Quantum chaos in optical microcavities

J. Wiersig

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Collaborations

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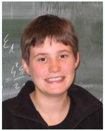
J. Main (Stuttgart)
H. Schomerus (Lancaster)

Supported by the DFG research group “Scattering Systems with Complex Dynamics”

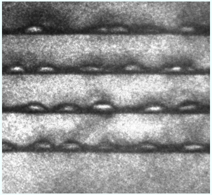
Prof. J. Wiersig

Dr. habil. G. Kasner



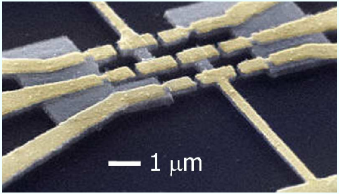
Dipl.−Phys.
J. Unterhinninghofen

Light−matter interaction in semiconductor nanostructures



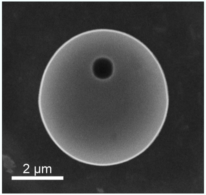
D. Hommel/A. Rosenauer et al., Bremen

Nonlinear dynamics and charge transport in nanostructures



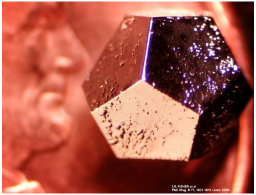
S.W. Cho and Y.D. Park, Seoul

Optical properties of microcavities



D. Heitmann/T. Kipp et al., Hamburg

Quasicrystals



I.R. Fischer et al., Iowa

Prof. J. Wiersig

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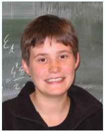

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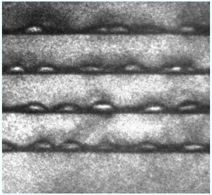

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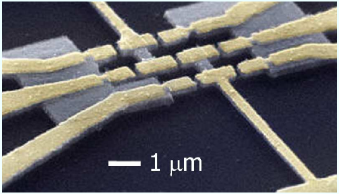

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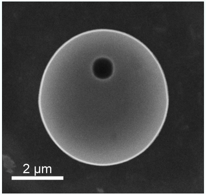

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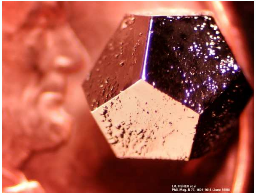
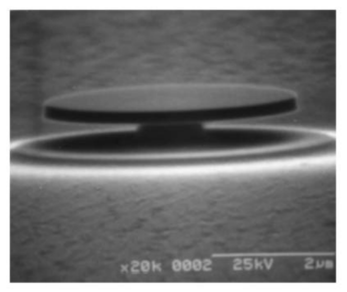


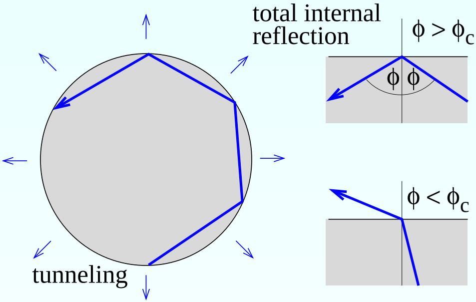
I.R. Fischer et al., Iowa

Outline

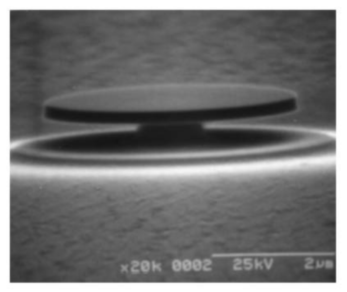
Introduction to optical microcavities



Bell labs



- Light confinement due to TIR



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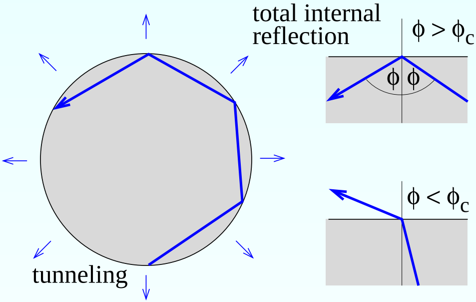
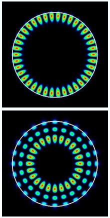
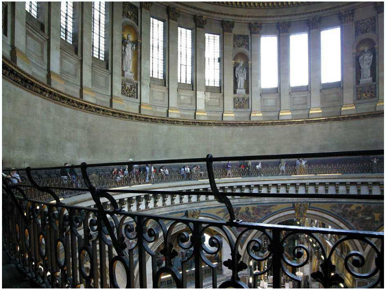


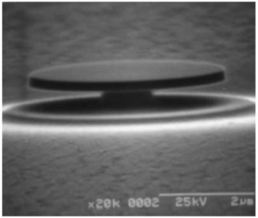
- Light confinement due to TIR
- Whispering-gallery modes
- Light emission due to tunneling

High quality factor $Q = \omega\tau$
Uniform far-field pattern

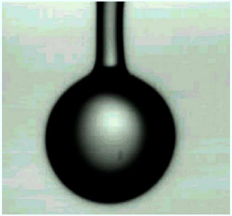




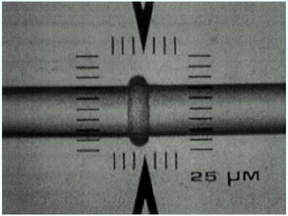
microdisk
Bell labs

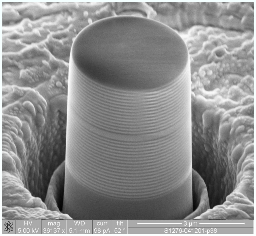


microsphere
H. Wang et al.

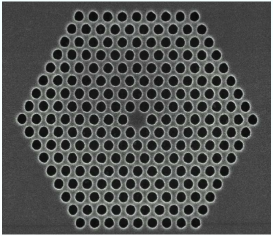


microtorus
V.S. Ilschenko et al.





VCSEL−micropillar
D. Hommel et al.



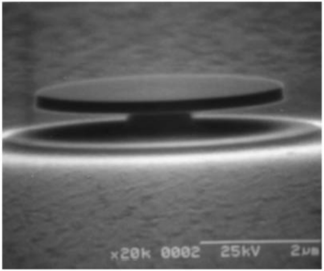
photonic crystal defect cavity
C. Reese et al.

Strong light confinement to a very small volume

- Individual optical modes
- Control over light-matter interaction
- ...

Applications

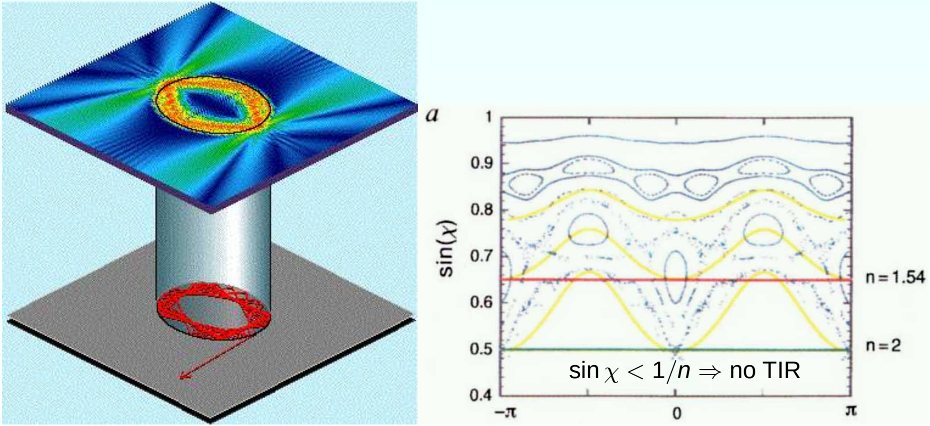
- Microlasers
- Single-photon sources
- Quantum computers
- Sensors
- Filters
- ...



