Lifetime distributions in open quantum systems: beyond ballistic chaotic decay

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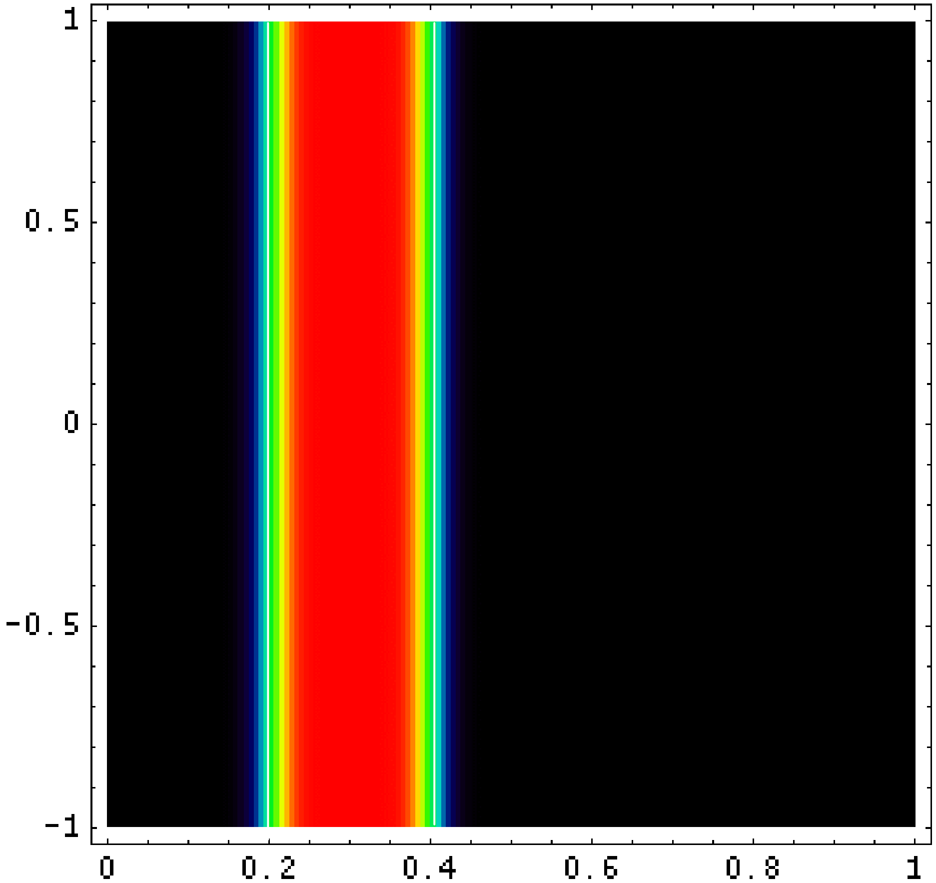
With J Tworzydło, M Kopp, J Wiersig, J Main, J Keating, M Novaes

Stroboscopic scattering theory:

round-trip operator *F*, *dim F=M= 1/h; opening operator P=(MxN) ,*

internal space: projector $Q=1-PP^T$



inject a particle:

exit: P^TFP

$P^TF(QF)P$

$P^TF(QF)^2P$

$P^TF(QF)^3P$

$P^TF(QF)^4P$...

$$\text{FT} \Rightarrow \text{S matrix}$$

$$S(\varepsilon) = P^T\left(e^{-i\varepsilon} - FQ\right)^{-1} FP$$

Resonances: $QFQ\psi = e^{-i\varepsilon}\psi; \quad e^{-i\varepsilon} \equiv \lambda; \quad \varepsilon = E - i\Gamma/2$

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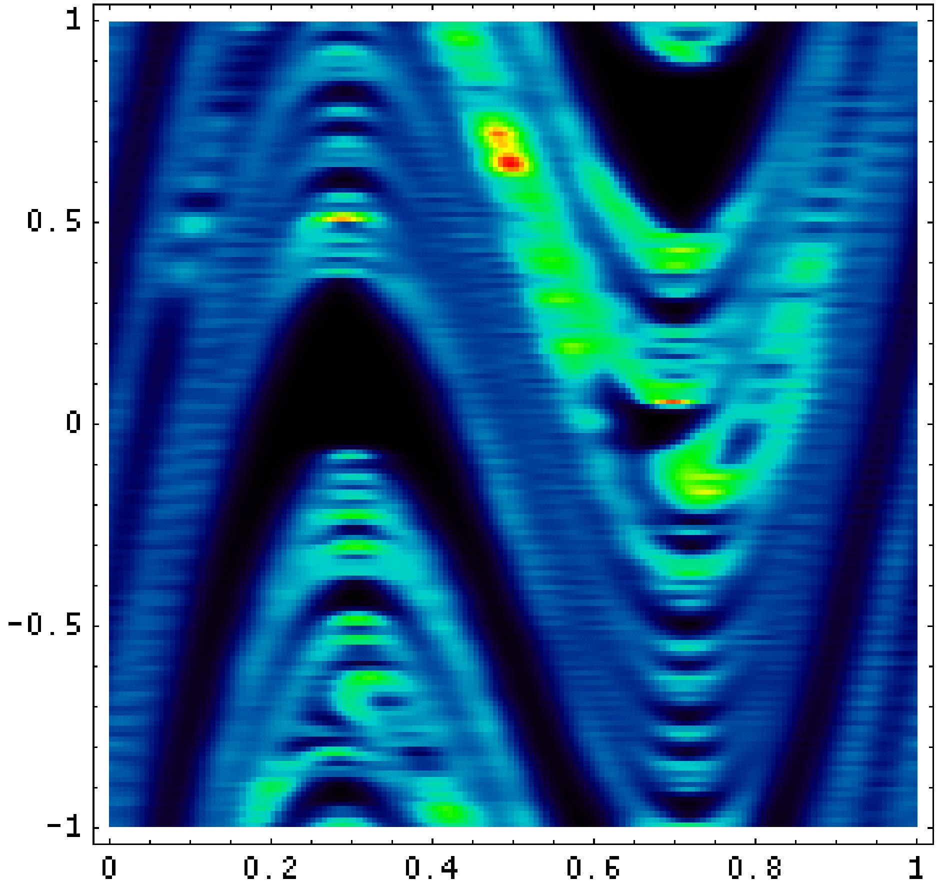

