:

Main Theorem. *If (X, L) is strongly K-stable relative to T, then (X, L) is asymptotically Chow-stable relative to T.*

2. Relative Chow-stability

For the maximal compact subgroup T_c of T, we put $\mathfrak{t}_c := \mathrm{Lie}(T_c)$. For every positive integer ℓ, we consider the space $V_\ell := H^0(X, L^{\otimes \ell})$ endowed with a Hermitian metric ρ_ℓ such that the infinitesimal action of $\mathfrak{t}_c$ on V_ℓ preserves the metric ρ_ℓ. Put $\mathfrak{t} := \mathrm{Lie}(T)$ and $n := \dim X$. Since the infinitesimal action of $\mathfrak{t}$ on X lifts to an infinitesimal action of $\mathfrak{t}$ on L, we view $\mathfrak{t}$ as a Lie subalgebra, denoted by $\mathfrak{t}_\ell$, of $\mathfrak{sl}(V_\ell)$ by taking the traceless part. Let $(\mathfrak{t}_\ell)_{\mathbb{Z}}$ be the kernel of the exponential map

$$\mathfrak{t}_\ell \ni y \mapsto \exp(2\pi\sqrt{-1}\, y) \in \mathrm{SL}(V_\ell).$$

Let $\mathfrak{z}_\ell$ denote the centralizer of $\mathfrak{t}_\ell$ in $\mathfrak{sl}(V_\ell)$, and we consider a symmetric bilinear form $\langle\ ,\ \rangle_\ell$ on $\mathfrak{sl}(V_\ell)$ defined by

$$\langle u, v\rangle_\ell \;=\; \mathrm{Tr}(uv)/\ell^{n+2}, \qquad u, v \in \mathfrak{sl}(V_\ell),$$

2010 *Mathematics Subject Classification.* Primary 32Q26; Secondary 14L24, 53C25.
*Supported by JSPS Grant-in-Aid for Scientific Research (A) No. 20244005.
**Supported by JSPS Grant-in-Aid for Young Scientiists (B) No. 23740063.

whose asymptotic limit as $\ell \to \infty$ plays an important role (cf. [9]) in the study of relative K-stability for test configurations. We now consider the set $\mathfrak{t}_\ell^\perp$ of all $u \in \mathfrak{z}_\ell$ such that

$$\langle u, v\rangle_\ell = 0 \qquad \text{for all } v \in \mathfrak{t}.$$

By the infinitesimal $\mathfrak{t}_\ell$-action on V_ℓ, we can write the vector space V_ℓ as a direct sum of $\mathfrak{t}_\ell$-eigenspaces

$$V_\ell \; = \; \bigoplus_{k=1}^{m_\ell} V(\chi_{\ell;k}),$$

for mutually distinct additive characters $\chi_{\ell;k} \in \mathrm{Hom}((\mathfrak{t}_\ell)_{\mathbb{Z}}, \mathbb{Z})$, $k = 1, 2, \dots, m_\ell$, where $V(\chi_{\ell;k})$ denotes the space of all $\sigma \in V_\ell$ such that

$$u\,\sigma \; = \; \chi_{\ell;k}(u)\,\sigma \qquad \text{for all } u \in (\mathfrak{t}_\ell)_{\mathbb{Z}}.$$

Since T_c acts isometrically on (V_ℓ, ρ_ℓ), the subspaces $V(\chi_{\ell;k})$ and $V(\chi_{\ell;k'})$ are orthogonal if $k \neq k'$. For the Lie subalgebra $\mathfrak{s}_\ell$ of $\mathfrak{sl}(V_\ell)$ defined by

$$\mathfrak{s}_\ell \; = \; \bigoplus_{k=1}^{m_\ell} \mathfrak{sl}(V(\chi_{\ell;k})),$$