Bell labs

Directed light emission from deformed disks A. Levi *et al.*, APL 62, 561 (1993)

Open quantum billiard (ray-wave correspondence)



J.U. Nöckel and A.D. Stone, Nature 385, 45 (1997)

Quantum billiard: energy eigenstate

$$\left[\nabla^2 + k^2\right] \psi(x, y) = 0$$

and $\psi(x, y) = 0$ outside, $k = \sqrt{\frac{2mE}{\hbar^2}} \in \mathbb{R}$.

Quantum billiard: energy eigenstate

$$\left[\nabla^2 + k^2\right] \psi(x,y) = 0$$

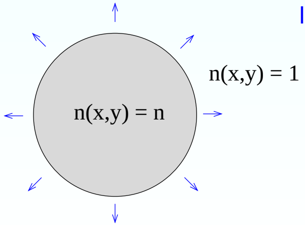
and $\psi(x,y) = 0$ outside, $k = \sqrt{\frac{2mE}{\hbar^2}} \in \mathbb{R}$.

Optical microcavity, (TM polarized) mode : $E_z = \mathrm{Re}[\psi(x,y)e^{-i\omega t}]$

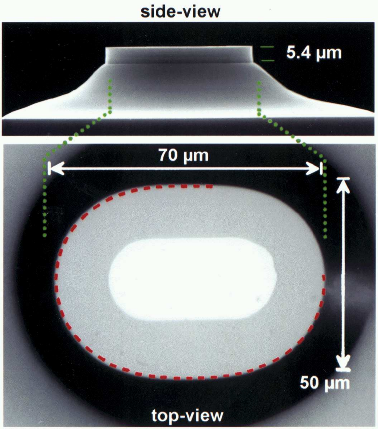
$$\left[\nabla^2 + n(x,y)^2 k^2\right] \psi(x,y) = 0$$

and continuity of ψ and $\nabla\psi$ + outgoing wave conditions, $k = \omega/c \in \mathbb{C}$.

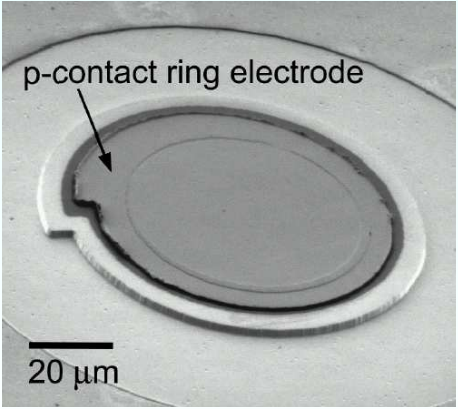
lifetime $\tau = -\frac{1}{2\mathrm{Im}(\omega)}$



A. Levi *et al.*, APL 62, 561 (1993)



C. Gmachl *et al.*, Science **280**, 1556 (1998)



M. Kneissl *et al.*, APL **84**, 2485 (2004)

Improved directionality but *Q*-factor is strongly reduced
Ultimate goal: unidirectional light emission from high-*Q* modes

Avoided resonance crossings

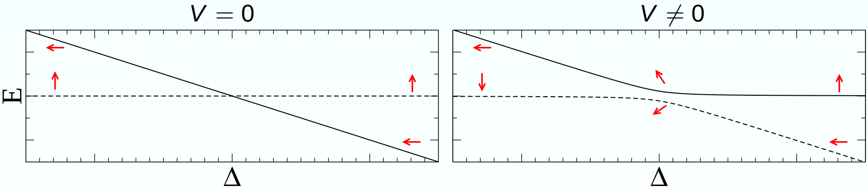
$$H = \begin{pmatrix} E_1 & V \\ W & E_2 \end{pmatrix}$$

$E_i \in \mathbb{R}$, $W = V^*$

Eigenvalues of H

$$E_\pm = \frac{E_1 + E_2}{2} \pm \sqrt{\frac{(E_1 - E_2)^2}{4} + VW}$$

Vary a parameter Δ

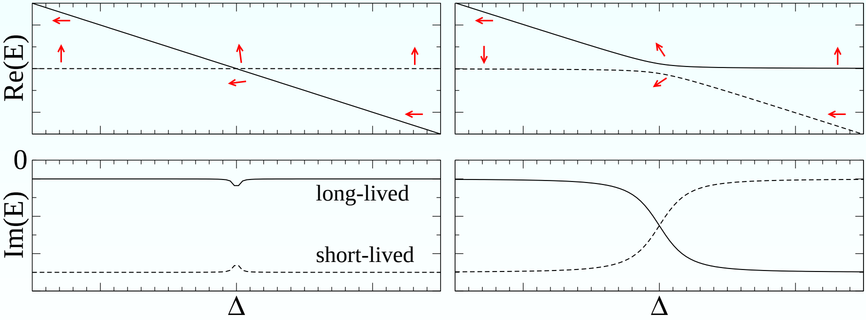


Hybridization (mixing) of eigenstates near avoided level crossing

$$E_{\pm} = \frac{E_1 + E_2}{2} \pm \sqrt{\frac{(E_1 - E_2)^2}{4} + VW}$$

$E_i \in \mathbb{C}$, $W = V^*$: internal coupling

$|V| < V_c$ (weak coupling) $\qquad$ $|V| > V_c$ (strong coupling)

