WINTER BRAIDS III
School on braids and low-dimensional topology

Institut Fourier – Université de Grenoble
17–20 December, 2012

Organizing committee:
P. Bellingeri (Univ. Caen), V. Florens (Univ. Pau), J.B. Meilhan (Univ. Grenoble)

- Program -

	Monday 17th	Tuesday 18th	Wednesday 19th	Thursday 20th
8h30-9h00	Registration			
9h00-10h00	Bodin I	Wagner II	Meilhan II	Habiro III
Coffee break				
10h30-11h30	Wagner I	Bodin II	Wagner III	Meilhan III
11h30-12h30	Habiro I	Maldonado / Guerville	Queffelec / Panagiotou	Guaschi III
Lunch				
14h15-15h15	Meilhan I	Guaschi I	Bodin III	Cisneros / Korinman
15h15-16h15	Suzuki / Buzunarriz	Habiro II	Guaschi II	
Coffee break				
16h45-17h45	Goundaroulis / Bouschbacher	Caruso / Aguilera	Moussard / Pereiro	

- Abstracts -

Mini courses

Arnaud Bodin (Univ. Lille)

Curves, knots, singularities and polynomials

The topic of these lectures is the topology of the family of curves given by the equations {$P(x,y)=c$} where $P(x,y)$ is a polynomial in two variables. We will present several aspects of the subject: differential geometry, algebraic geometry with special regards to topology.
Lecture I - Curves and polynomials maps
Lecture II - Knots and links
Lecture III - Deformations of singularities

John Guaschi (LMNO, Univ. Caen)

Surface Braid Groups

Surface braid groups occur naturally as generalisations of the Artin braid groups and of the fundamental group.
In this mini-course, we shall start by comparing different definitions of surface braid groups. The topological definition, in terms of configuration spaces, brings into play the short exact sequences of pure braid groups (due to Fadell and Neuwirth) that play a fundamental rôle in the theory. This allows us to prove basic results about surface braid groups, compute their centre, and show that they are torsion free for most surfaces.
We shall also discuss the relationship between braid and mapping class groups of surfaces, and embeddings of braid groups of different surfaces. The braid groups of the sphere and the projective plane are particularly interesting, not least because they possess elements of finite order and are linear. We shall discuss the structure of these braid groups in more detail, and we will classify the isomorphism classes of their finite subgroups.

Kazuo Habiro (RIMS, Univ. Kyoto)

Quantum Fundamental Groups

The fundamental group of a topological space is one of the most fundamental invariants in algebraic topology and it plays a vital role in 3-dimensional topology as well.
First I recall basic properties of fundamental groups such as functoriality and the Van Kampen theorem. Then I describe the definition of the "quantum fundamental group" of a 3-manifold M, which is defined by using the isotopy classes of bottom tangles in M or alternatively the isotopy classes of embedding of handlebodies into M.
Although this is not a group in the usual sense, many properties of fundamental groups can be translated into the setting of quantum fundamental groups. I plan to explain the "Van Kampen theorem" for quantum fundamental groups, and, if time permits, "representation varieties" derived from quantum fundamental groups.

JB Meilhan (IF, Univ. Grenoble)

An introduction to Milnor invariants

The purpose of these lectures is to give an overview of a family of link invariants introduced by Milnor in the 50's, and which are in some sense a far-reaching generalization of the linking number.
We will first closely follow Milnor's work to define these invariants, prove a few important properties, and work out a few examples. Then we will attempt to give a better understanding of these Milnor invariants by presenting several more recent results.
The lectures will be mostly self-contained, and require no particular expertise in knot theory.

Emmanuel Wagner (Univ. Bourgogne)

Link invariants, braid groups and categorification

We give an introduction to the categorification process in low-dimensional topology.
We will focus mainly on two examples : a categorification of the braid group introduced by Rouquier, and a categorification of the Burau representation of the braid group introduced by Khovanov-Seidel.
We will develop the first example a little bit further and following Khovanov see how one can derive from it a categorification of the HOMFLY-PT polynomial invariant of links.