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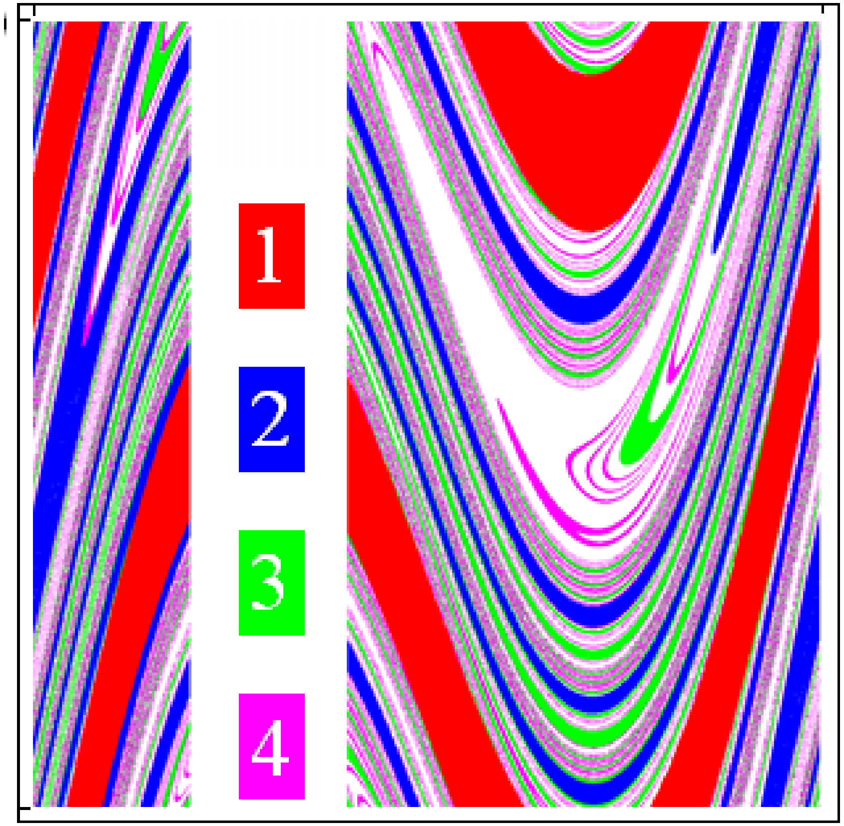
Stroboscopic scattering theory:

Qm-cl correspondence

Goal: exploit this for resonance states



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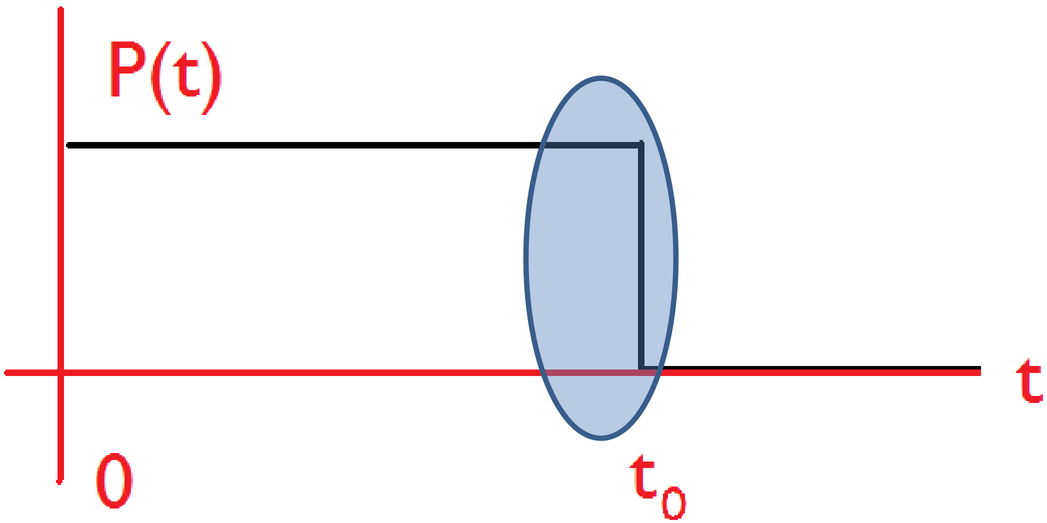
Challenge: quasi-deterministic decay



$$\lim_{\Gamma\to\infty} \frac{1}{1+e^{\Gamma(t-t_0)}}$$

- Nominally diverging decay rates: $|\lambda| = \exp(\operatorname{Im}\varepsilon) = 0$
- Resonance wave functions quasi-degenerate (defective eigensystem)

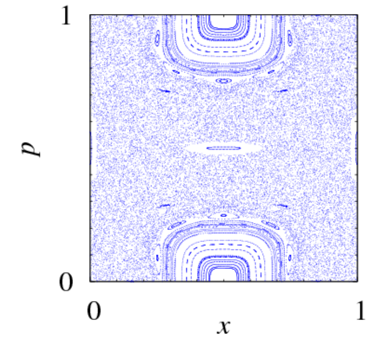
illustration: standard map/kicked rotator

(classical)

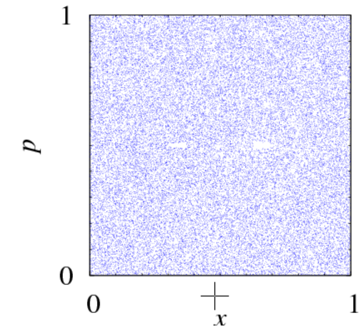
$$x_{n+1} = x_n + p_n \qquad (\text{mod}1)$$

$$p_{n+1} = p_n + \frac{K}{2\pi}\sin(2\pi x_{n+1}) \qquad (\text{mod}1)$$

K=2



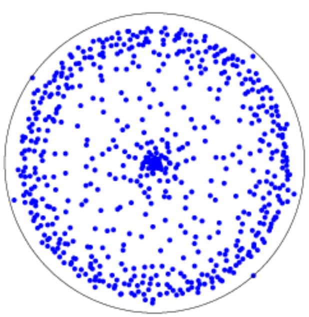
K=7.5



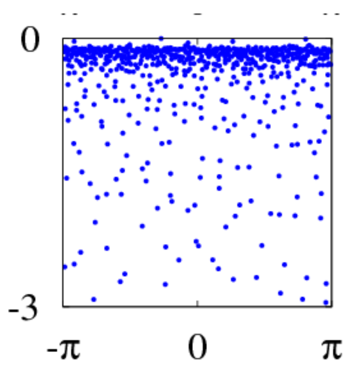
(qm)

$$F_{nm} = \frac{1}{\sqrt{iM}}\exp[\tfrac{i\pi}{M}(m-n)^2 - \tfrac{iMK}{2\pi}(\cos 2\pi \tfrac{m}{M})]$$

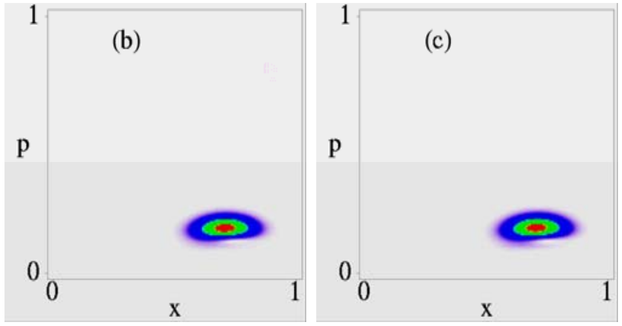
K=7.5, M=1280, N=256



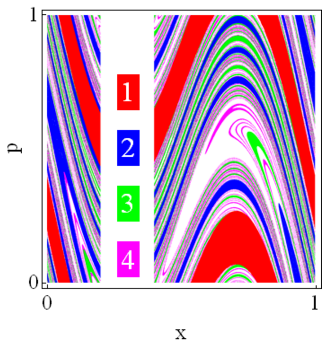
λ



ε

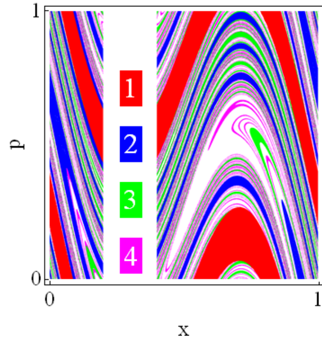


Resonances wave functions

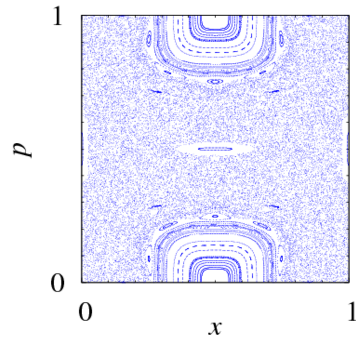


Escape zones

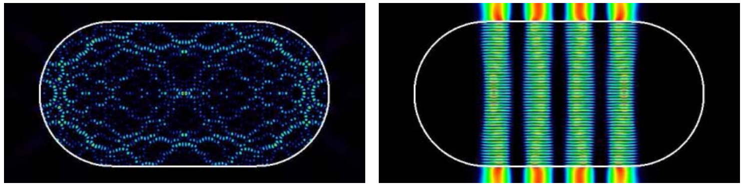
- Classically chaotic systems (with J Tworzydło): fractal Weyl law (see M Zworski)
  - Goal: reinstate phase space rules