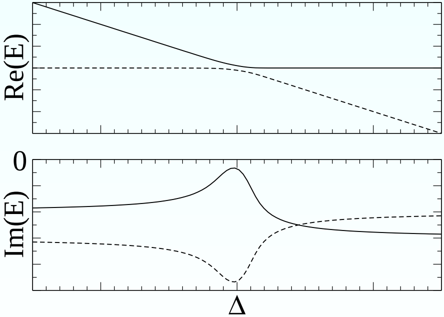


Small mixing of eigenstates

$$E_\pm = \frac{E_1 + E_2}{2} \pm \sqrt{\frac{(E_1 - E_2)^2}{4} + VW}$$

$E_i \in \mathbb{C}$, $W \neq V^*$: external coupling

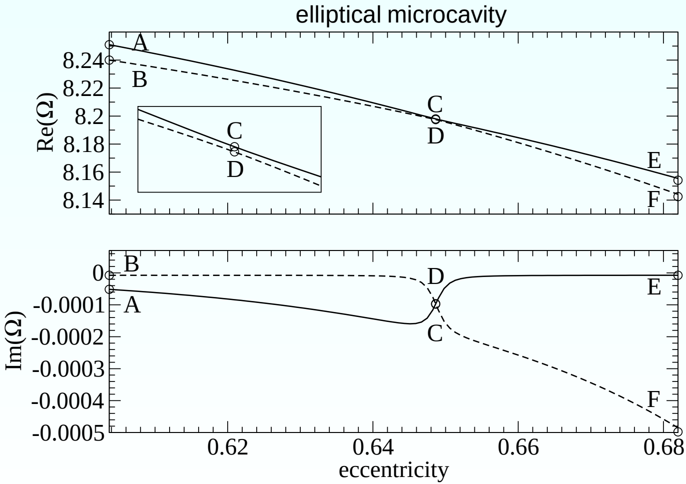
strong coupling

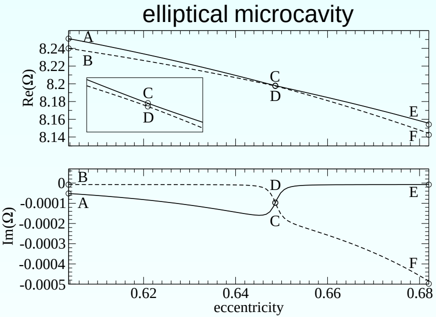


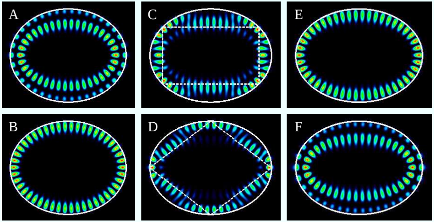
Formation of short- and long-lived modes

Boundary element method J. Wiersig, J. Opt. A: Pure Appl. Opt. **5**, 53 (2003)

Normalized frequency $\Omega = \omega R/c = kR$

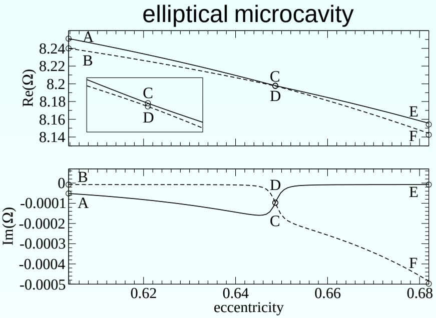


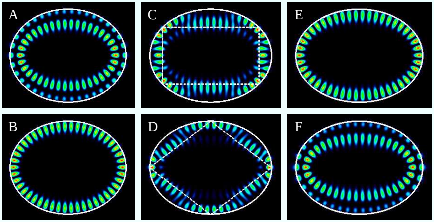


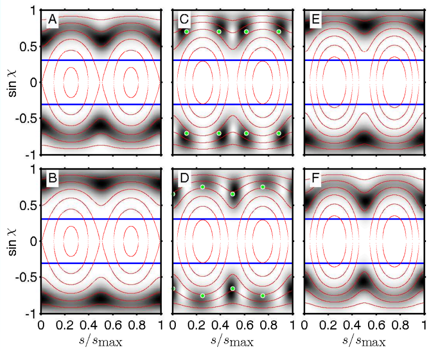


- Formation of scarlike modes

  J. Wiersig, PRL **97**, 253901 (2006)







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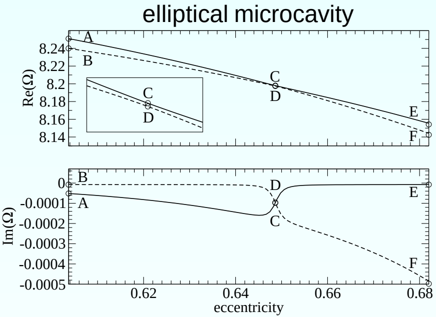
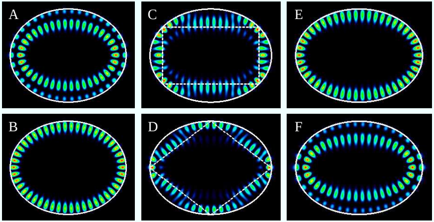
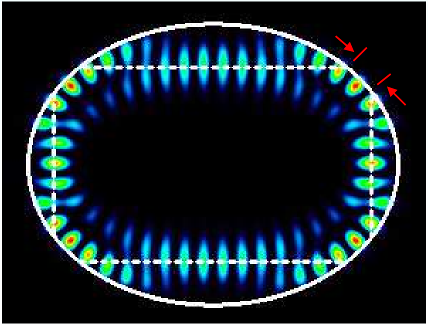






- Formation of scarlike modes

  J. Wiersig, PRL **97**, 253901 (2006)

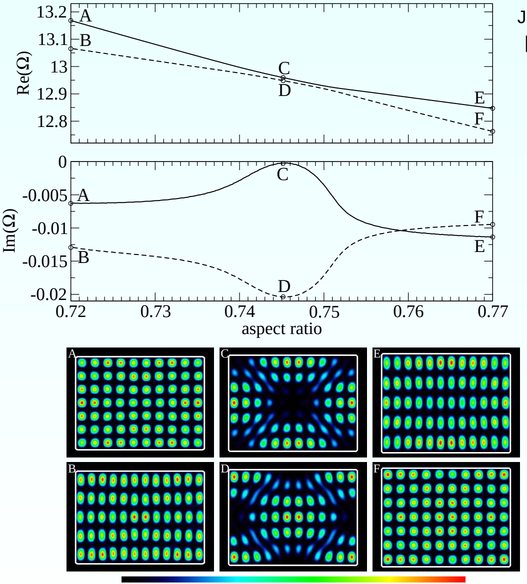
- Augmented ray dynamics including the Goos-Hänchen shift

  J. Unterhinninghofen, J. Wiersig,

  and M. Hentschel, PRE **78**, 016201 (2008)

Avoided resonance crossings

Formation of long-lived, scarlike modes



J. Wiersig, PRL **97**, 253901 (2006)

Long-lived mode *C*:

- $Q \approx 23000$ (increase by more than one order of magnitude)
- no diffraction at corners

Avoided resonance crossings

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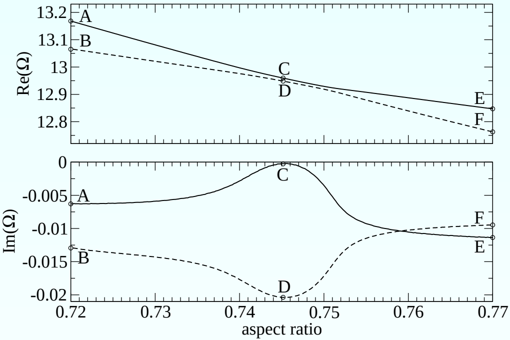
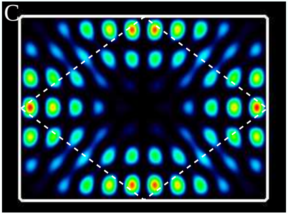




J. Wiersig, PRL **97**, 253901 (2006)

Long-lived mode *C*:

