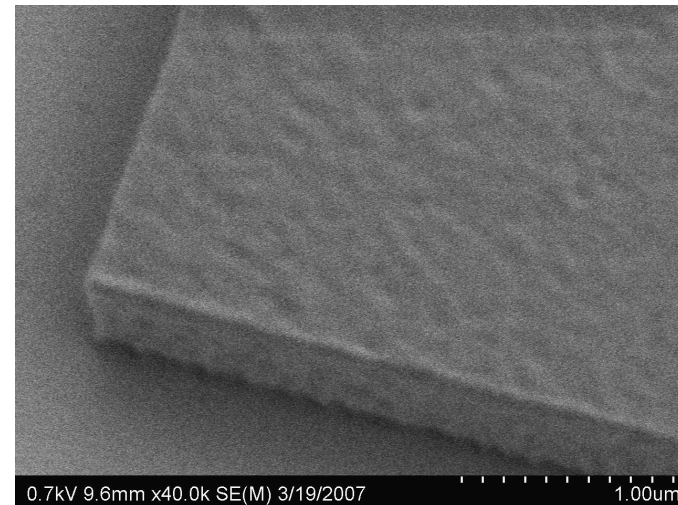
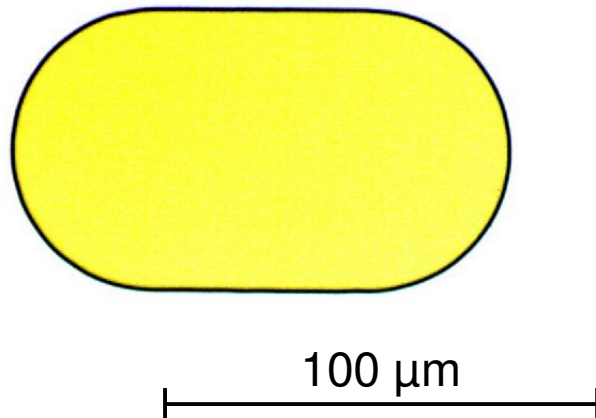




Organic micro-lasers



Melanie LEBENTAL



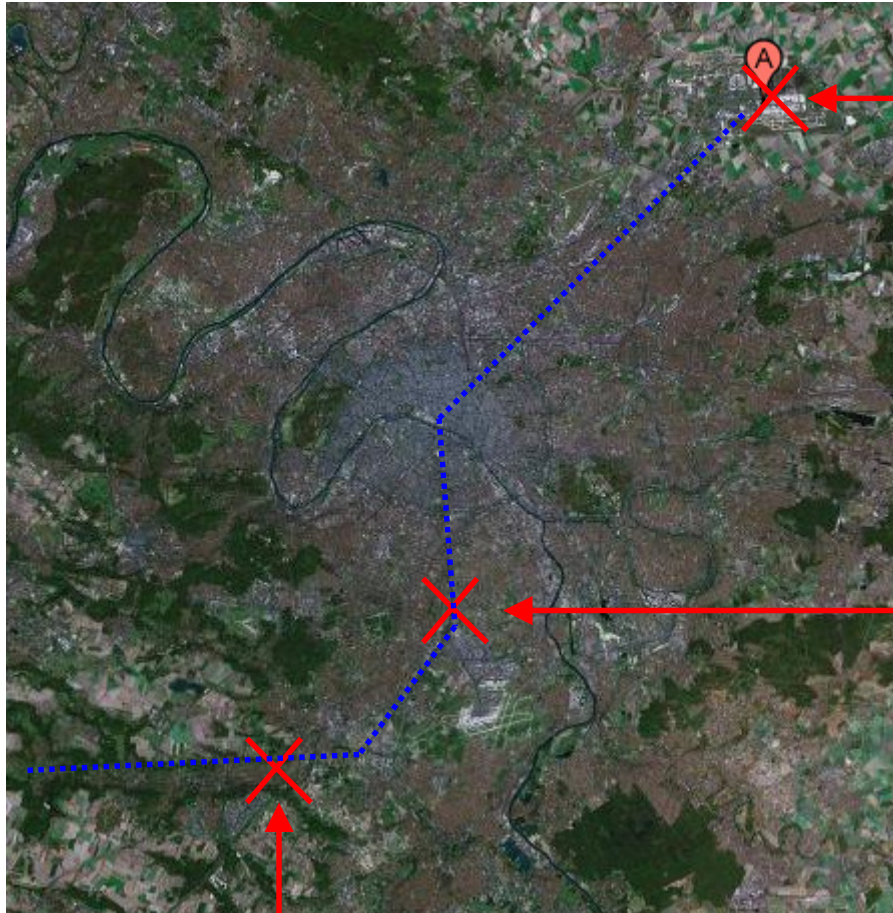
N. Djellali, S. Lozenko, I. Gozhyk, J. Lautru, I. Ledoux, and J. Zyss

Laboratory for Quantum and Molecular Photonics (LPQM)

ENS of Cachan

Resonances in Mathematical Physics, January 2009

To put Cachan on the map...