- Mixed phase space (with M Kopp): … fractal Weyl law …
  - Goal: test character of chaotic component

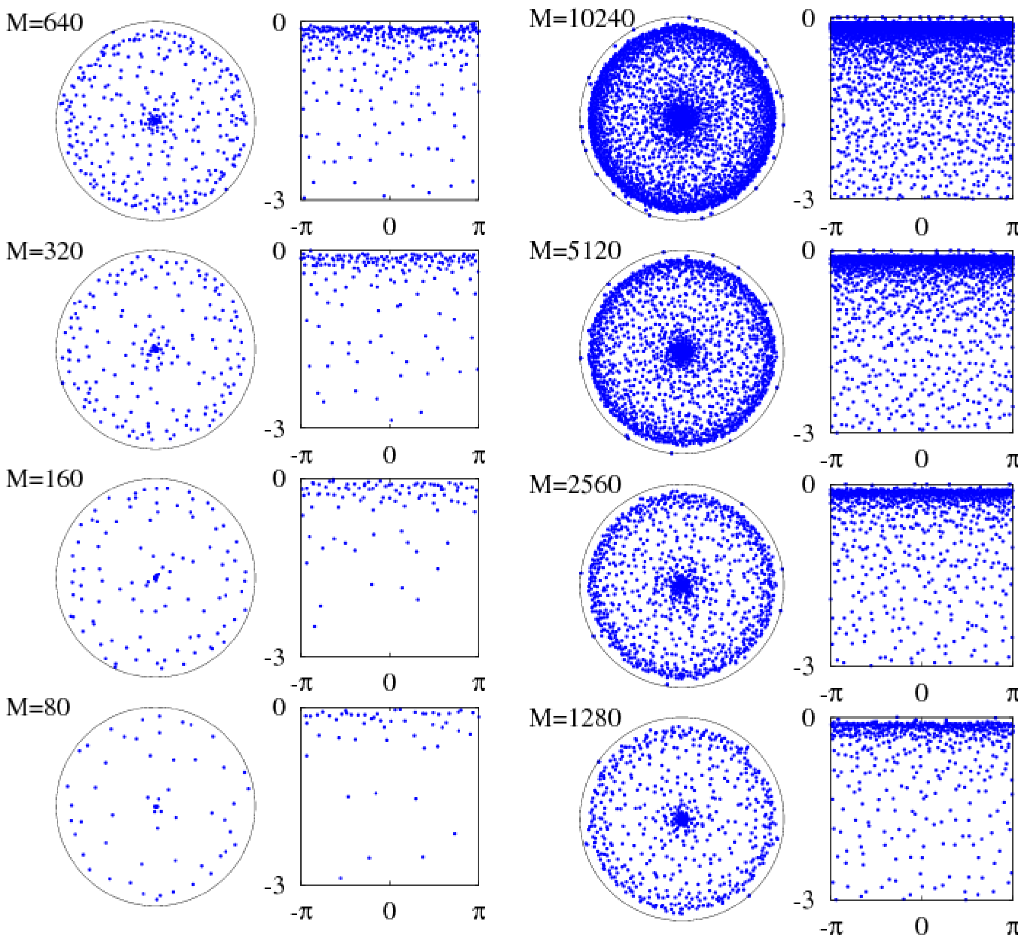


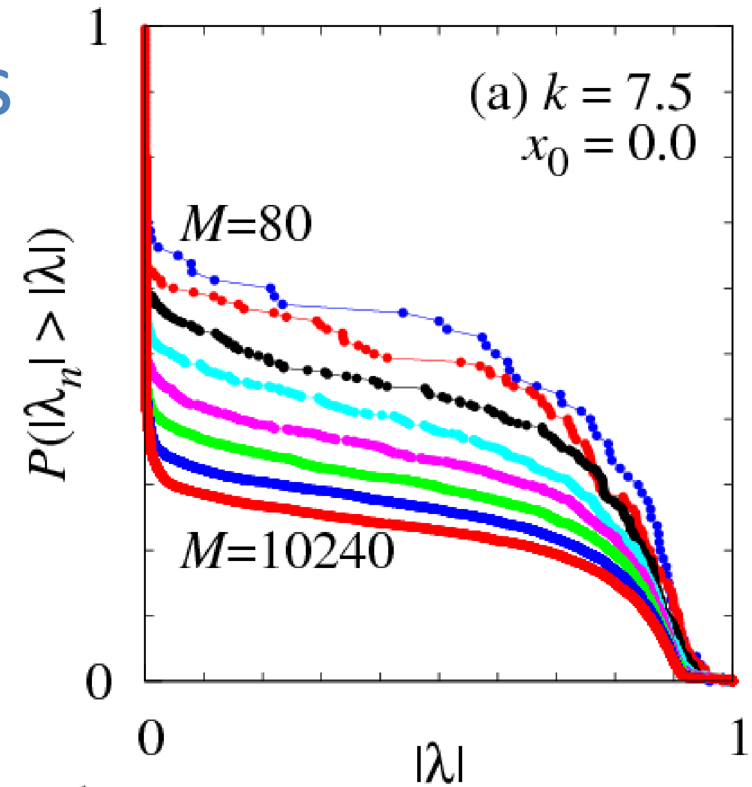
- Refractive escape (with J Wiersig; J Keating and M Novaes): dielectric resonators
  - Goal: generalization and comparison to realistic systems