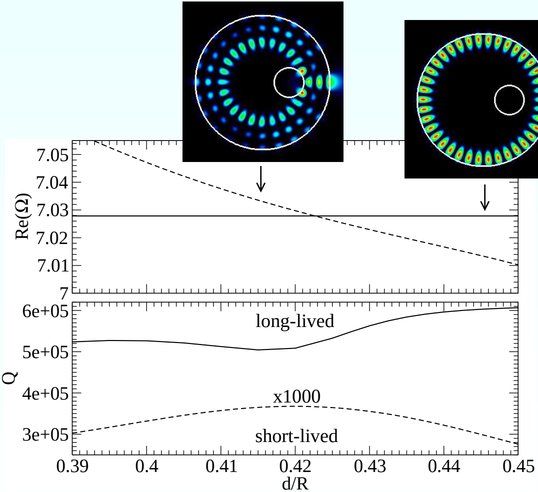
- $Q \approx 23000$ (increase by more than one order of magnitude)
- no diffraction at corners
- scarlike mode pattern

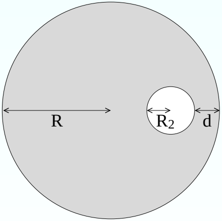
At the avoided resonance crossing a long-lived, scarlike mode is formed

Avoided resonance crossings

Unidirectional light emission from high-Q modes

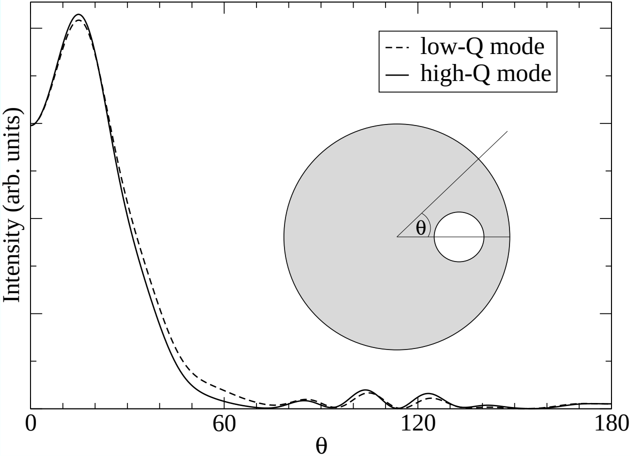
Normalized frequency $\Omega = \omega R/c = kR$, quality factor $Q = -\frac{\text{Re}(\Omega)}{2\,\text{Im}(\Omega)}$

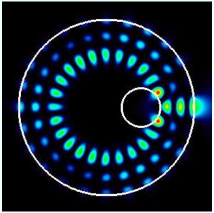


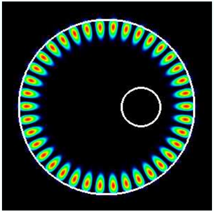


Weak internal coupling: weak hybridization of modes

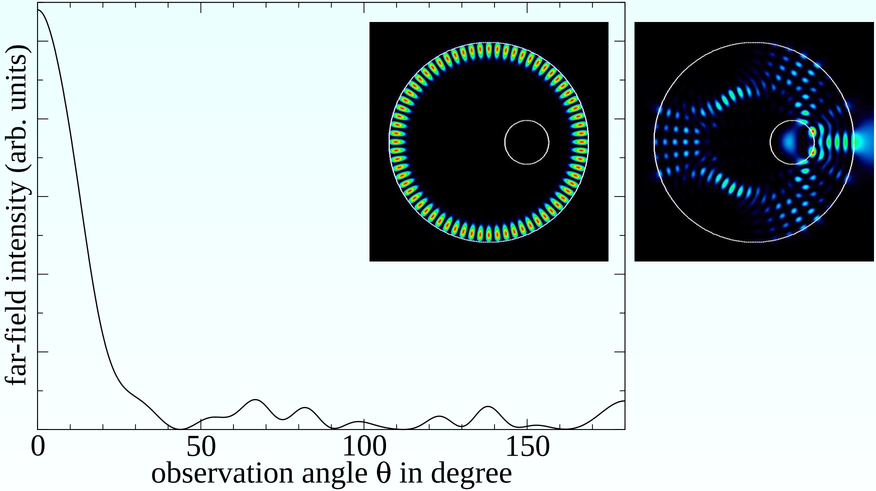
Far-field pattern is dominated by the short-lived component







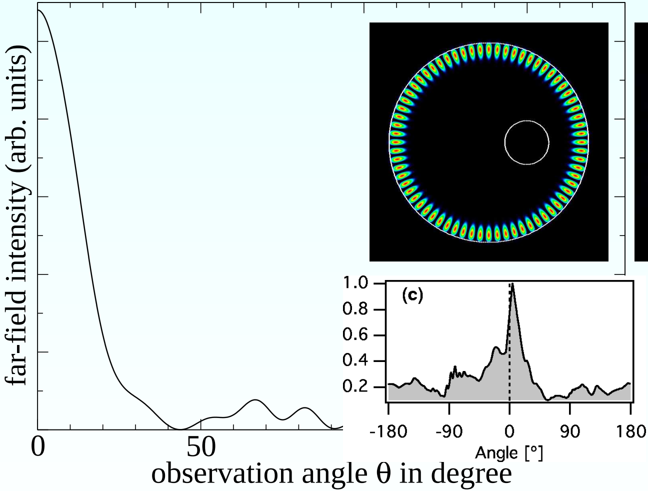
Hybridized whispering-gallery mode has $Q = 550000$ and unidirectional emission

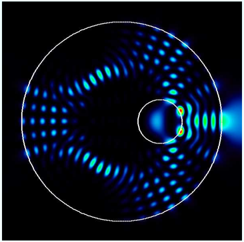


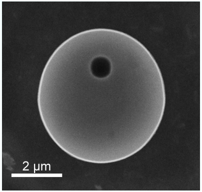
Small angular divergence and ultra-high $Q > 10^8$

Avoided resonance crossings

Unidirectional light emission from high-Q modes







F. Wilde PhD thesis (2008)
Heitmann group, Hamburg

Small angular divergence and ultra-high $Q > 10^8$

Unidirectional light emission
and
universal far-field patterns

Problem:

in the case of multimode lasing we may have modes with different directionality

Unidirectional light emission and universal far-field patterns

Problem:

in the case of multimode lasing we may have modes with different directionality

"Universal" far-field pattern due to unstable manifold

H.G.L Schwefel *et al.*, J. Opt. Soc. Am. B **21**, 923 (2004)

S.-Y. Lee *et al.*, Phys. Rev. A **72**, 061801(R) (2005)

S.-B. Lee *et al.*, Phys. Rev. A **75**, 011802(R) (2007)

S. Shinohara and T. Harayama, Phys. Rev. E **75**, 036216 (2007)

Unidirectional light emission and universal far-field patterns

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S.-B. Lee *et al.*, Phys. Rev. A **75**, 011802(R) (2007)

S. Shinohara and T. Harayama, Phys. Rev. E **75**, 036216 (2007)

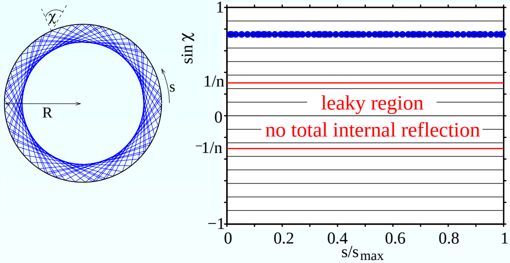
Our goal:

