Graduate Student Seminar

Summer School in Mathematics, Institut Fourier, Grenoble

June 16 – July 4, 2008

Duration of talks: 30 minutes

Principal organizer: Evgeny Smirnov (Universität Bonn)

Gwyn Bellamy

Representation theory of rational Cherednik algebras at $t = 0$

I will give an overview of the representation theory of the rational Cherednik algebras at t = 0 and show how much of this theory is governed by certain "cuspidal" representation, that I shall introduce.

Lesya Bodnarchuk

Simple vector bundles on degenerations of elliptic curves

In my talk I discuss the problem of classification of simple coherent sheaves on certain degenerations of elliptic curves. Indecomposable vector bundles on smooth elliptic curves were classified in 1957 by Atiyah. An approach to study coherent sheaves on singular projective curves was suggested in works of Burban, Drozd and Greuel. In particular, it was shown that the category of coherent sheaves on a nodal cubic curve and on cycles of projective lines is tame, and all other degenerations of elliptic curves are wild. However, it turns out that all plane cubic curves are brick-tame. As a main technical tool to prove this result we use representation theory of boxes (differential biquivers), worked out in 70s by the Kiev representation school to formalize and generalize the matrix problems.

Martin Doubek

Diagram cohomology via operads

I will give elementary introduction to operads with emphasis on minimal models. I will show how these give rise to A_∞ (and other) algebras, canonical cohomology theories. Finally I discuss colored operad as a way to deal with diagrams (of algebras).

Stanislav Fedotov

Affine algebraic groups with periodic components

A connected component of an affine algebraic group is called periodic if all its elements have finite order. We give a characterization of periodic components in terms of automorphisms with finite number of fixed points. It is also discussed which connected groups have finite extensions with periodic components. The results are applied to the study of the normalizer of a maximal torus in a simple algebraic group.

Sachin Gautam

Cluster algebras and Grassmannians of type G_2

I will introduce cluster algebras as a convenient tool for studying dual semi-canonical basis and discuss a conjecture of Christof Geiß, Bernard Leclerc and Jan Schröer which gives a cluster algebra structure to coordinate ring of partial flag varieties. I will also give a proof of this conjecture in the case G_2.

Brian Jurgelewicz

McKay correspondence for curves of genus $g > 2$

Let X be a smooth projective non-hyperelliptic curve of genus at least 3. Let G be the automorphism group of X. Let T^* be the cotangent bundle of X. Then T^* is a quasi-projective variety on which G acts symplectically. One may ask for a full strong exceptional collection for the derived category of G-equivariant coherent sheaves on T^*. We will discuss what progress has been made towards this and related questions.

Oskar Kedzierski

Resolutions of $1/r(1, a, r - a)$ singularity and quiver representations

Two natural resolutions of cyclic terminal quotient singularity, G-Hilbert scheme and Danilov resolution, can be described in terms of θ-stable McKay quiver representations for suitable stability parameter θ. I will show how to compute such parameter for the Danilov resolution and discuss some McKay correspodence type results holding in this case.

Apoorva Khare

Decomposing representations of wreath products of semisimple Lie algebras

Let R be the wreath product of $U(\mathfrak{g})$ with S_n, for some n and some complex semisimple Lie algebra $\mathfrak{g}$. We first classify all finite-dimensional R-modules. Next, we compute the center of R, and classify all central characters on these modules (and others). The common theme here is an analogue of the BGG Category $\mathcal{O}$ for R (and a far larger class of smash product algebras); we prove that this is a highest weight category, with block decomposition and (a modified form of) BGG Reciprocity.

Ryan Kinser

Rank functors and representation rings of quivers

I will discuss the recently constructed functor which takes a representation of an arbitrary (finite) quiver Q, and returns a representation of Q for which the maps over all arrows are isomorphisms. The common dimension of the resulting vector spaces at each vertex is a numerical invariant of the representation. Combining these functors with pulling back representations along well chosen maps of directed graphs allows one to construct other numerical invariants of representations. These include, as the simplest cases, the dimension vector of a representation, the ranks of any composition of maps, dimensions of intersections of images, and so forth. We call the functors giving rise to these invariants "rank functors", although in general they measure something more complicated than the rank of any one map.

There is a natural tensor product on representations of Q, which allows one to construct a representation ring $R(Q)$ à la Grothendieck. The rank functors above commute with direct sum and tensor product of representations (addition and multiplication in $R(Q)$), and fix the identity, hence induce ring homomorphisms from $R(Q)$ to the integers, called rank functions. In more recent work, when Q is a tree quiver with a unique sink, I use combinatorial methods to construct all rank functions on Q and show that ring $R(Q)$, modulo its ideal of nilpotents, is finitely generated as an abelian group.

The basics of quivers covered in Brion's lectures during the first week will assumed; any other necessary concepts will be presented during the talk.

Roland Olbricht

Nori's construction and algebraic geography

The finite dimensional associative algebras with unit of dimension d can be parametrized by their structure constants into a variety Alg_d. Unfortunately, this variety has quite a lot of singularities as well as complicated combinatorics. It has been investigated up to dimension 7. We will present another new tool which enables us to spot a large singularity inside the irreducible component of 3×3-matrices inside Alg_9.

Jaimal Thind

Coxeter elements and periodic Auslander–Reiten quiver

Traditionally, to study a root system R one starts by choosing a set of simple roots $\Pi \subset R$ (or equivalently, polarization of the root system into positive and negative parts) which is then used in all constructions and proofs. We discuss a different approach, starting not with a set of simple roots but with a choice of a Coxeter element C in the Weyl group. We show that for a simply-laced root system a choice of C gives rise to a natural construction of the Dynkin diagram, in which vertices of the diagram correspond to C-orbits in R; moreover, it gives an identification of R with a certain subset $\hat{I}$ of $I \times \mathbb{Z}_{2h}$, where h is the Coxeter number. The set $\hat{I}$ has a natural quiver structure; we call it the periodic Auslander-Reiten quiver. This gives a combinatorial construction of the root system associated with the Dynkin diagram I: roots are vertices of $\hat{I}$, and the root lattice and the inner product admit an explicit description in terms of $\hat{I}$. (This is joint work with A. Kirillov Jr)

Fábio Xavier Penna

Foliations of codimension one and degree zero in $\mathbb{CP}^n$

The study of the irreducible components of the space of 1-codimensional holomorphic foliations in $\mathbb{CP}^n$, with a fixed degree, is a central problem in complex dynamics. I will discuss the simple case of degree zero. We will see that we have only one irreducible component, the Grassmannian of lines of $\mathbb{P}(\mathbb{C}^{n+1*})$.