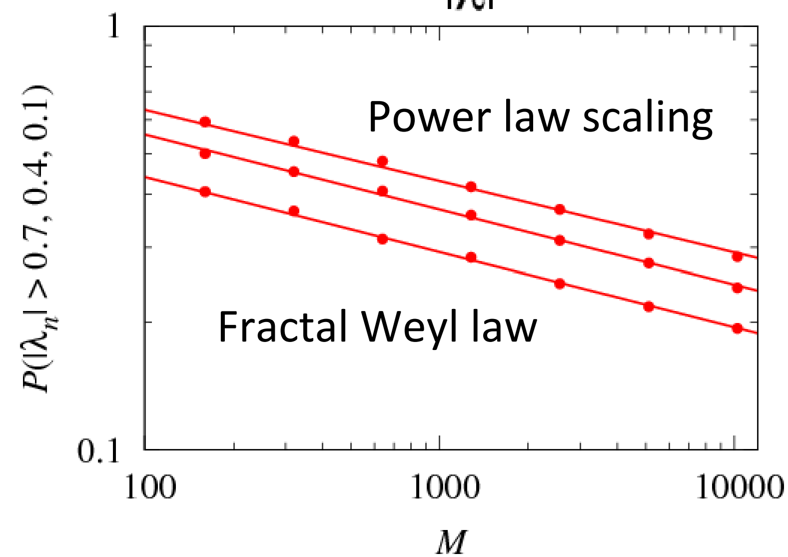


Classically chaotic systems

Resonance distribution







Classically chaotic systems

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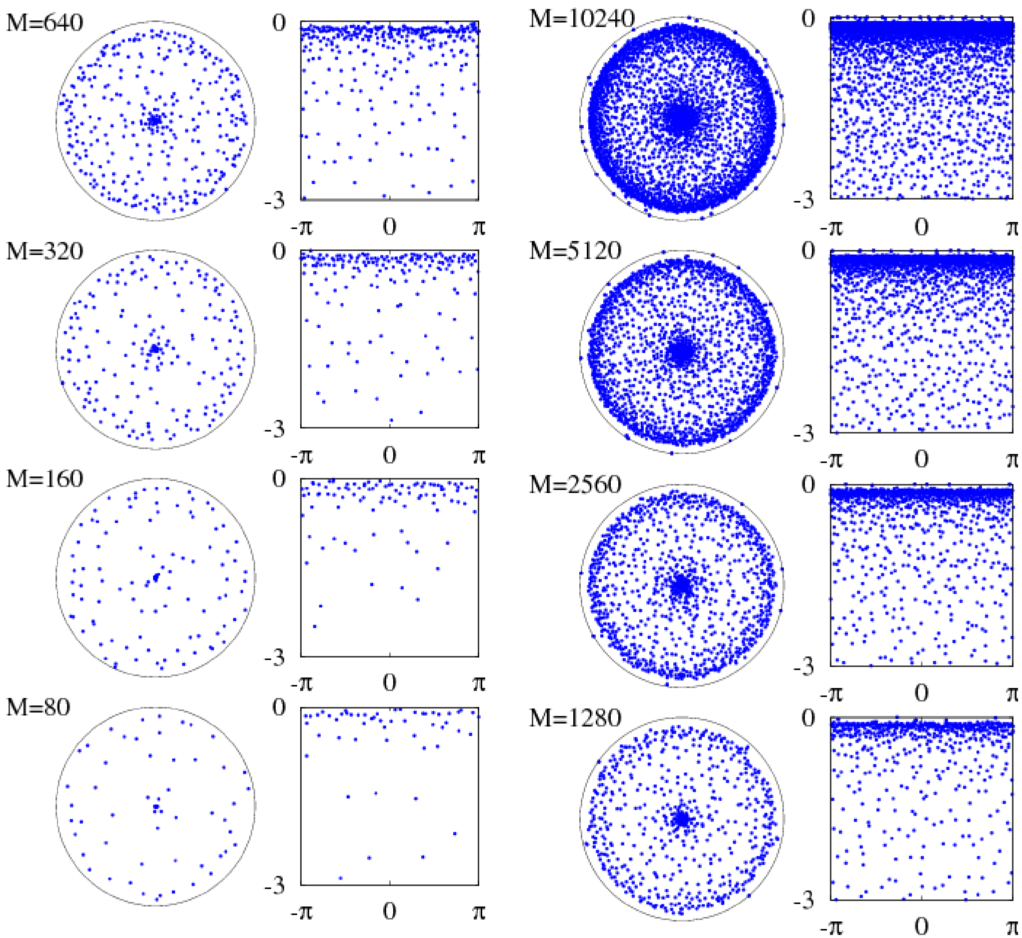
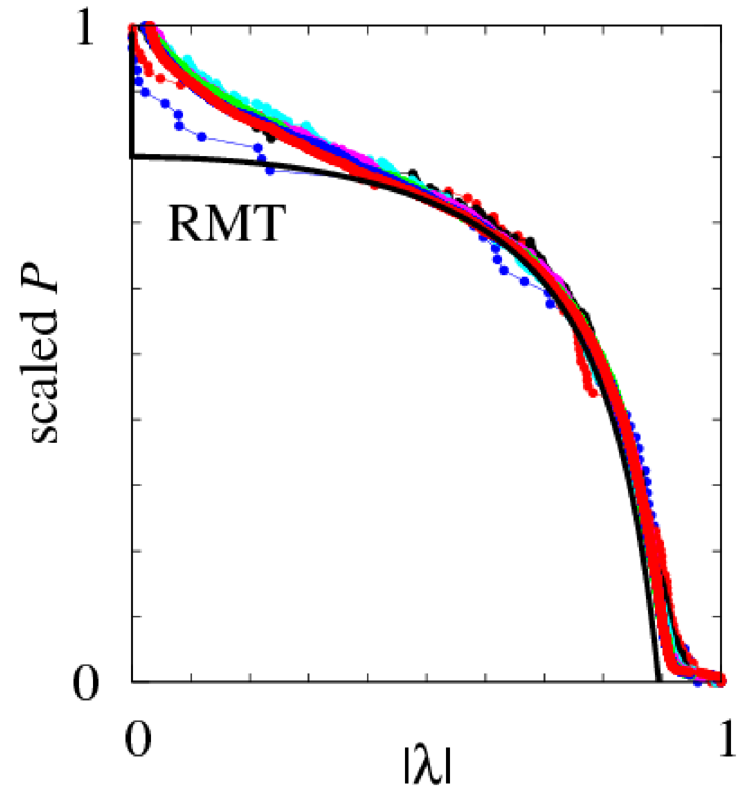
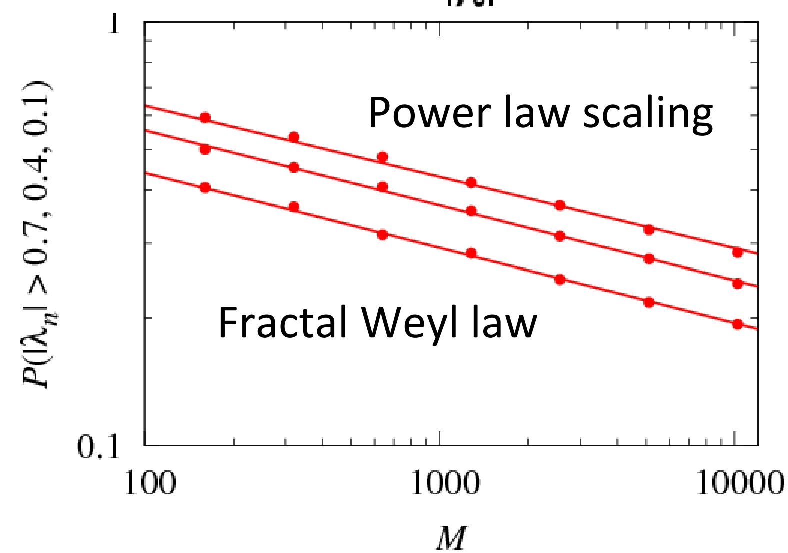