we consider the associated algebraic subgroup $S_\ell := \Pi_{k=1}^{m_\ell} \mathrm{SL}(V(\chi_{\ell;k}))$ of $\mathrm{SL}(V_\ell)$. Let $Z(S_\ell)$ be the centralizer of S_ℓ in $\mathrm{SL}(V_\ell)$. Then the Lie algebra $\mathfrak{z}_\ell$ is written as a direct sum of Lie subalgebras

$$\mathfrak{z}_\ell \; = \; \mathfrak{z}(\mathfrak{s}_\ell) \oplus \mathfrak{s}_\ell$$

where $\mathfrak{z}(\mathfrak{s}_\ell) := \mathrm{Lie}(Z(S_\ell))$. For the Lie subalgebra $\mathfrak{t}'_\ell := \mathfrak{t}_\ell^\perp \cap \mathfrak{z}(\mathfrak{s}_\ell)$ of $\mathfrak{z}(\mathfrak{s}_\ell)$, we consider the associated algebraic subtorus T'_ℓ of $Z(S_\ell)$. Then

$$T_\ell^\perp \; := \; T'_\ell \cdot S_\ell$$

is a reductive algebraic subgroup of $\mathrm{SL}(V_\ell)$ with the Lie algebra $\mathfrak{t}_\ell^\perp$. Let $(\mathfrak{t}_\ell^\perp)_{\mathbb{Z}}$ denote the set of all $u \in (\mathfrak{t}_\ell^\perp)_{\mathbb{Z}}$ in the kernel of the exponential map

$$\mathfrak{z}_\ell \ni u \mapsto \exp(2\pi\sqrt{-1}u) \in \mathrm{SL}(V_\ell)$$

such that the circle group $\{\,\exp(2\pi s\sqrt{-1}u)\,;\, s \in \mathbb{R}\,\}$ acts isometrically on (V_ℓ, ρ_ℓ). For each $u \in (\mathfrak{t}_\ell^\perp)_{\mathbb{Z}}$, by varying $s \in \mathbb{C}$, let

$$\psi_u : \mathbb{G}_m \to \mathrm{SL}(V_\ell), \qquad \exp(2\pi s\sqrt{-1}) \mapsto \exp(2\pi s\sqrt{-1}u),$$

be the algebraic one-parameter group generated by u, where $\mathbb{G}_m$ denotes the 1-dimensional algebraic torus $\mathbb{C}^*$. Let X_ℓ be the image of X under the Kodaira embedding

$$\Phi_\ell \; : \; X \to \mathbb{P}^*(V_\ell)$$

associated to the complete linear system $|L^{\otimes \ell}|$ on X. For the degree d_ℓ of X_ℓ in $\mathbb{P}^*(V_\ell)$, we put $W_\ell^* := \{\mathrm{Sym}^{d_\ell}(V_\ell^*)\}^{\otimes n+1}$. Let $\hat{X}_\ell \in W_\ell^*$ be the Chow form for the irreducible reduced algebraic cycle X_ℓ on $\mathbb{P}^*(V_\ell)$, so that the associated point $[\hat{X}_\ell]$ in $\mathbb{P}^*(W_\ell)$ is the Chow point for X_ℓ. Then the action of $T_\ell^\perp$ on V_ℓ induces an action of $T_\ell^\perp$ on W_ℓ^* and also on $\mathbb{P}^*(W_\ell)$.

Definition 2.1. (1) $(X, L^{\otimes \ell})$ is called *Chow-stable relative to* T, if the following conditions are satisfied:
(a) The isotropy subgroup of $T_\ell^\perp$ at $[\hat{X}_\ell]$ is finite;
(b) The orbit $T_\ell^\perp \cdot \hat{X}_\ell$ in W_ℓ^* is closed.

(2) (X, L) is called *asymptotically Chow-stable relative to* T, if there exists a positive integer ℓ_0 such that $(X, L^{\otimes \ell})$ are Chow-stable relative to T for all positive integers ℓ satisfying $\ell \geq \ell_0$.