Charles-de-Gaulle
Airport

..... RER B (urban train)

ENS of Cachan

*Coll: E. Bogomolny, R.
Dubertrand, C. Schmit
(LPTMS)*

Orsay University

Outline

Organic micro-**lasers** as test-beds
for wave chaos of **open** systems

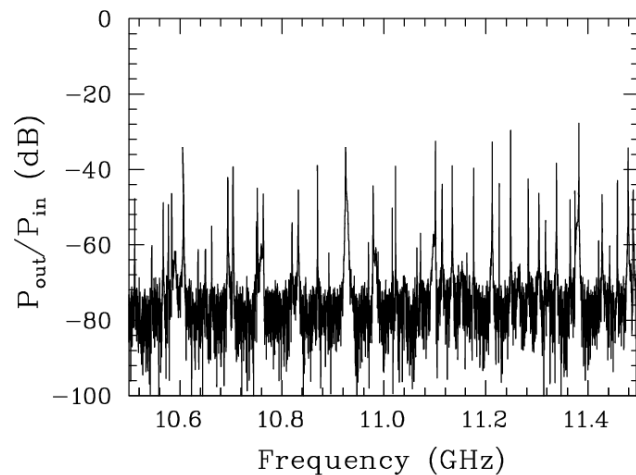
- I Micro-lasers and wave chaos
- II Existing tools (*what we can do*)
- III Open questions

I Wave chaos for closed systems

Conjectures

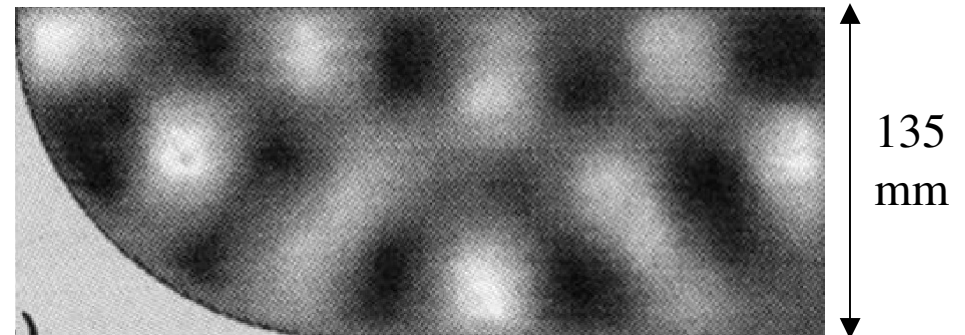
Spectra: random
matrix theory

Bohigas et al. , PRL 52 1 (1984)



Wavefunctions: random
superposition

Berry, J. Phys. A 10 2083 (1977)

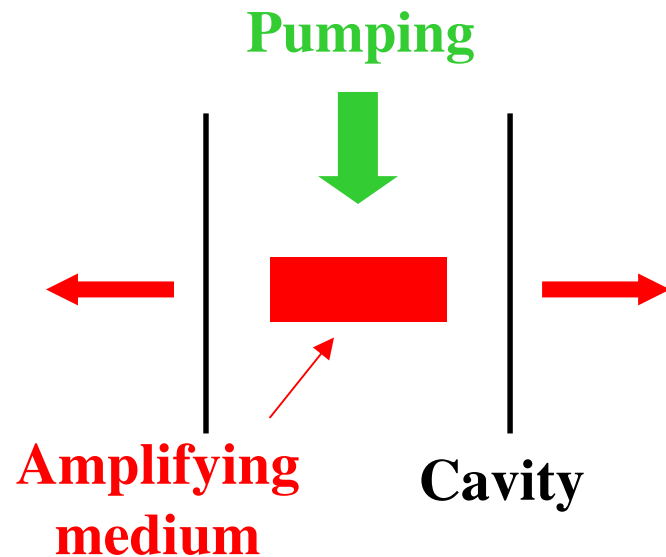


Experiments with metallic stadium-shaped microwave cavities

Alt et al., PRE 60, 2851-2857 (1999)

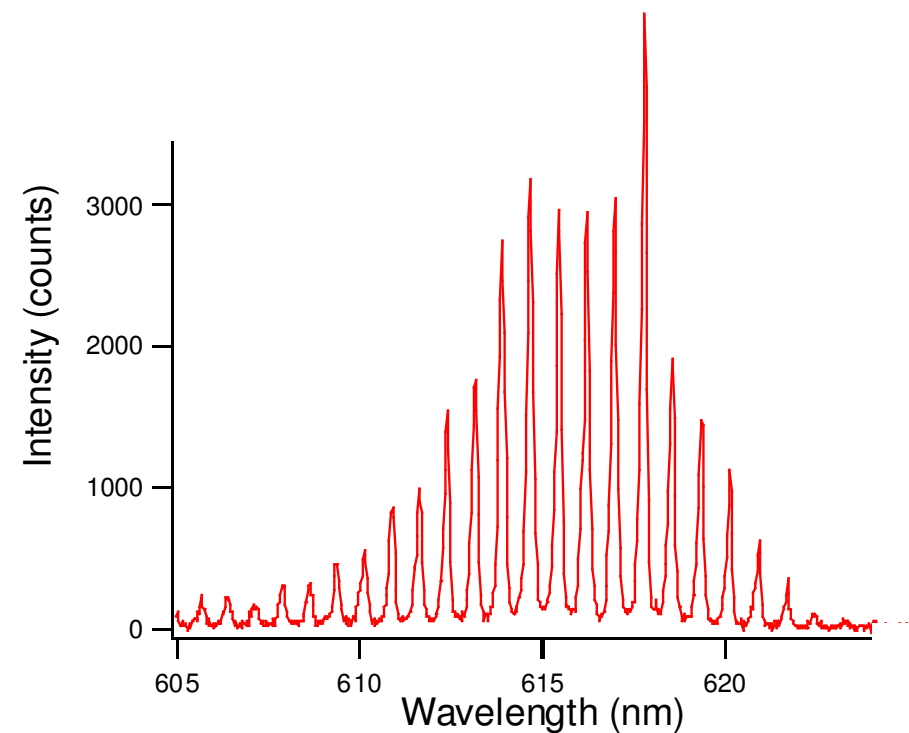
Stein et al., PRL 75, 53-56 (1995)