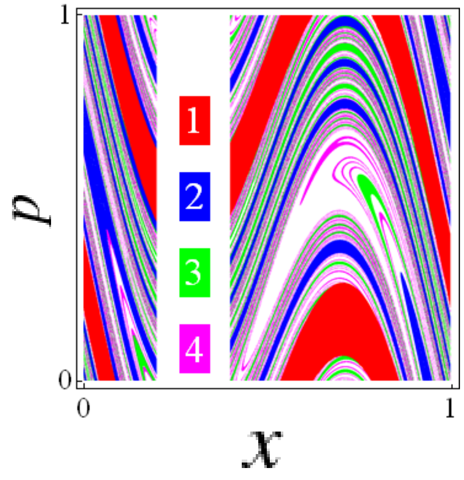




Try to count short-living states

A. identify short-lived deterministic dynamics in phase space

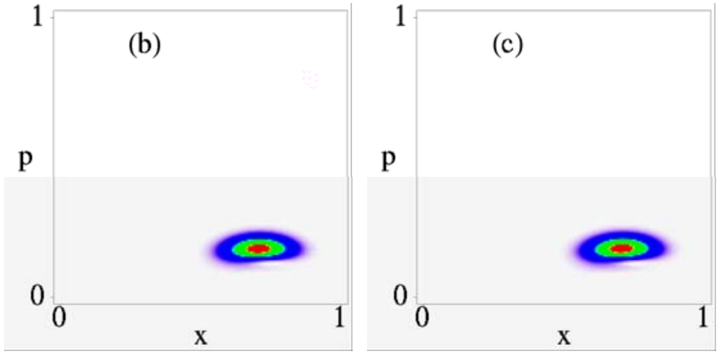
$$QFQ\psi_n = 0 \quad (\lambda_n = 0, \quad \Gamma_n = \infty)$$



- Define $\mathcal{P}$=P^{T} P=1-Q
- trivially: $Q\mathcal{P}$=0 → N states on opening ($\mathcal{P}_o = \mathcal{P}$)

- semicl.: preimage: projector $\mathcal{P}_1 = P_1 P_1^{T}$
- naïve Weyl: dim = area/Planck= M • area

problem: underestimates no. of states
reason: operator not self-adjoint,
states nonorthog., highly degenerate

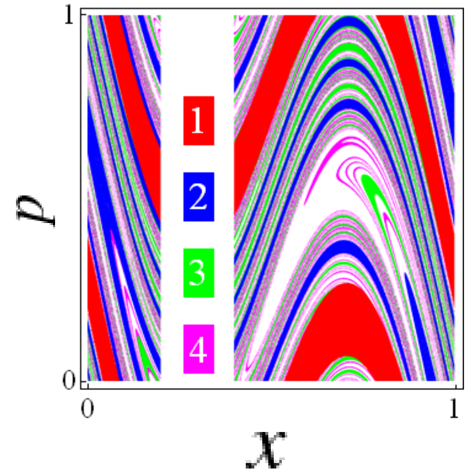


B. Cure degeneracy

$QFQ\psi_n^{(1)} = 0 \quad (\lambda_n = 0)$: consider $QFQ\psi_n^{(t+1)} = \lambda_n \psi_n^{(t+1)} + \psi_n^{(t)} = \psi_n^{(t)}$

- 2nd preimage, projector $\mathcal{P}_2 = P_2 P_2^{\mathrm{T}}$
- 3rd preimage, projector $\mathcal{P}_3 = P_3 P_3^{\mathrm{T}}$
- tth preimage, projector $\mathcal{P}_t = P_t P_t^{\mathrm{T}}$
- semiclassical propagation:

$$(QFQ)^t \mathcal{P}_t = 0, \quad \mathcal{P}_t \mathcal{P}_s = 0 \quad (t \neq s)$$



C. Requires: areas $A \approx \exp(-\Lambda t) > 1/M \;\Rightarrow t < \dfrac{1}{\Lambda}\ln(M) \equiv t_{Ehr}$

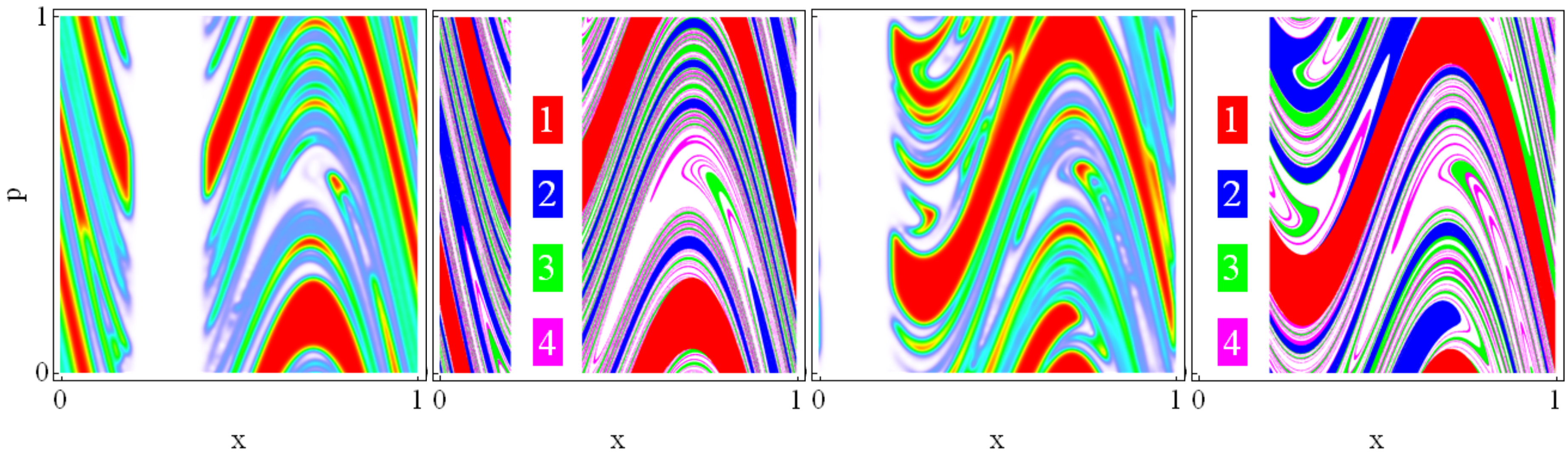
Weyl: $\sum_{t < t_{Ehr}} \mathrm{rank}\,\mathcal{P}_t = M(1 - e^{-t_{Ehr}/t_{dwell}})$

D. Remaining states (long living): $Me^{-t_{Ehr}/t_{dwell}} \propto M^{1-1/\Lambda t_{dwell}}$

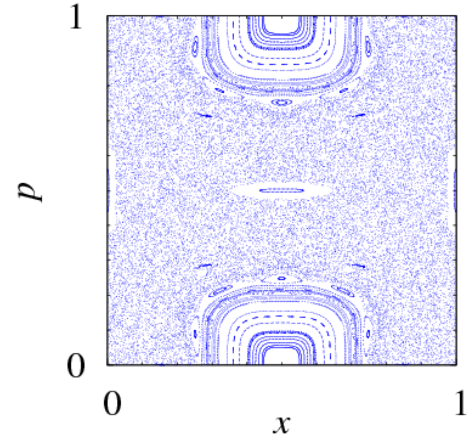
What have we done? A semiclassical partial Schur decomposition!