- unidirectional emission
- high Q-factors

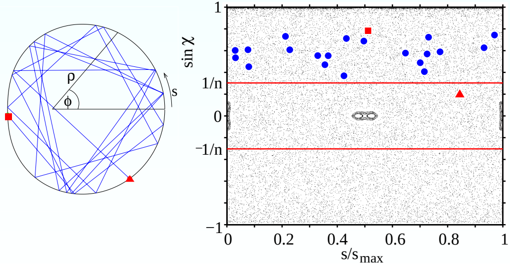
J. Wiersig and M. Hentschel, PRL **100**, 033901 (2008)

$$\rho(\phi) = R(1 + \varepsilon \cos \phi)$$

ε = 0

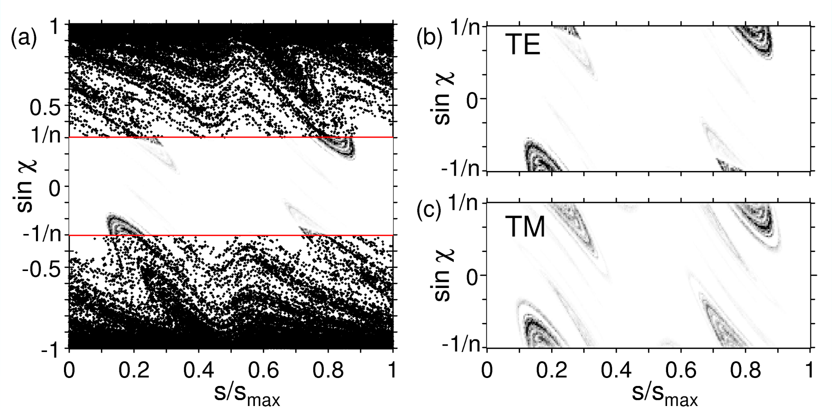


ε = 0.43

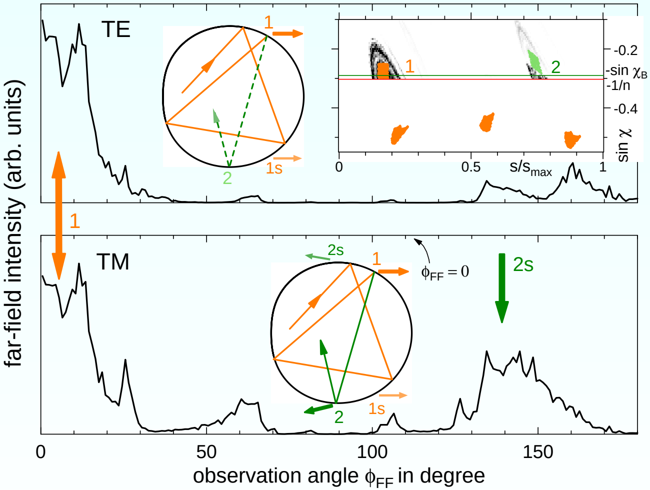


Chaotic repeller: set of points in phase space that never visits the leaky region both in forward and backward time evolution

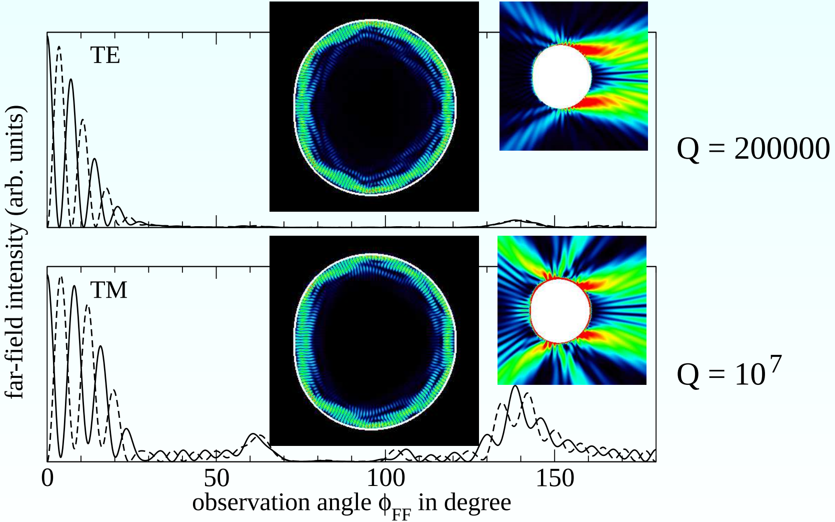
Unstable manifold: the set of points that converges to the repeller in backward time evolution (weight according to Fresnel's laws)



Subtle differences between TE and TM polarization



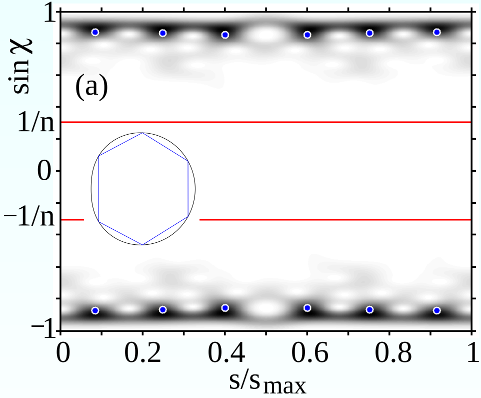
Difference between TE and TM modes is due to the Brewster angle

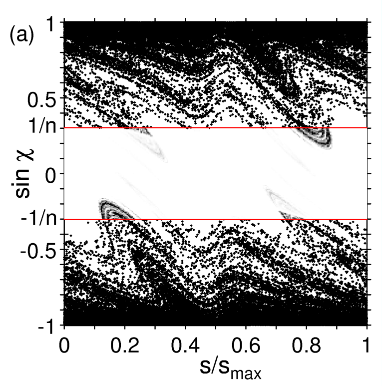


all high-*Q* TE modes show unidirectional emission (universal far-field pattern)

Unidirectional light emission and universal far-field patterns

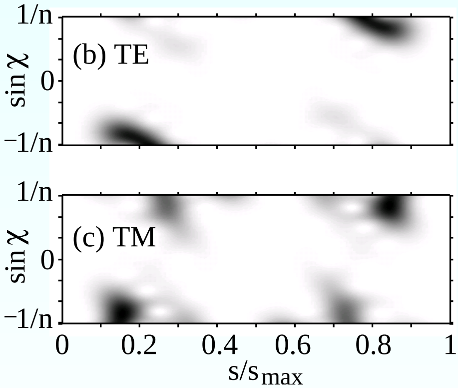
Husimi representation

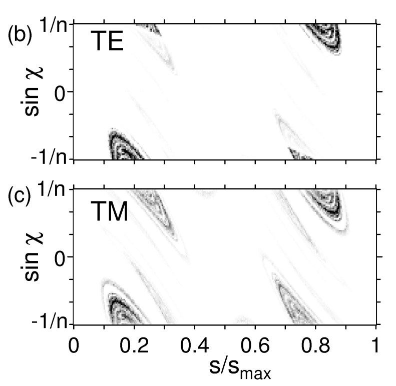




Scarring ensures high *Q*-factors

Unidirectional light emission and universal far-field patterns
Husimi magnification