I Lasing: basic ingredients

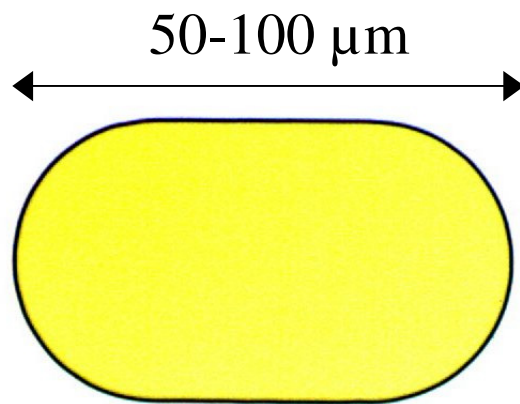


No direct connexion between
pumping and **emission**

- Directions of emission ?
- Spectrum ?



Plastic micro-lasers

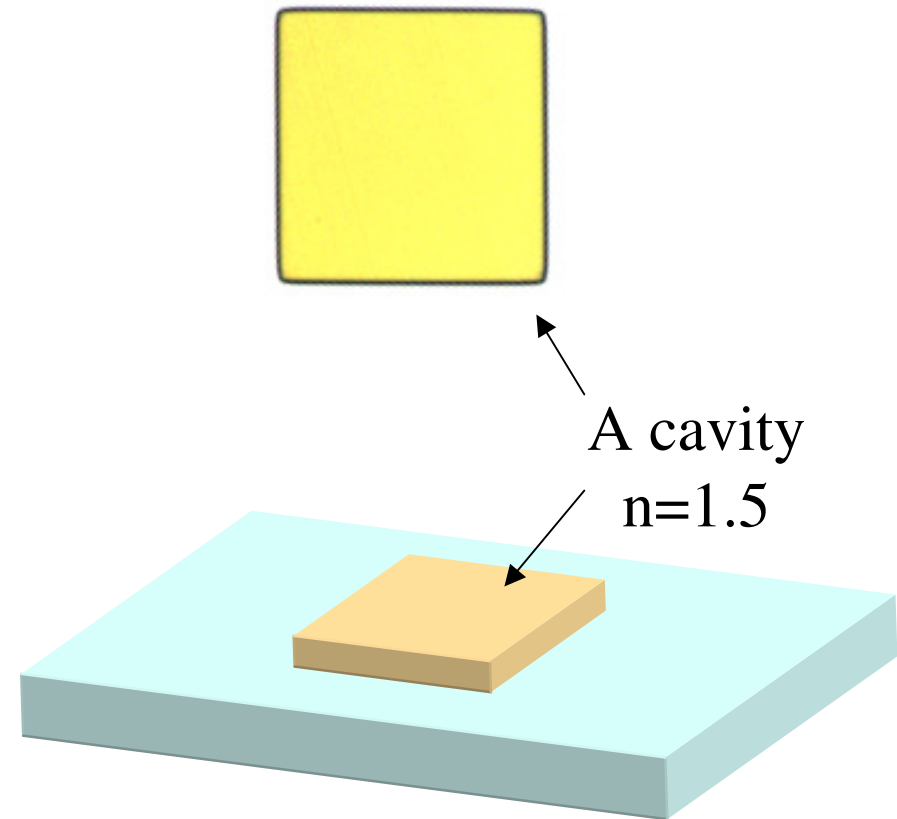


Photography

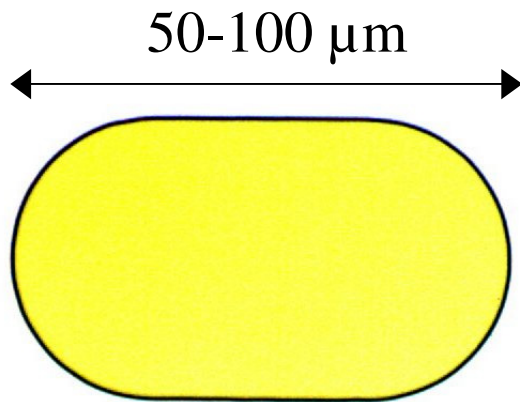
Matrix host: polymer (PMMA)

Guest: **laser dye** (DCM)

$\lambda \sim 0.6 \mu\text{m}$



Plastic micro-lasers



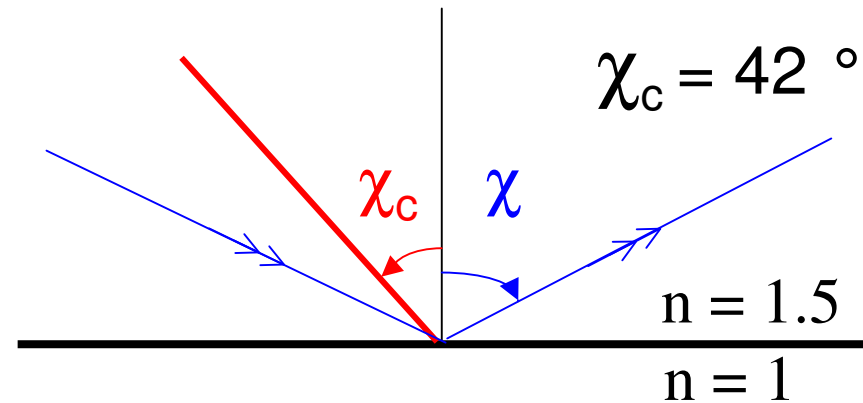
Photography

Matrix host: polymer (PMMA)