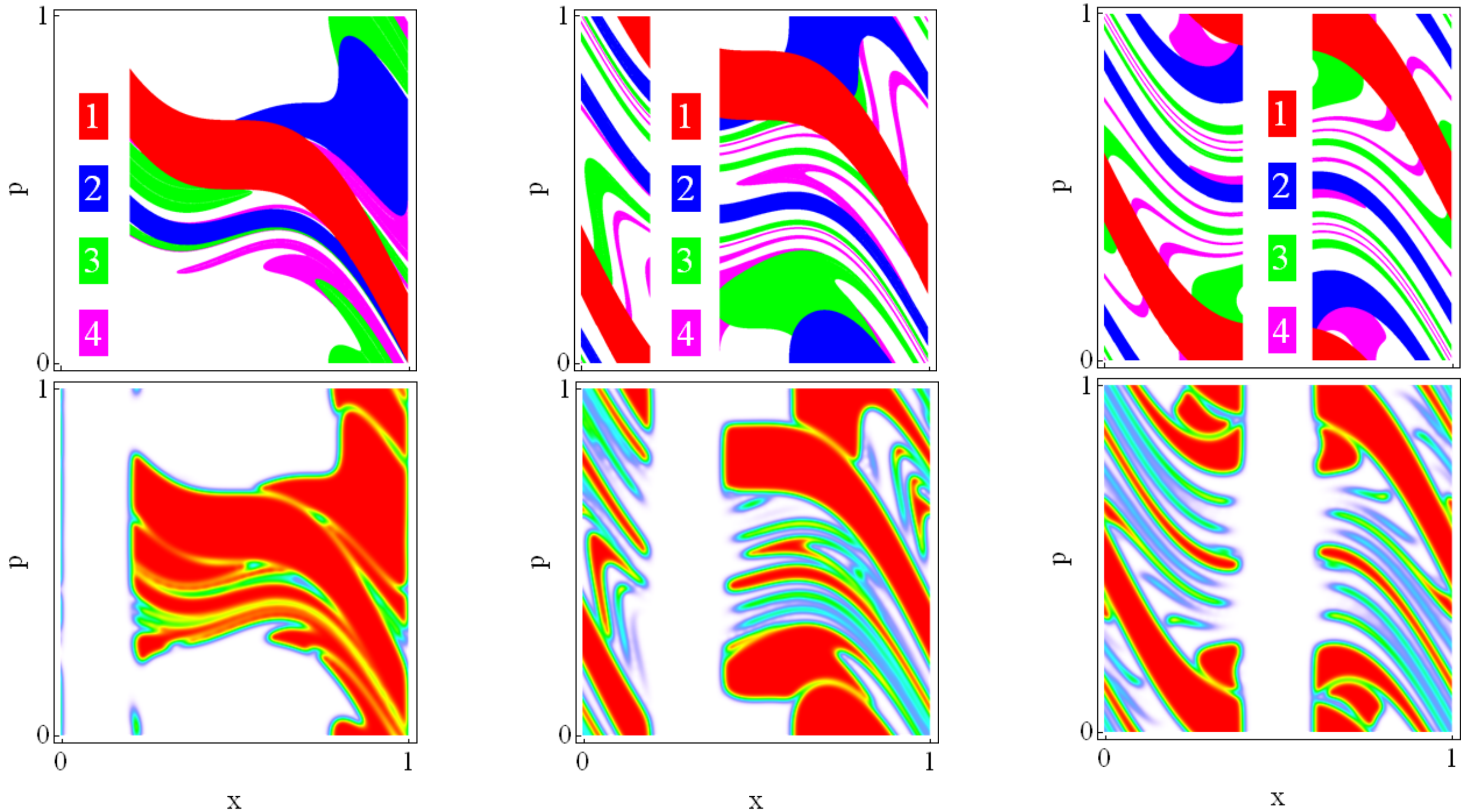
$\mathcal{P}_t$: part of orthogonal basis U in $QFQ = UTU^{+}$
where T is triangular with evals on diagonal.

Test: Husimi rep. of Schur vectors ($|\lambda_n|<0.1$, M=1280)