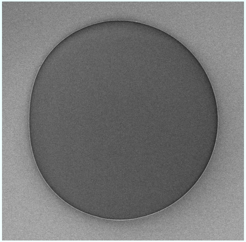


Unidirectional emission is due to the unstable manifold

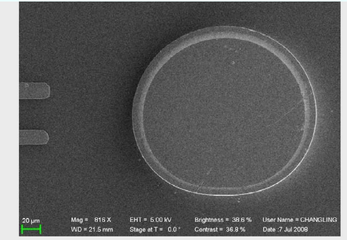
Robust:

- not sensitive to internal mode structure and cavity size
- works in a broad regime of shape parameter ε and refractive index

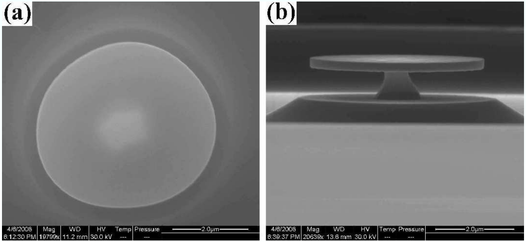
Theory: J. Wiersig and M. Hentschel, PRL **100**, 033901 (2008)



Harayama et al., Kyoto



Capasso et al., Harvard



Cao et al., Yale

Fractal Weyl law

Density of states

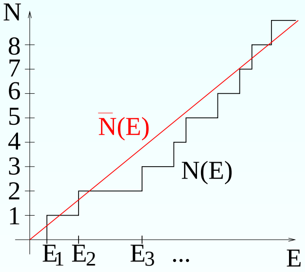
$$\rho(E) = \sum_{i=1}^{\infty} \delta(E - E_i) \qquad ; E_1 \leq E_2 \leq \dots$$

Integrated density of states

$$N(E) = \int_{-\infty}^{E} \rho(E')dE' = \#\{i|E_i \leq E\}$$

split $N(E)$ into a smooth part and a fluctuating part

$$N(E) = \bar{N}(E) + N_{\text{fluc}}(E)$$



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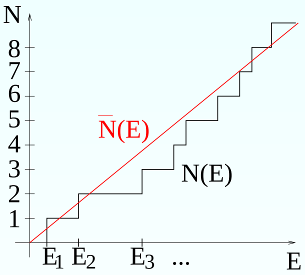
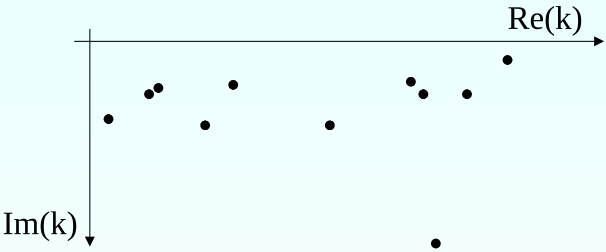
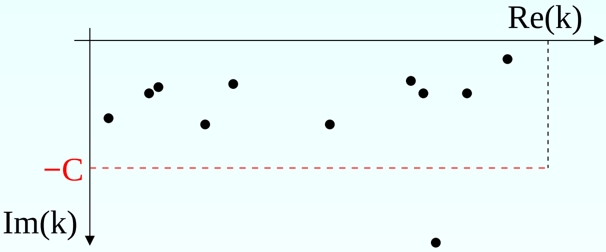


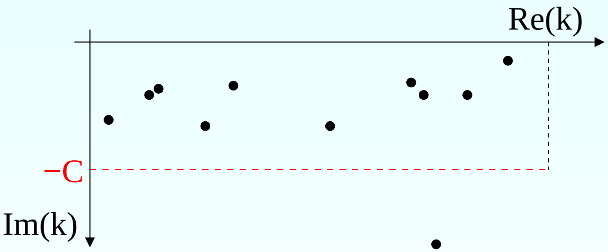
Weyl's law for 2D billiard with area A ($\hbar^2/2m = 1$)

$$\bar{N}(E) = \frac{A}{4\pi}E \sim k^2$$


Re(k)
Im(k)



$$N(k) = \{k_n : \mathrm{Im}(k_n) > -C,\ \mathrm{Re}(k_n) \le k\}.$$



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Conjecture: fractal Weyl law for open chaotic systems

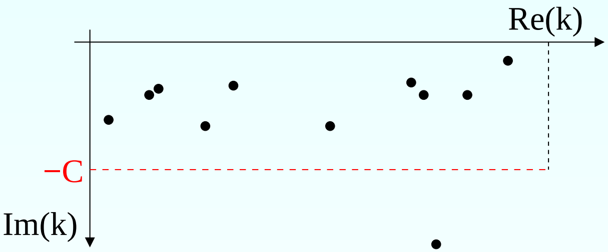
J. Sjöstrand, Duke Math. J. **60**, 1 (1990), M. Zworski, Invent. Math. **136**, 353 (1999)

$$\bar{N}(k) \sim k^{\alpha}\ .$$

with non-integer exponent

$$\alpha = \frac{D+1}{2}$$

where D is the fractal dimension of the chaotic repeller



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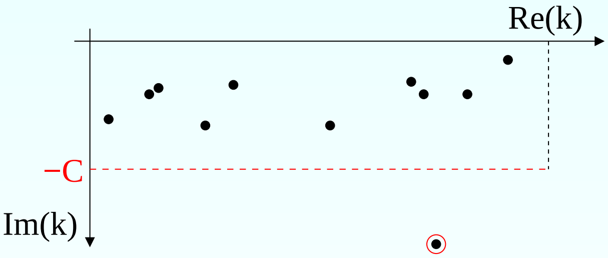

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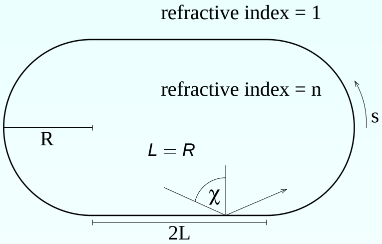
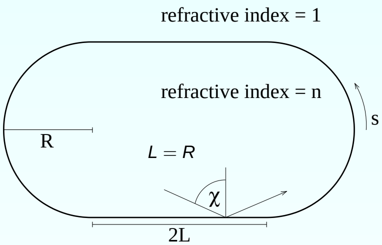
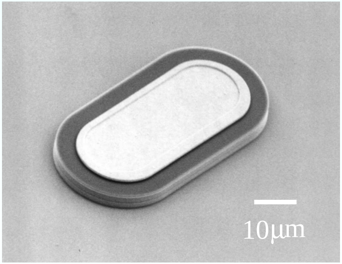
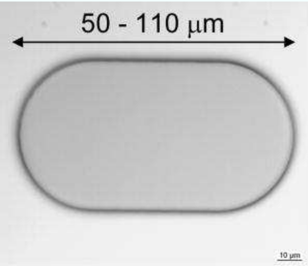

refractive index = 1
refractive index = n
s
R
$L = R$
χ
2L





$n = 3.3$ (GaAs): weakly open

T. Fukushima and T. Harayama, IEEE J. Sel. Top. Quantum Electron., **10**, 1039 (2004)



$n = 1.5$ (polymer): strongly open

M. Lebenthal *et al.*, Appl. Phys. Lett., **88**, 031108 (2006)

Boundary element method J. Wiersig, J. Opt. A: Pure Appl. Opt. **5**, 53 (2003)