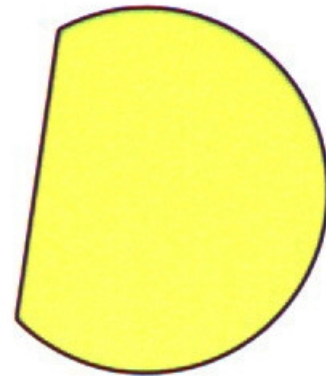
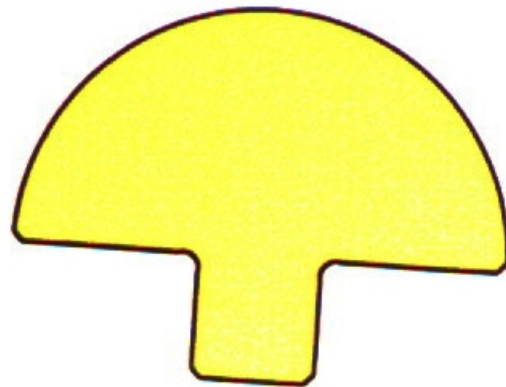
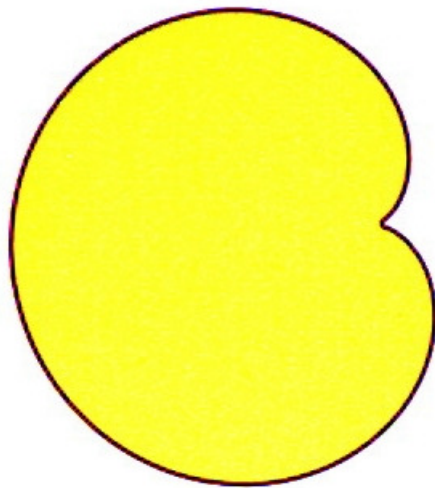
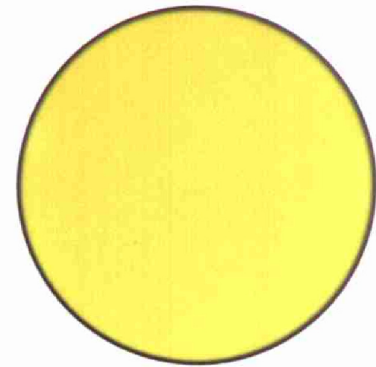
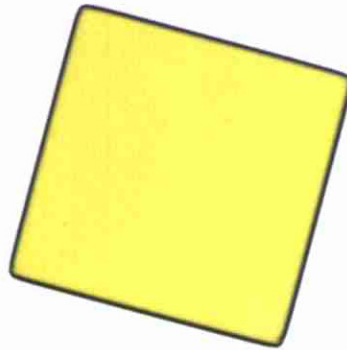
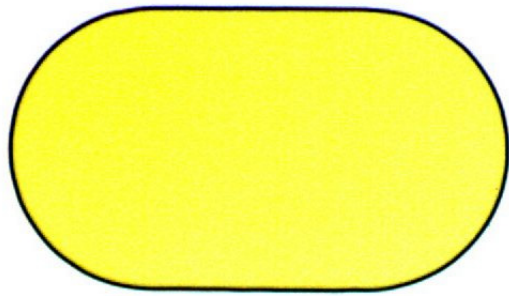
Guest: **laser dye** (DCM)

$\lambda \sim 0.6 \mu\text{m}$

- Cheap
- Easy
- Low refractive index
- Modular (mask)