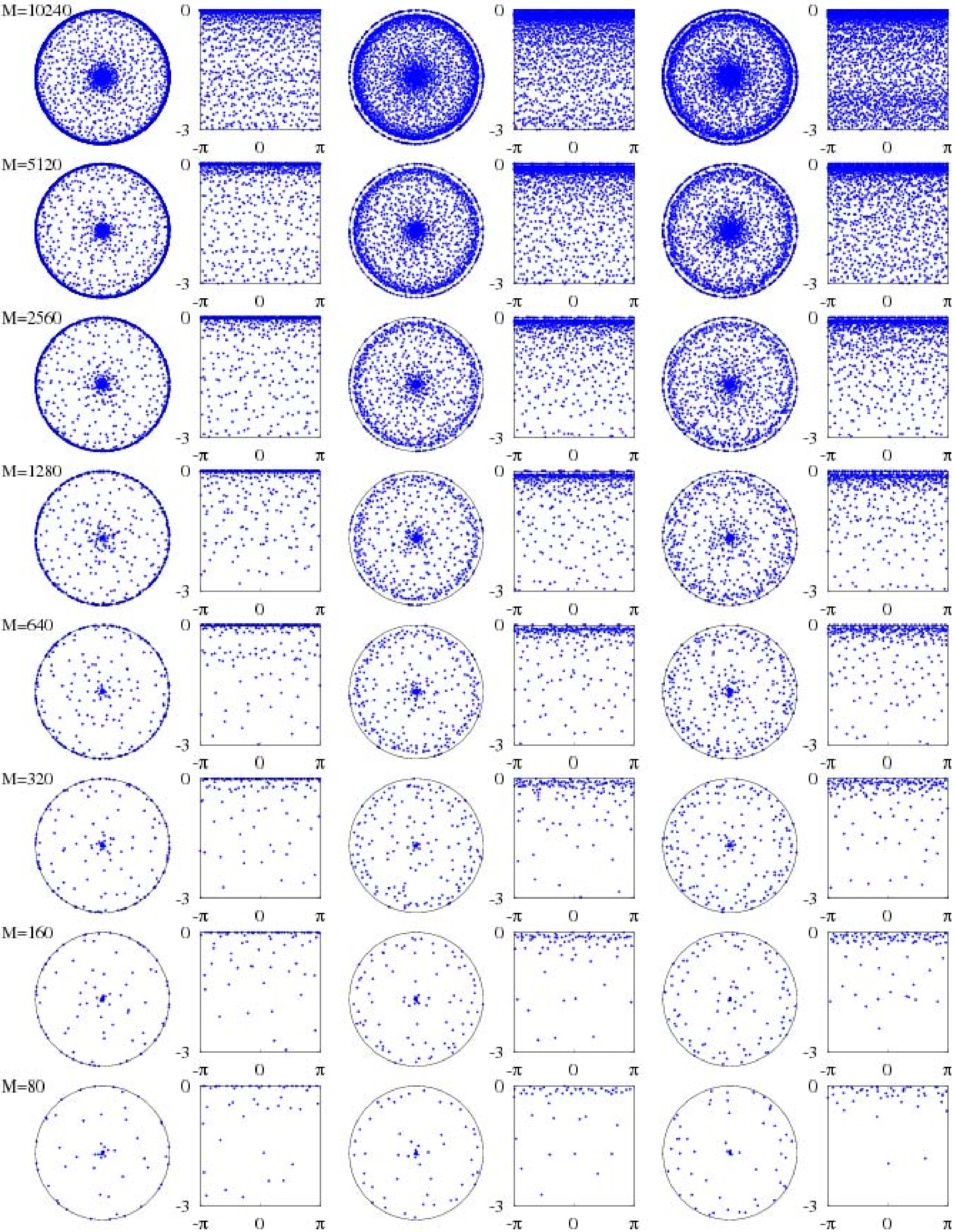
Mixed phase space





Position of leads is important; coupled islands: fast decay

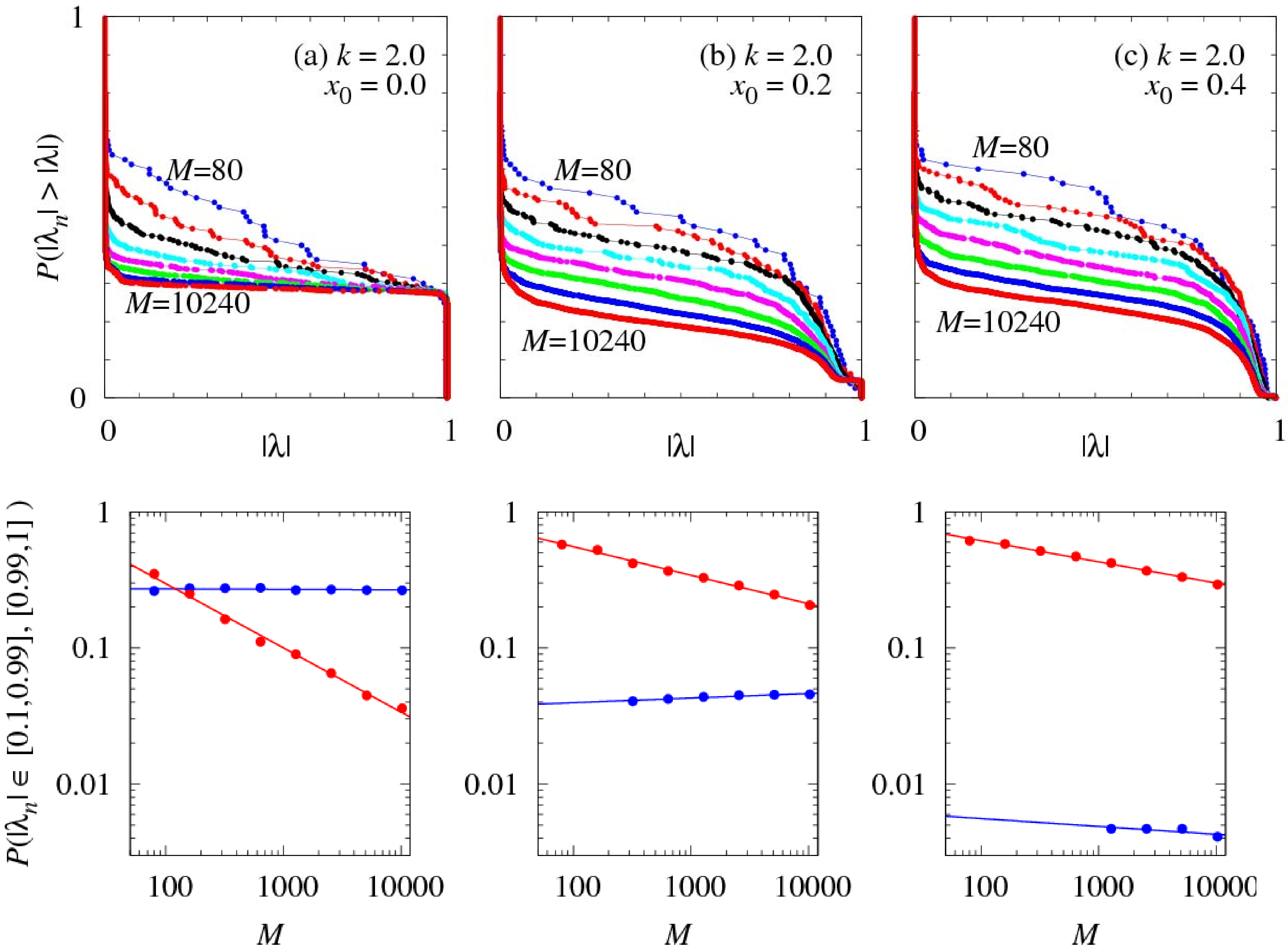
Uncoupled islands: slow tunneling escape



Two accumulation regions:

$$|\lambda| \approx 0{,}1$$

- uncoupled islands (long-living states): just the ordinary Weyl law…
- idea: fix both upper and lower cut-off of lifetimes

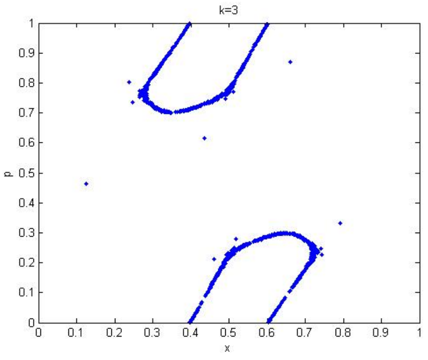


(a) $k = 2.0$ $x_0 = 0.0$
(b) $k = 2.0$ $x_0 = 0.2$
(c) $k = 2.0$ $x_0 = 0.4$
M=80
M=10240
$P(|\lambda_n| > |\lambda|)$
$|\lambda|$
$P(|\lambda_n| \in [0.1,0.99], [0.99,1])$
M

Slightly unexpected…

Time domain studies: classical part of mixed phase space is quite unlike a fully chaotic phase space:

Power law decay $\propto t^{-\alpha}$ vs
exponential decay $\propto \exp(-t/t_{dwell})$
Origin: sticking to islands
(see eg Cristadoro/Ketzmerick PRL 08)



Possible explanations:
a)The fractal Weyl law actually breaks down for much larger M
b)Sticking just contributes to the long-living states
c)Areas also power-law distributed?

Generalization: nonballistic escape

Applications: q-dots w/tunnel barriers, dielectric resonators

Stroboscopic scattering operator

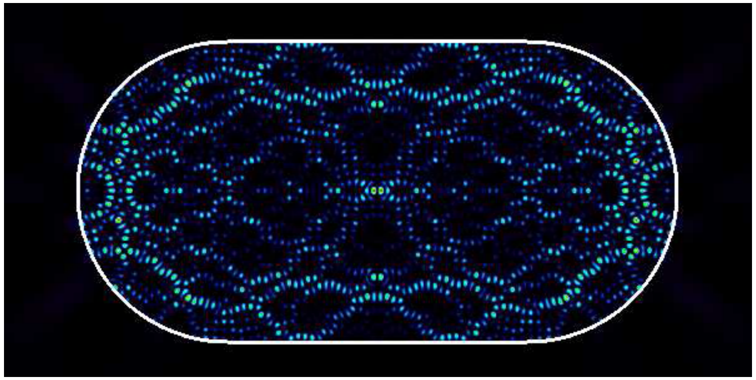
$$S(\varepsilon) = R' + T'\left(e^{-i\varepsilon} - FR\right)^{-1} FT$$