Computing sufficiently many resonances in the complex plane is extremely difficult

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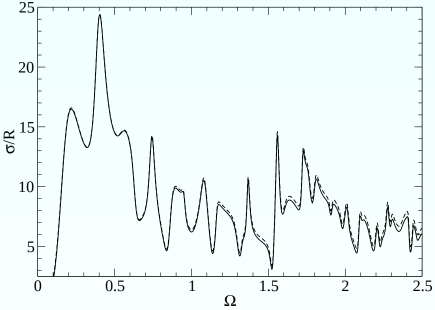

Normalized frequency $\Omega = kR$

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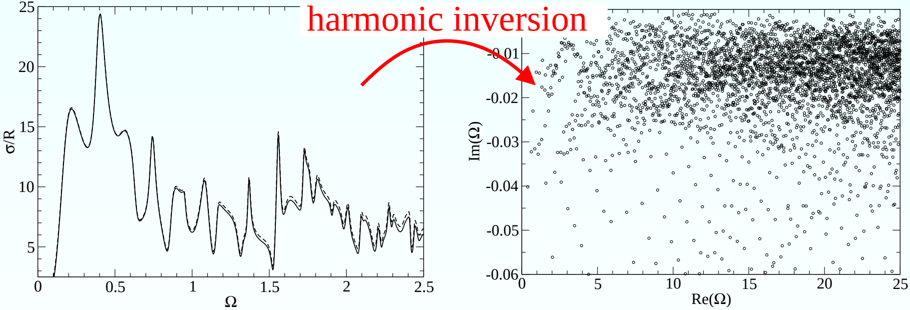
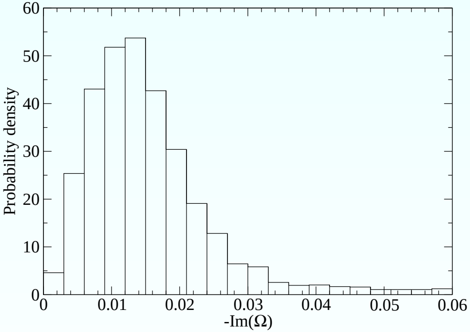
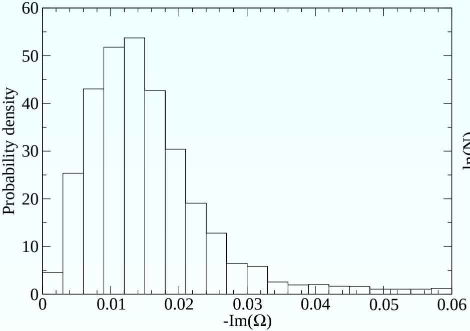


Normalized frequency $\Omega = kR$

Boundary element method + harmonic inversion → statistics of resonances



Probability density
-Im(Ω)
0
10
20
30
40
50
60
0
0.01
0.02
0.03
0.04
0.05
0.06


Probability density
60
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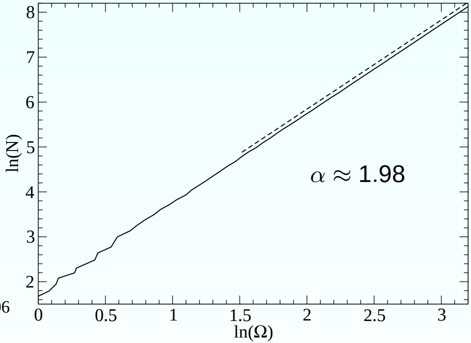
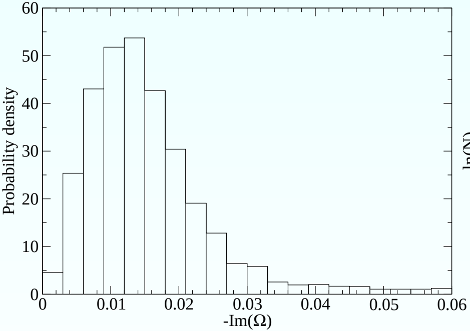
10
0
0
0.01
0.02
0.03
0.04
0.05
0.06
-Im(Ω)

cutoff $C = 0.06$


ln(N)
8
7
6
5
4
3
2
0
0.5
1
1.5
2
2.5
3
ln(Ω)
$\alpha \approx 1.98$



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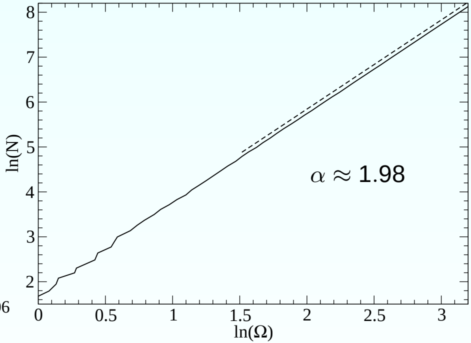
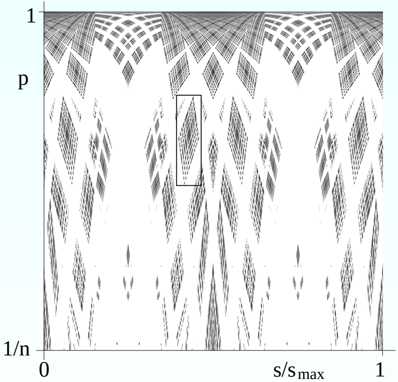


$\alpha \in [1.96, 2.02]$ for $C \in [0.03, 0.1]$

Chaotic repeller: set of points in phase space that never visits the leaky region both in forward and backward time evolution



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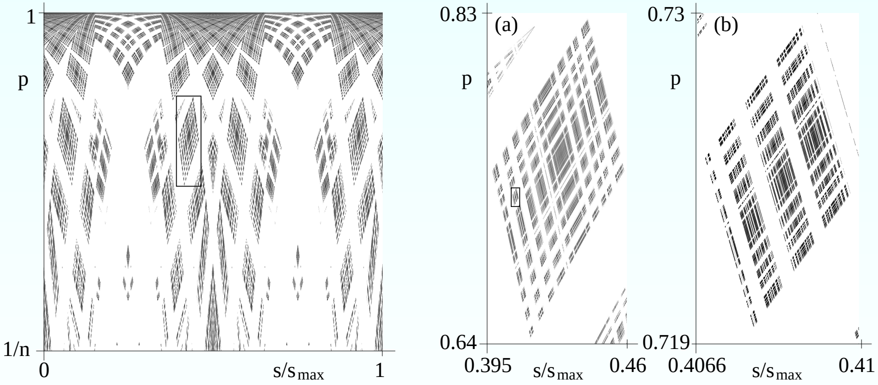