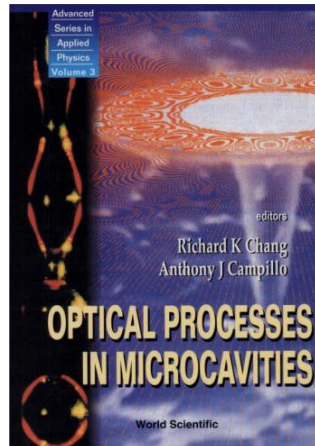
I Microlasers: cavity shapes



Photographies from an optical microscope

I Practical applications

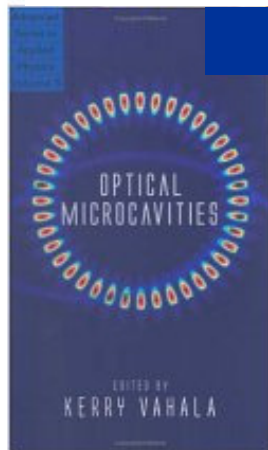
*Chang
Campillo
1996*



Optical telecommunications

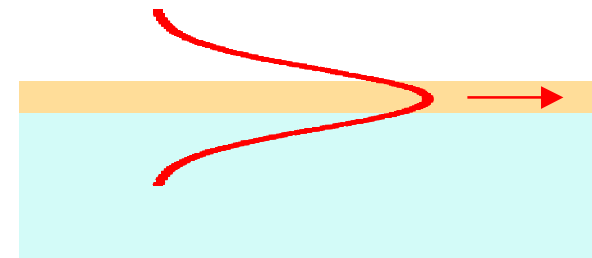
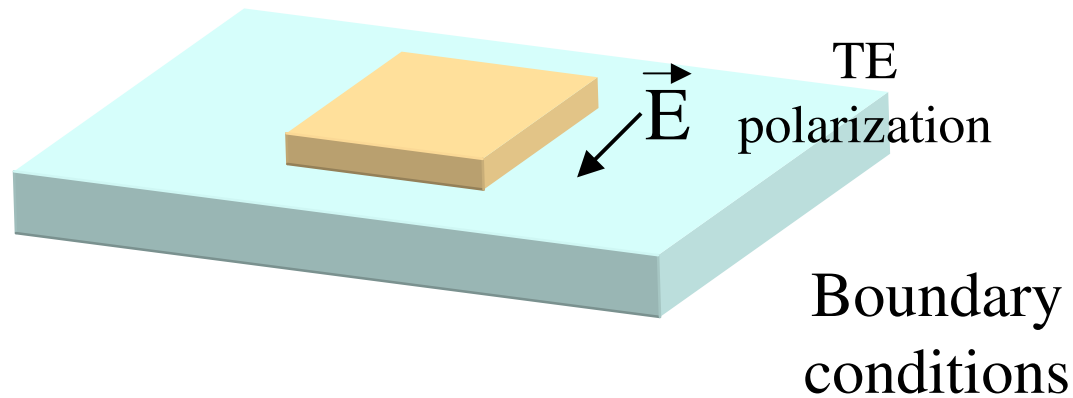
Chemical/biological sensors

*Vahala
2004*



*Practical applications of micro-resonators
in optics and photonics, Matsko, 2009.*

I From electromagnetism to wave chaos



Inside

$$(\Delta + n^2 k^2) \begin{cases} \vec{E} \\ \vec{B} \end{cases} = 0$$

Outside

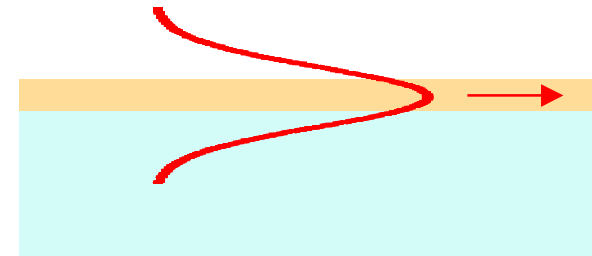
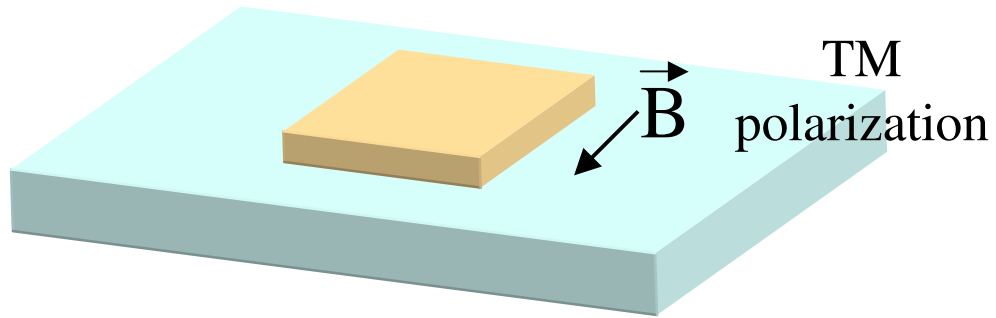
$$(\Delta + k^2) \begin{cases} \vec{E} \\ \vec{B} \end{cases} = 0$$

Effective index approximation

TE \Rightarrow $\Psi = B_z$

Passive cavity (no laser)

I From electromagnetism to wave chaos



Inside

$$(\Delta_{xy} + n_{eff}^2 k^2) \Psi = 0$$

Outside

$$(\Delta_{xy} + k^2) \Psi = 0$$

Effective index approximation

$$\text{TE} \Rightarrow \Psi = B_z$$

$$\text{TM} \Rightarrow \Psi = E_z$$

Passive cavity (no laser)

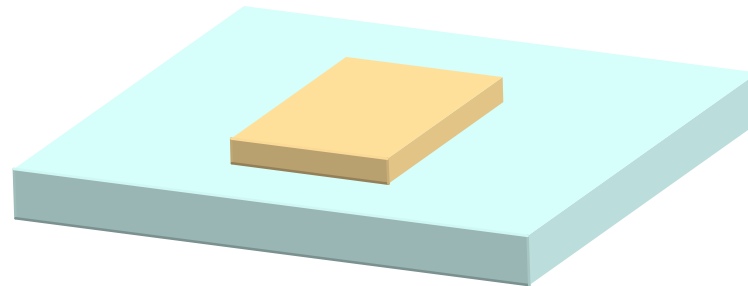
Outline

I Micro-lasers and wave chaos



Cavity shape \rightarrow Billiard (*open*)