3. Test configurations

Let $u \in (\mathfrak{t}_\ell^\perp)_{\mathbb{Z}}$. For the complex affine line $\mathbb{A}^1 := \{z \in \mathbb{C}\}$, we consider the algebraic subvariety $\mathcal{X}^u$ of $\mathbb{A}^1 \times \mathbb{P}^*(V_\ell)$ obtained as the closure of

$$\bigcup_{t \in \mathbb{C}^*} \{t\} \times \psi_u(t) X_\ell$$

in $\mathbb{A}^1 \times \mathbb{P}^*(V_\ell)$, where $\mathrm{SL}(V_\ell)$ acts naturally on the set $\mathbb{P}^*(V_\ell)$ of all hyperplanes in V_ℓ passing through the origin. We now put $\mathcal{L}^u := \mathrm{pr}_2^* \mathcal{O}_{\mathbb{P}^*(V_\ell)}(1)$, where $\mathrm{pr}_2 : \mathcal{X}^u \to \mathbb{P}^*(V_\ell)$ is the restriction to $\mathcal{X}^u$ of the projection to the second factor: $\mathbb{A}^1 \times \mathbb{P}^*(V_\ell) \to \mathbb{P}^*(V_\ell)$. The triple

$$\mu \;=\; (\mathcal{X}^u, \mathcal{L}^u, \psi_u),$$

is called a *test configuration for* (X, L) *generated by* u, where we call ℓ the *exponent* of the test configuration μ. If $u = 0$, then μ is called *trivial.*

For μ as above, taking the fiber $\mathcal{X}_0^u$ of $\mathcal{X}^u$ over the origin in $\mathbb{A}^1$, we consider the Chow weight $q_\ell(u)$ for $\mathcal{X}_0^u$ sitting in $\{0\} \times \mathbb{P}^*(V_\ell)$ ($\cong \mathbb{P}^*(V_\ell)$), i.e., the weight at $\hat{\mathcal{X}}_0^u$ of the $\mathbb{G}_m$-action induced by ψ_u, where $\hat{\mathcal{X}}_0^u \in W_\ell^*$ denotes the Chow form for $\mathcal{X}_0^u$ viewed as an algebraic cycle on $\mathbb{P}^*(V_\ell)$.

Definition 3.1. (1) $(X, L^{\otimes \ell})$ is called *weakly Chow-stable relative to* T, if $q_\ell(u) < 0$ for all $0 \neq u \in (\mathfrak{t}_\ell^\perp)_{\mathbb{Z}}$.

(2) (X, L) is called *asymptotically weakly Chow-stable relative to* T, if there exists a positive integer ℓ_0 such that $(X, L^{\otimes \ell})$ is weakly Chow-stable relative to T for all positive integers ℓ satisfying $\ell \geq \ell_0$.

Remark 3.2. (1) If $(X, L^{\otimes \ell})$ is weakly Chow-stable relative to T, then by [2], Theorem 3.2, the orbit $T_\ell^\perp \cdot \hat{X}_\ell$ is closed in W_ℓ^*.

(2) If $(X, L^{\otimes \ell})$ is Chow-stable relative to T, then by the Hilbert-Mumford stability criterion, $(X, L^{\otimes \ell})$ is weakly Chow-stable relative to T.

4. Strong relative K-stability

For materials in this section, see [5] and [6]. For a maximal algebraic torus $\bar{T}$ in $\mathrm{Aut}(X)$ containing T, we fix a Hermitian metric h for L such that $\omega := c_1(L; h)$ is a Kähler form preserved by the action of the maximal compact subgroup $\bar{T}_c$ of $\bar{T}$. Then each V_ℓ admits a Hermitian structure ρ_ℓ preserved by the T_c-action such that

$$\langle \sigma, \tau \rangle_{\rho_\ell} := \int_X (\sigma, \tau)_h \, \omega^n, \qquad \sigma, \tau \in V_\ell,$$

where $(\sigma, \tau)_h$ is the pointwise Hermitian pairing of σ and τ on X by the Hermitian metric $h^{\otimes \ell}$. In this section, following [5], we explain how we define the Donaldson-Futaki invariant F_1 for a sequence of test configurations