Chaotic repeller is a fractal with box-counting dimension $d \approx 1.68$

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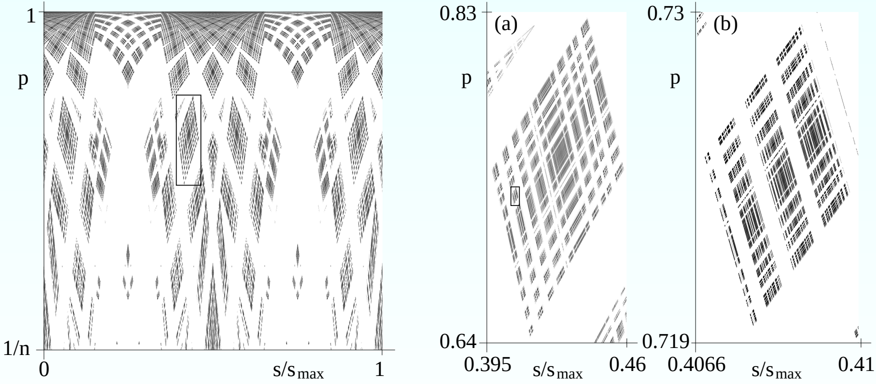
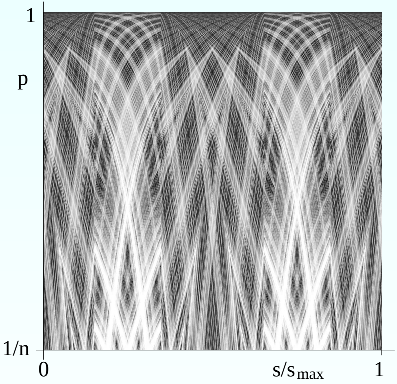


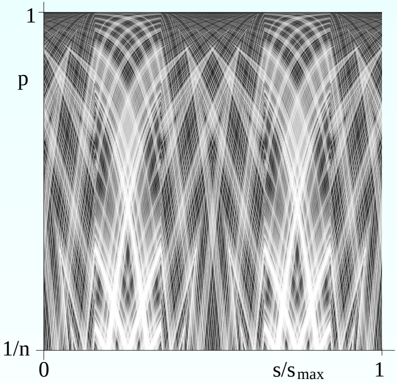
Chaotic repeller is a fractal with box-counting dimension $d \approx 1.68$

$\rightarrow \alpha = \frac{d+2}{2} = 1.84$, i.e. fractal Weyl law fails!

Account for partial escape due to Fresnel's laws $\rightarrow$ real-valued $I(s,p) \in [0,1]$



Account for partial escape due to Fresnel's laws → real-valued $I(s,p) \in [0,1]$

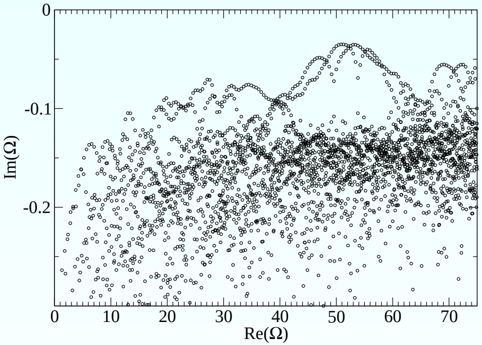


Multifractal: infinite set of fractal dimensions $d(q)$ with real q

- box counting dimension $d(0) \approx 1.986 \rightarrow \alpha \approx 1.99$
- information dimension $d(1) \approx 1.913 \rightarrow \alpha \approx 1.96$
- correlation dimension $d(2) \approx 1.877 \rightarrow \alpha \approx 1.94$

$d(0)$ is consistent with fractal Weyl law ($\alpha \in [1.96, 2.02]$)

n = 1.5 (polymer)



$\alpha \in [1.68, 1.88]$

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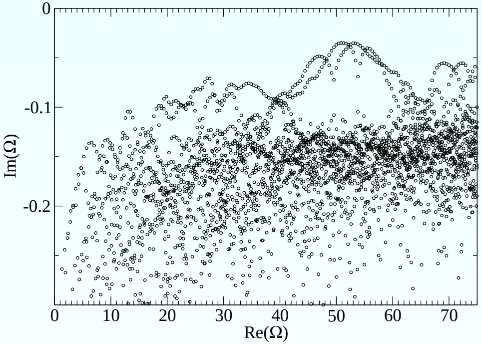

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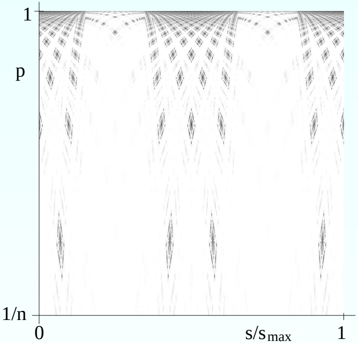

$d(0) \approx 1.512$
$d(1) \approx d(2) \approx 1.593$

The predicted exponent, 1.76 and 1.78, is consistent with the fractal Weyl law

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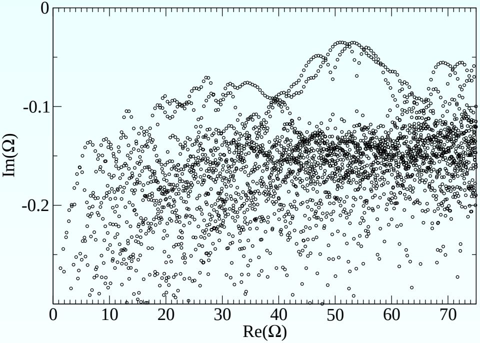

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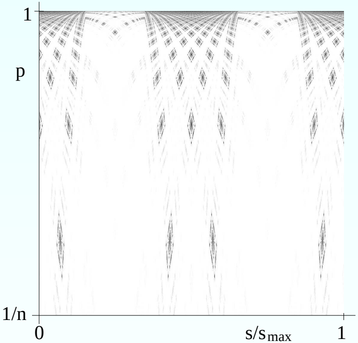

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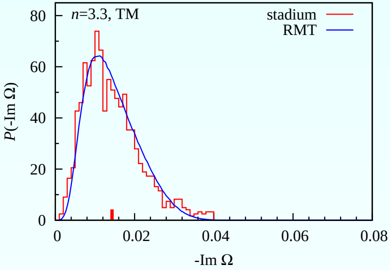
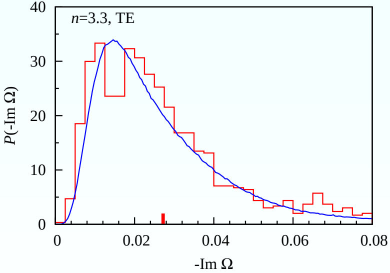
Conjecture: the fractal Weyl law applies to optical microcavities if the concept of the chaotic repeller is extended by including Fresnel's laws

J. Wiersig and J. Main, PRE **77**, 036205 (2008)

Lifetime statistics

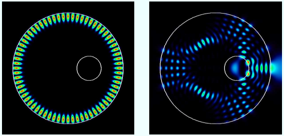




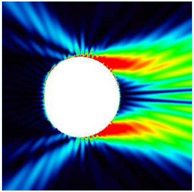
Good agreement with random-matrix theory; see talk by H. Schomerus

H. Schomerus, J. Wiersig, and J. Main, submitted (2008)

- Optical microcavities as open quantum billiards
- Avoided resonance crossings
    - Avoided crossings despite integrability
    - Formation of long-lived, scarlike modes
    - Unidirectional light emission from high-Q modes



- Unidirectional light emission and universal far-field patterns



- Fractal Weyl law