Short talks

Marta Aguilera (Universidad de Sevilla)
Garside tools for pseudo-Anosov braids

It has been shown that, in a conjugacy class of reducible braids, the simplest braids from the Garside structure point of view, have round or almost round reduction curves (Gonzalez-Meneses and Wiest, 2011). For the pseudo-Anosov case, similar properties seem to hold on the train tracks of those Garside-simplest braids of a conjugacy class.
In this talk, we will discuss these properties in the case of B_3. Given the normal form of a Garside-simplest braid, we can give a train-track for it, as well as the associated matrix and therefore the dilatation factor. We will also see a bijection between the set of Garside-simplest braids and the monoid $SL_2(N)$ (see Handel, 1997), that can be extended to a map onto the set of all posible dilatation factors. We will finish commenting other properties that arise for higher number of strand."

Fabien Bouschbacher (Université de Strasbourg)
Shear coordinates on the super-Teichmüller space

Thurston introduced a global parametrization on the Teichmüller space of Riemann surfaces with punctures, called shear coordinates, given by the datum of a positive number on each edge of an ideal triangulation of the surface. We will first recall the definition of super-riemann surface and of super-Teichmüller space; then we will construct the super analogs of Thurston's shear coordinates on these spaces.

Sandrine Caruso (Université Rennes 1 – IRMAR)
Pseudo-Anosov braids with a large super summit set

Garside theory provides useful tools for solving the conjugacy problem in braids groups, but no polynomial time algorithm is known yet. Among the invariants of conjugacy classes that have been explored for the purpose of finding fast algorithms, one of the best known is the so-called super summit set. Unfortunately, there exist some families of braids whose size of the super summit set is exponential in the canonical length. An example with reducible braids was given by Gonzalez-Meneses in 2010. In this talk we present an example of a family of pseudo-Anosov braids with such a large super summit set.

Bruno Cisneros de la Cruz (Institut de Mathématiques de Bourgogne)
Virtual braids: from algebra to topology

The aim of my talk is to explain, in a first glance, how and why virtual braids are defined and then how we can construct a topological representantion of this combinatorial objects.

Dimos Goundaroulis (National Technical University of Athens)
A Markov Trace on the Yokonuma-Temperley-Lieb Algebra

In this talk we will introduce the Yokonuma-Temperley-Lieb algebra as a quotient of the Yokonuma-Hecke algebra. We will discuss certain properties of this algebra such as dimension and canonical basis. Further, we will present necessary and sufficient conditions for the Markov trace defined on the Yokonuma-Hecke algebras by J. Juyumaya to pass through to the quotient algebras.

Benoît Guerville (UPPA)
On the boundary manifold of a complex line arrangement

Let A be a line arrangement in the projective plane CP^2. The topology of A, determined by the pair (CP^2, A) is the topological type of the complement. Rybnikov used the fundamental group to show that the combinatorics do not determine the topology. In this talk, we consider the boundary manifold, defined as the boundary of a close regular neighborhood of A in CP^2 and study the inclusion map on the complement. Moreover, we explain how to compute the map induced by this inclusion on the fundamental groups.
Joint work with V.Florens and M. Marco-Buzunariz.

Julien Korinman (Institut Fourier)
TQFT and mapping class group

I wish to present shortly the concept of a TQFT and its applications for studying the MCG of surfaces. I won't focus the talk on my result but rather give a short overview of the recent overall progress.

Miguel Maldonado (Universidad Autonoma de Zacatecas)
On labeled configuration spaces and the homology of some braid groups

We consider the labeled configuration space *C(M;X)* to express the homology of unordered configuration spaces of a surface M in terms on the homology of Artin braid groups. This work is inspired by a homotopical approximation to the punctured mapping class groups of non-orientable surfaces.

Miguel Angel Marco-Buzunariz (Universidad Complutense de Madrid)
Braid groups in Sage

We present a framework for working with the braid group in the computar algebra system Sage. It is a part of a bigger project that aims to include the posibility of working with finitely presented groups in general.

Delphine Moussard (Institut Fourier)
Rational Blanchfield forms and null LP-surgeries

A QSK-pair is a pair *(M,K)* made of a rational homology sphere *M* and a null-homologous knot *K* in M. A null Lagrangian-preserving surgery on such a pair *(M,K)* is a replacement of a null-homologous rational homology handlebody in *M\K* by another such handlebody with identical Lagrangian. We will see that a null Lagrangian-preserving surgery induces a canonical isomorphism beetween the rational Alexander modules endowed with their Blanchfield forms of the involved QSK-pairs, and that, conversely, such an isomorphism can be realised by a sequence of null Lagrangian-preserving surgeries up to multiplication by a power of t.