$$\mu_j = (\mathcal{X}^{u_j}, \mathcal{L}^{u_j}, \psi_{u_j}), \qquad j = 1, 2, \dots,$$

generated by $u_j \in (\mathfrak{t}_{\ell_j}^\perp)_{\mathbb{Z}}$, where the positive integer ℓ_j, called the exponent of μ_j, is required to satisfy $\ell_j \to +\infty$ as $j \to \infty$. Let $\mathcal{M}$ be the set of all such sequences $\{\mu_j\}$. For the image X_{ℓ_j} of X under the Kodaira embedding

$$\Phi_{\ell_j} : X \to \mathbb{P}^*(V_{\ell_j}),$$

we consider its associated Chow form $\hat{X}_{\ell_j} \in W_{\ell_j}^* := \{\mathrm{Sym}^{d_{\ell_j}}(V_{\ell_j}^*)\}^{\otimes n+1}$. Let $b_{j,\alpha}$, $\alpha = 1, 2, \dots, N_{\ell_j}$, be the weights of the $\mathbb{G}_m$-action on V_{ℓ_j} induced by ψ_{u_j}. We then define the norms $\|\mu_j\|_1$ and $\|\mu_j\|_\infty$ by

$$\begin{cases} \|\mu_j\|_1 := \Sigma_{\alpha=1}^{N_{\ell_j}} |b_{j,\alpha}|/\ell_j^{n+1}, \\ \|\mu_j\|_\infty := \max\{ |b_{j,1}|, |b_{j,2}|, \dots, |b_{j,N_{\ell_j}}| \}/\ell_j. \end{cases}$$

Let $\delta(\mu_j)$ denote $\|\mu_j\|_\infty / \|\mu_j\|_1$ or 1 according as $\|\mu_j\|_\infty \neq 0$ or $\|\mu_j\|_\infty = 0$. If $\|\mu_j\|_\infty \neq 0$, we write $t \in \mathbb{R}_+$ as $t = \exp(s/\|\mu_j\|_\infty)$ for some $s \in \mathbb{R}$, while we require no relations between s and t if $\|\mu_j\|_\infty$ vanishes. Since $\mathrm{SL}(V_{\ell_j})$ acts naturally on $W_{\ell_j}^*$, we define a function $f_{u_j} = f_{u_j}(s)$ in s on $\mathbb{R}$ by

$$(4.1) \qquad f_{u_j}(s) := \delta(\mu_j) \ell_j^{-n} \log \|\psi_{u_j}(t) \cdot \hat{X}_{\ell_j}\|_{\mathrm{CH}(\rho_{\ell_j})}, \qquad s \in \mathbb{R},$$

where $W^*_{\ell_j} \ni w \mapsto \|w\|_{\mathrm{CH}(\rho_{\ell_j})} \in \mathbb{R}_{\geq 0}$ is the Chow norm for $W^*_{\ell_j}$ (see [10]). Taking the derivative $\dot{f}_{u_j}(s) := df_{u_j}/ds$, we define $F_1(\{\mu_j\}) \in \mathbb{R} \cup \{-\infty\}$ by

$$F_1(\{\mu_j\}) \ := \ \lim_{s\to-\infty} \{ \varliminf_{j\to\infty} \dot{f}_{u_j}(s) \}.$$

Definition 4.2 (cf. [6]). (1) (X, L) is called *strongly K-semistable relative to* T, if $F_1(\{\mu_j\}) \leq 0$ for all $\{\mu_j\} \in \mathcal{M}$.

(2) Let (X, L) be strongly K-semistable relative to T. Then (X, L) is called *strongly K-stable relative to* T, if for every $\{\mu_j\} \in \mathcal{M}$ satisfying $F_1(\{\mu_j\}) = 0$, there exists a j_0 such that for all $j \geq j_0$, μ_j is trivial, i.e., $u_j = 0$.