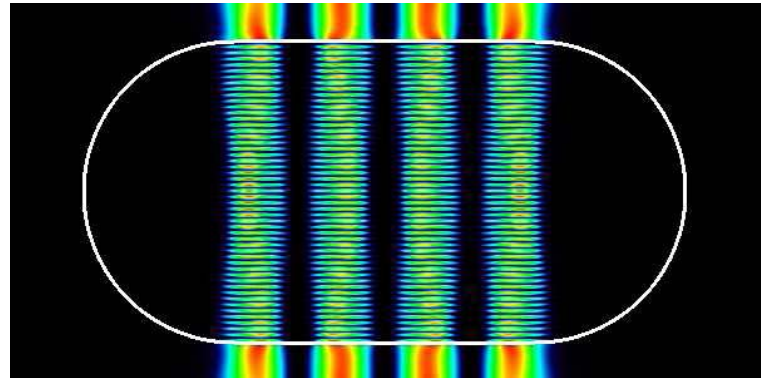
For dielectric resonators:

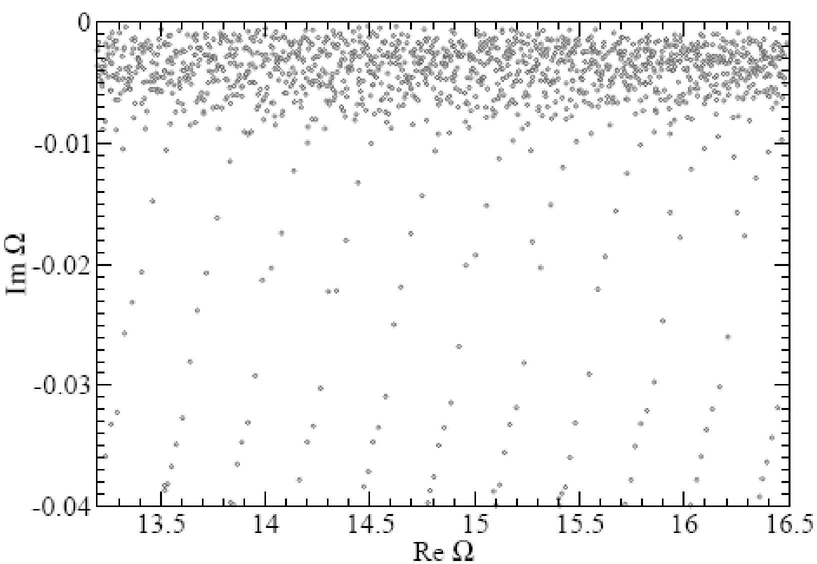
$$S(\omega) = -R + T\left(e^{-i\omega\tau} - FR\right)^{-1} FT$$

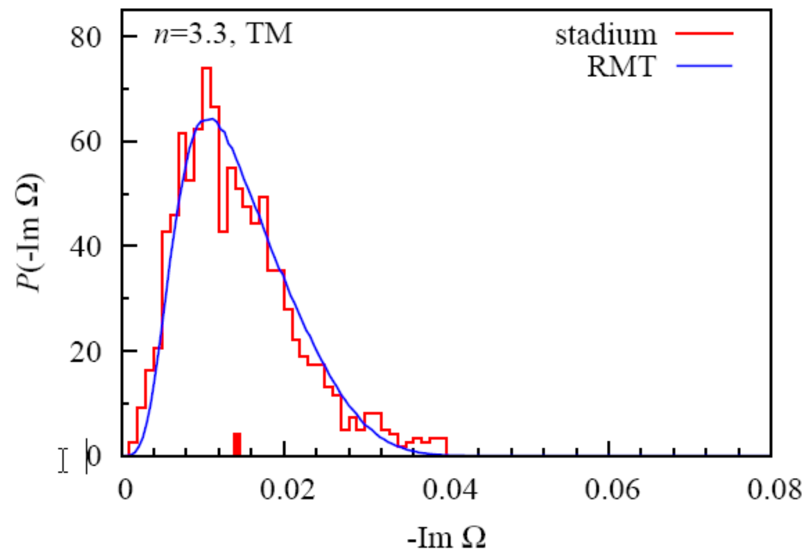
with frequency ω, traversal time $\tau = n\,\pi\,A\,/\,v\,C$ (Sabine's law),
and *R*, *T* determined by Fresnel reflection coefficients.
(n: refractive index; A: area, C: perimeter, v: velocity)
Also, M=N=dim S= $\omega\, C/v\, \pi$ (Weyl's law applied to the boundary)

Compare realistic resonator to random matrix theory (RMT)





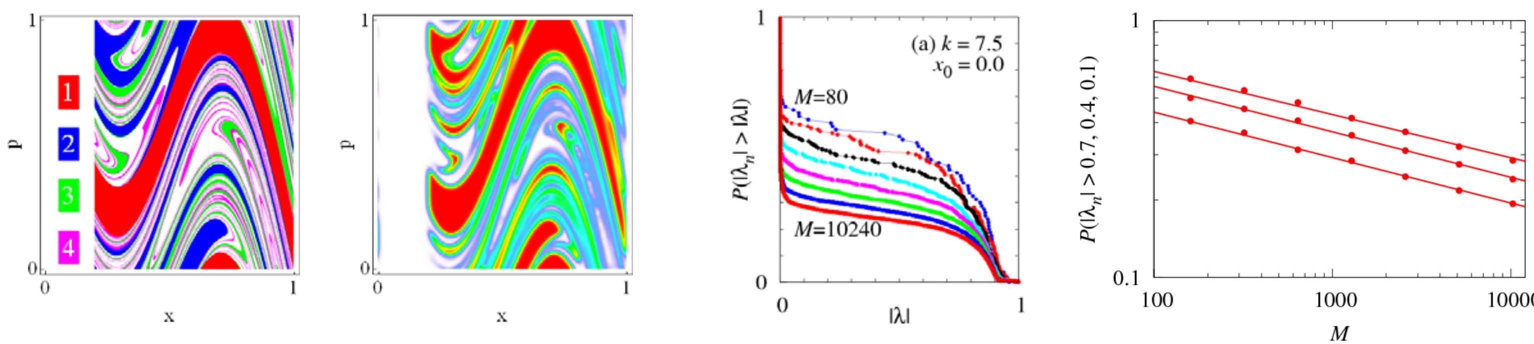




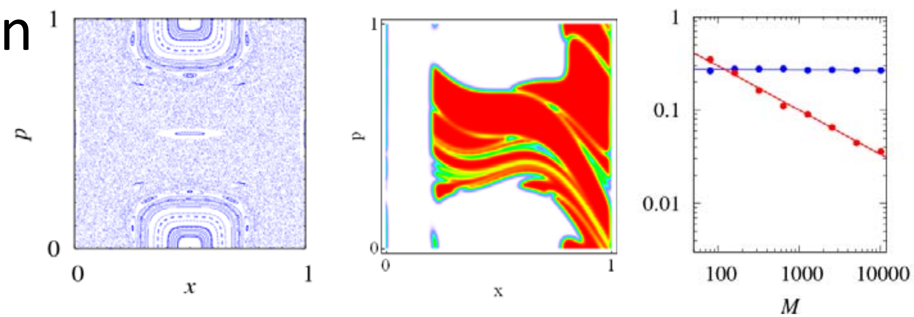
Bands of short-living states (origin: bouncing ball motion)
Requires to renormalize M and τ! Here done independent from fluctuations by using mean level spacing and decay rate of long-living states.

Summary

- Phase space rules can be resurrected by semiclassical Schur decomposition; links fractal Weyl law to Ehrenfest time



- Fractal Weyl law also exists in generic dynamical systems (mixed phase space)



- Stroboscopic scattering theory succeeds to describe realistic (autonomous) systems