Eleni Panagiotou (Isaac Institute for Math. Sciences, Cambridge)
Topological methods for measuring the entanglement in polymer melts

Polymer melts are dense systems of macromolecules. In such dense systems the conformational freedom and motion of a chain is significantly affected by entanglement with other chains which generates obstacles of topological origin to its movement. In this talk we will discuss methods by which one may quantify and extract entanglement information from a polymer melt configuration using tools from knot theory. A classical measure of entanglement is the Gauss linking integral which is an integer topological invariant in the case of pairs of disjoint oriented closed chains in 3-space. For pairs of open chains, we will see that the Gauss linking integral can be applied to calculate an average linking number. In order to measure the entanglement between two oriented closed or open chains in a system with three-dimensional periodic boundary conditions (PBC) we use the Gauss linking number to define the periodic linking number. For a collection of open or closed chains in 3-space or in PBC, we define the linking matrix. The matrix’ s eigenvalues provide insight into the character of the entanglement.

Carolina de Miranda e Pereiro (UFSCar – Unicaen)
The Braid Group of the Torus and the Klein Bottle.

Let M be the Torus or the Klein bottle. We have defined the n-th configuration space

$$F_n(M) = \{(x_1,...,x_n) : x_i \text{ in } M, x_i \neq x_j, i \neq j\}.$$

The pure braid group of n strings of surface M is defined as $P_n(M)=\pi_1(F_n(M))$, the braid group of n strings is $B_n(M)=\pi_1(F_n(M)/S_n)$, and we also define the mixed braid group $B_{n,m}(M)=\pi_1(F_{n,m}(M)/\{S_n \times S_m\})$. We have found presentation for these groups.
From the work of Fadell and Neuwirth, we know that the projection $p:F_{n+1}(M)\to F_n(M)$, given by $(x_1,...,x_{n+1}) \to (x_1,...,x_n)$ is locally trivial fibration with fiber $M\backslash\{x_1,...,x_n\}$. So, we have the Fadell-Neuwirth short exact sequence

$$1 \to \pi_1(M \setminus \{x_1,...,x_n\}) \to P_{n+1}(M) \to P_n(M) \to 1.$$

This sequence splits and we have found several explicit sections in terms of the generators of $P_n(M)$. We also consider the projection $q:F_{n+m}(M)/\{S_n \times S_m\} \to F_n(M)/S_n$ given by $(x_1,...,x_n, x_{n+1},...,x_{n+m}) \to ()$, then we have the short exact sequence

$$1 \to B_m(M \setminus \{x_1,...,x_n\}) \to B_{n,m}(M) \to B_n(M) \to 1.$$

We discuss when this sequence splits.

Hoel Queffelec (Univ. Paris 7 – IMJ)
Foam based knot homology via categorical skew Howe duality

One of the original motivation for categorifying quantum groups was to provide a representation theoretic explanation for the existence of Khovanov homology and other link homologies categorifying quantum link invariants. Just as the Jones polynomial is described representation theoretically by the quantum group *Uq(sl2)* and its two dimensional representation, the categorification of the Jones polynomial via Khovanov homology should be described in terms of the 2-representation theory of the categorified quantum group associated to *Uq(sl2).*
Although some ways to obtain such constructions already exist, they are generally highly technical and rarely straighforward. Our work proposes a direct construction of foam based sln-link homology theories for $n = 2$ or $n = 3$ intrinsically in terms of categorified quantum groups. The key step consists in categoryfying Cautis' work on links between Reshetikhin-Turaev invariants and skew Howe duality.
Joint work with Aaron Lauda and David Rose (University of Southern California)

Sakie Suzuki (RIMS, Kyoto Univ.)
Bing doubling and the colored Jones polynomials

The colored Jones polynomial is a quantum invariant associated with the quantized enveloping algebra of the Lie algebra sl_2.
We are interested in the relationship between topological properties of links and algebraic properties of the colored Jones polynomial. Bing doubling is an operation which gives a satellite of a knot. Habiro defined a certain series of the colored Jones polynomials to construct the unified WRT invariant of 3-manifolds.
Let *B(K)* the Bing double of a knot K. In this talk, we give an explicit formula to derive Habiro's colored Jones polynomials of *B(K)* from these of *K*. We aim to apply the result to study the unified WRT invariant.