Note that neither strong K-semistability relative to T nor strong K-stability relative to T depends on the choice of $\bar{T}$ and h (see [8]).

5. Proof of Main Theorem

In this section, we consider a polarized algebraic manifold (X, L) which is strongly K-stable relative to T. The proof is divided into two parts.

Step 1. We shall first show that (X, L) is asymptotically weakly Chow-stable relative to T. Assume the contrary for contradiction. Then we can find an increasing sequence of positive integer ℓ_j, $j = 1, 2, \ldots$, such that

$$\ell_j \to +\infty, \qquad \text{as } j \to \infty,$$

and that $(X, L^{\otimes \ell_j})$ is not weakly Chow-stable relative to T for any j. Then by Definition 3.1, to each j, we can assign a element $0 \neq u_j \in (\mathfrak{t}^{\perp}_{\ell_j})_{\mathbb{Z}}$ such that $q_{\ell_j}(u_j) \geq 0$. Recall that (see for instance [4], Appendix I)

$$q_{\ell_j}(u_j) \ = \ \|\mu_j\|_1 \, \ell_j^n \lim_{s\to-\infty} \dot{f}_{u_j}(s).$$

Since the function $\dot{f}_{u_j}(s)$ is non-decreasing in s for each j, it follows that

$$0 \ \leq \ \|\mu_j\|_1^{-1} \, \ell_j^{-n} q_{\ell_j}(u_j) \ \leq \ \dot{f}_{u_j}(s), \qquad -\infty < s < +\infty.$$

Hence $0 \leq \dot{f}_{u_j}(s)$ for each fixed $s \in \mathbb{R}$. Taking $\varliminf$ as $j \to \infty$, we have

$$0 \ \leq \ \varliminf_{j\to\infty} \dot{f}_{u_j}(s), \tag{5.1}$$

for every $s \in \mathbb{R}$. By taking limit of (5.1) as $s \to -\infty$, we obtain

$$0 \ \leq \ \lim_{s\to-\infty} \varliminf_{j\to\infty} \dot{f}_{u_j}(s) \ = \ F_1(\{\mu_j\}).$$

Since (X, L) is strongly K-stable relative to T, this inequality implies that $F_1(\{\mu_j\})$ vanishes. Again by strong K-stability of (X, L) relative to T, there exists a j_0 such that μ_j are trivial for all j with $j \geq j_0$ in contradiction to $u_j \neq 0$, as required.

Step 2. In view of (1) of Remark 3.2, we see from Step 1 above that the orbit $O_\ell := T_\ell^\perp \cdot \hat{X}_\ell$ is closed in W_ℓ^*. Hence O_ℓ is an affine algebraic subset of W_ℓ^*. Since O_ℓ is closed in W_ℓ^*, we here observe that:

$$O_\ell \cap \mathbb{C}\hat{X}_\ell \text{ is a finite set,} \tag{5.2}$$

where $\mathbb{C}\hat{X}_\ell$ is the one-dimensional vector subspace of W_ℓ^* generated by $\hat{X}_\ell$. Consider the identity components H_ℓ and H'_ℓ of the isotropy subgroups of the reductive algebraic group $T_\ell^\perp$ at the point $\hat{X}_\ell$ and $[\hat{X}_\ell]$, respectively. Since $\dim H_\ell = \dim H'_\ell$ by (5.2), it suffices to show that an ℓ_0 exists such that $\dim H_\ell = 0$ for all ℓ with $\ell \geq \ell_0$. Assume the contrary for contradiction. Then we have an increasing sequence of positive integers ℓ_j such that