Laser effect \rightarrow To fill the resonances
with photons



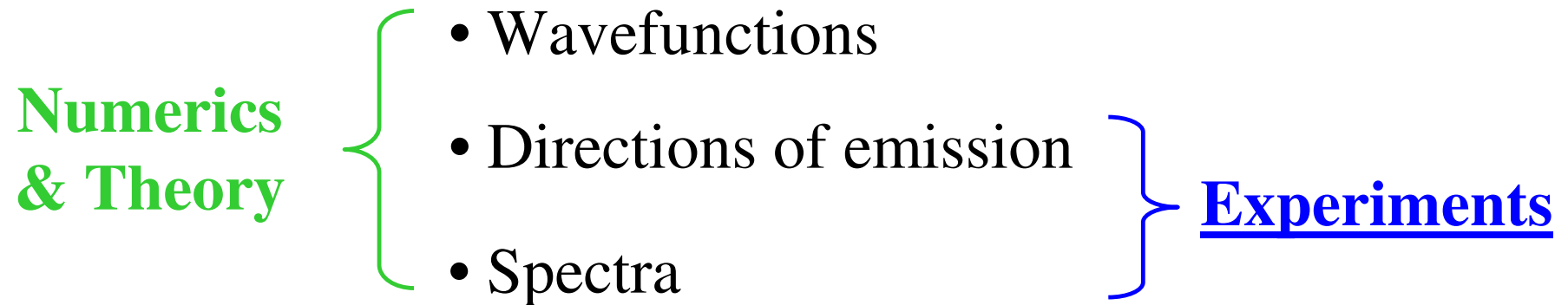
II Existing tools (*what we can do*)

III Open questions

Outline

I Micro-lasers and wave chaos

II Existing tools (*what we can do*)

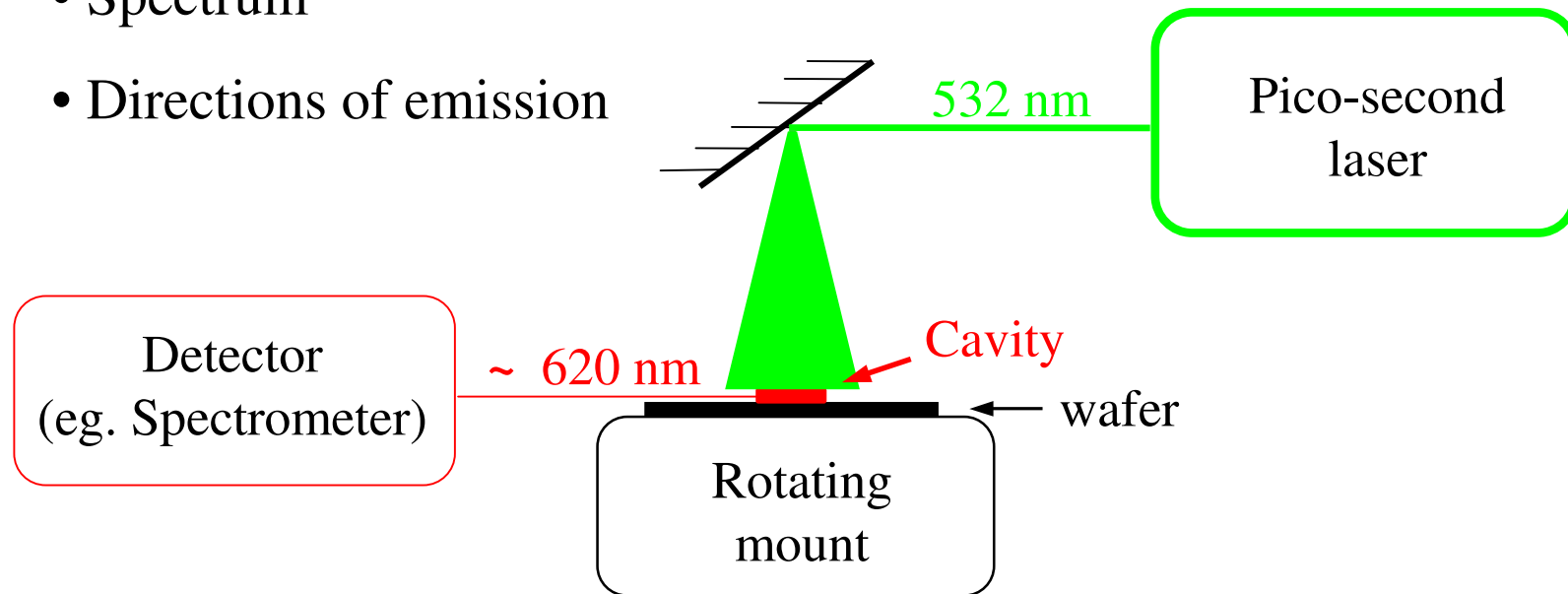


III Open questions

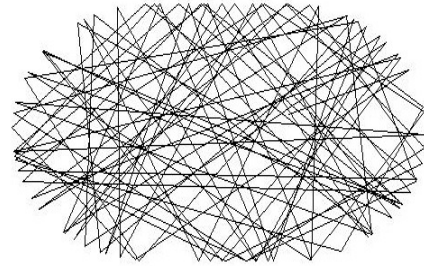
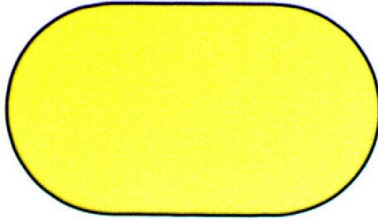
II Experimental set-up

Information about :