$$\dim H_{\ell_j} \; > \; 0, \qquad j = 1, 2, \dots,$$

and that $\ell_j \to +\infty$, as $j \to \infty$. Since by [7] the isotropy subgroup H_{ℓ_j} of the reductive algebraic group $T_{\ell_j}^\perp$ at the point $\hat{X}_{\ell_j}$ is a reductive algebraic group, the group H_{ℓ_j} contains a nontrivial algebraic torus $\mathbb{G}_m$. Since (5.2) allows us to obtain a natural isogeny $\iota : H_{\ell_j} \to \bar{H}_{\ell_j}$ from H_{ℓ_j} to an algebraic subgroup $\bar{H}_{\ell_j}$ of $\mathrm{Aut}(X)$, the image $\bar{H}_{\ell_j}$ also contains a nontrivial algebraic torus $G_j = \mathbb{G}_m$. For $\bar{T}$ in Section 4, replacing $\bar{T}$ by its conjugate in $\mathrm{Aut}(X)$ if necessary, we may assume that $\bar{T}$ contains G_j. For the maximal compact subgroup $(G_j)_c$ of G_j, we choose a generator $u_j \neq 0$ for the one-dimensional real Lie subalgebra

$$\sqrt{-1}\,(\mathfrak{g}_j)_c \; := \; \sqrt{-1}\ \mathrm{Lie}((G_j)_c)$$

in $\mathfrak{t}_{\ell_j}^\perp \cap H^0(X, \mathcal{O}(T_X))$ such that $\exp(2\pi\sqrt{-1}u_j) = \mathrm{id}_X$. Then for the algebraic group homomorphisms $\psi_{u_j} : \mathbb{G}_m \to T_\ell^\perp \subset \mathrm{SL}(V_{\ell_j})$ generated by u_j, we obtain the associated test configurations

$$\mu_{u_j} \; = \; (\mathcal{X}^{u_j}, \mathcal{L}^{u_j}, \psi_{u_j}), \qquad j = 1, 2, \dots,$$

for (X, L) generated by u_j. Let β_j be the weight of the $\mathbb{G}_m$-action by ψ_{u_j} at $\hat{X}_{\ell_j}$. Since $\psi_{u_j}(t) \cdot \hat{X}_{\ell_j} = t^{\beta_j} \hat{X}_{\ell_j}$, by differentiating the functions $f_{u_j}(s)$ in (4.1) with respect to s, we obtain

$$\dot{f}_{u_j}(s) \; = \; \ell_j^{-n} \beta_j / \|\mu_{u_j}\|_1, \qquad -\infty < s < +\infty. \tag{5.3}$$

Replacing u_j by $v_j := -u_j$, we also have the test configurations

$$\mu_{v_j} \; = \; (\mathcal{X}^{v_j}, \mathcal{L}^{v_j}, \psi_{v_j}), \qquad j = 1, 2, \dots,$$

for (X, L) generated by v_j. Replace u_j by v_j in (4.1). Then by differentiating the functions $f_{v_j}(s)$ with respect to s, we obtain

$$(5.4) \qquad \dot{f}_{v_j}(s) \; = \; -\ell_j^{-n}\beta_j/\|\mu_{v_j}\|_1, \qquad -\infty < s < +\infty.$$

Note that $\|\mu_{u_j}\|_1 = \|\mu_{v_j}\|_1$. The right-hand side of (5.3) and the right-hand side of (5.4) are both bounded from above by a positive constant independent of j (see [5], Section 3). Hence, replacing $\{u_j\,;\, j = 1, 2, \dots\}$ by its subsequence if necessary, we may assume that

$$\{\, \ell_j^{-n}\beta_j/\|\mu_{u_j}\|_1 \,;\, j = 1, 2, \dots, \}$$

is a convergent sequence. Let γ be its limit. Then by (5.3) and (5.4), $F_1(\{\mu_{u_j}\}) = \gamma = -F_1(\{\mu_{v_j}\})$. Since (X, L) is strongly K-stable relative to T, the inequalities $F_1(\{\mu_{u_j}\}) \leq 0$ and $F_1(\{\mu_{v_j}\}) \leq 0$ hold, and hence

$$\gamma = 0.$$

Again by strong K-stability of (X, L) relative to T, we see that μ_{u_j} are trivial for $j \gg 1$, so that $u_j = 0$ for $j \gg 1$ in contradiction, as required.

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