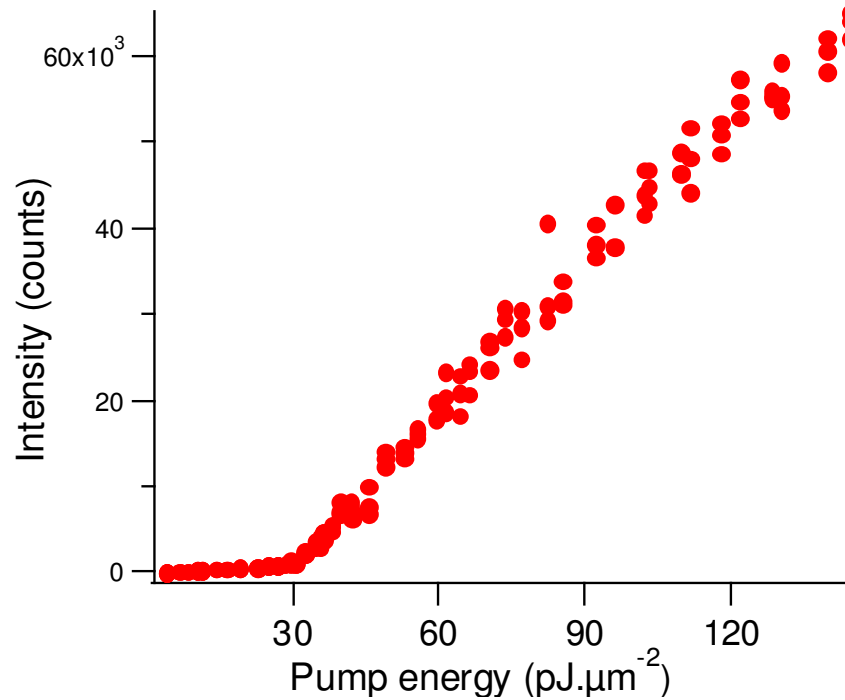
- Lasing
- Spectrum
- Directions of emission



II Lasing ?



Laser threshold

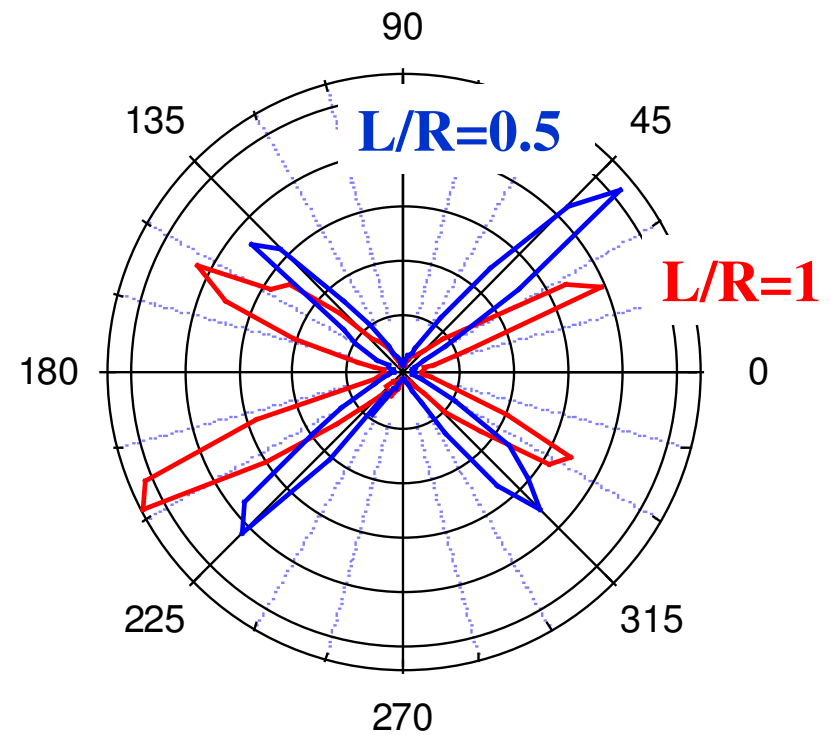
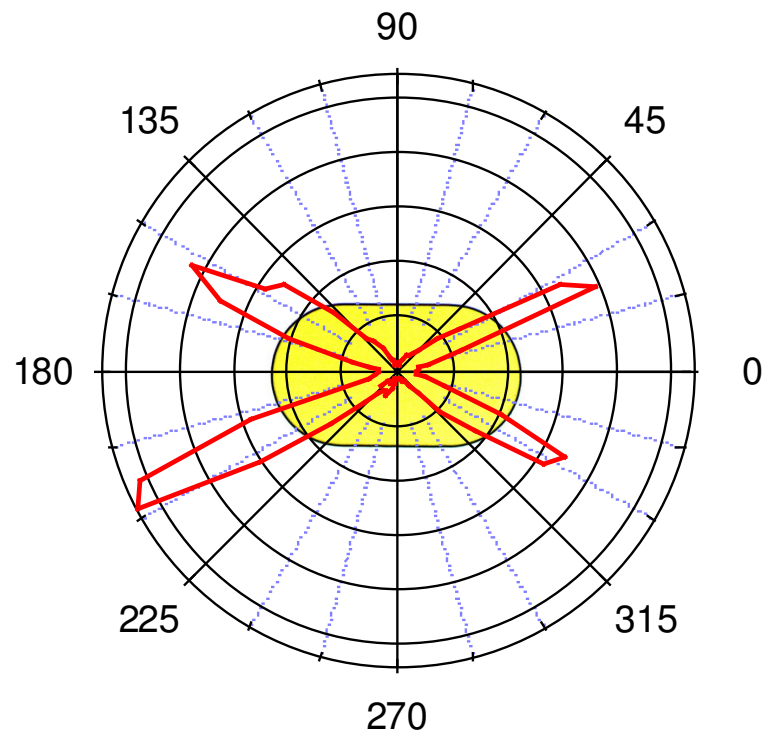
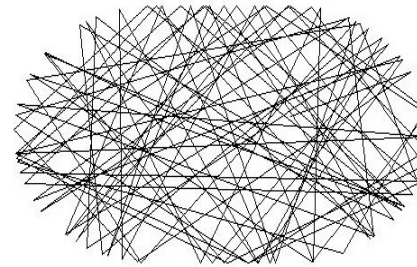
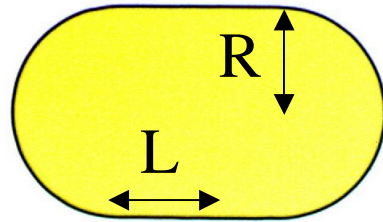


- Ease for detection

- Coherence

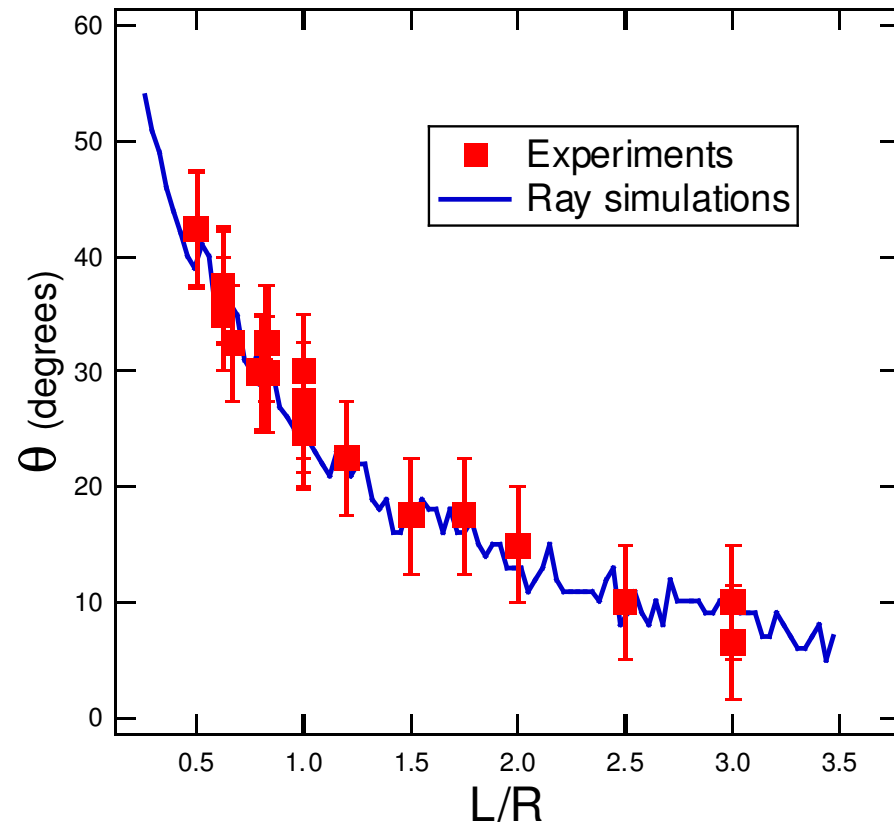
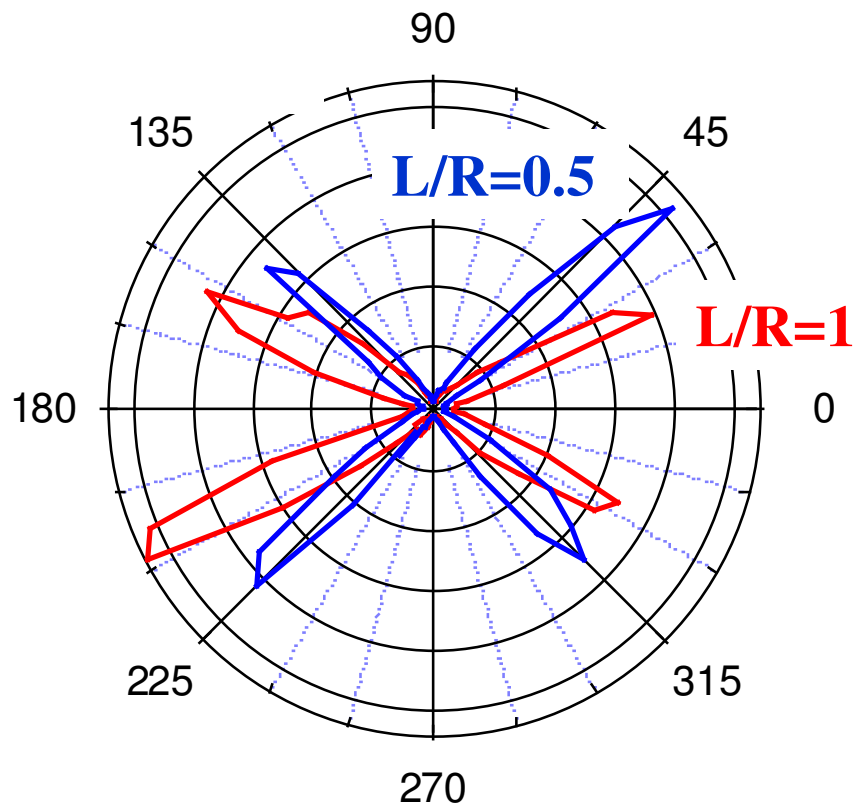
$$kR_{\text{exp}} \sim 200 - 1000$$

II Stadium: directions of emission



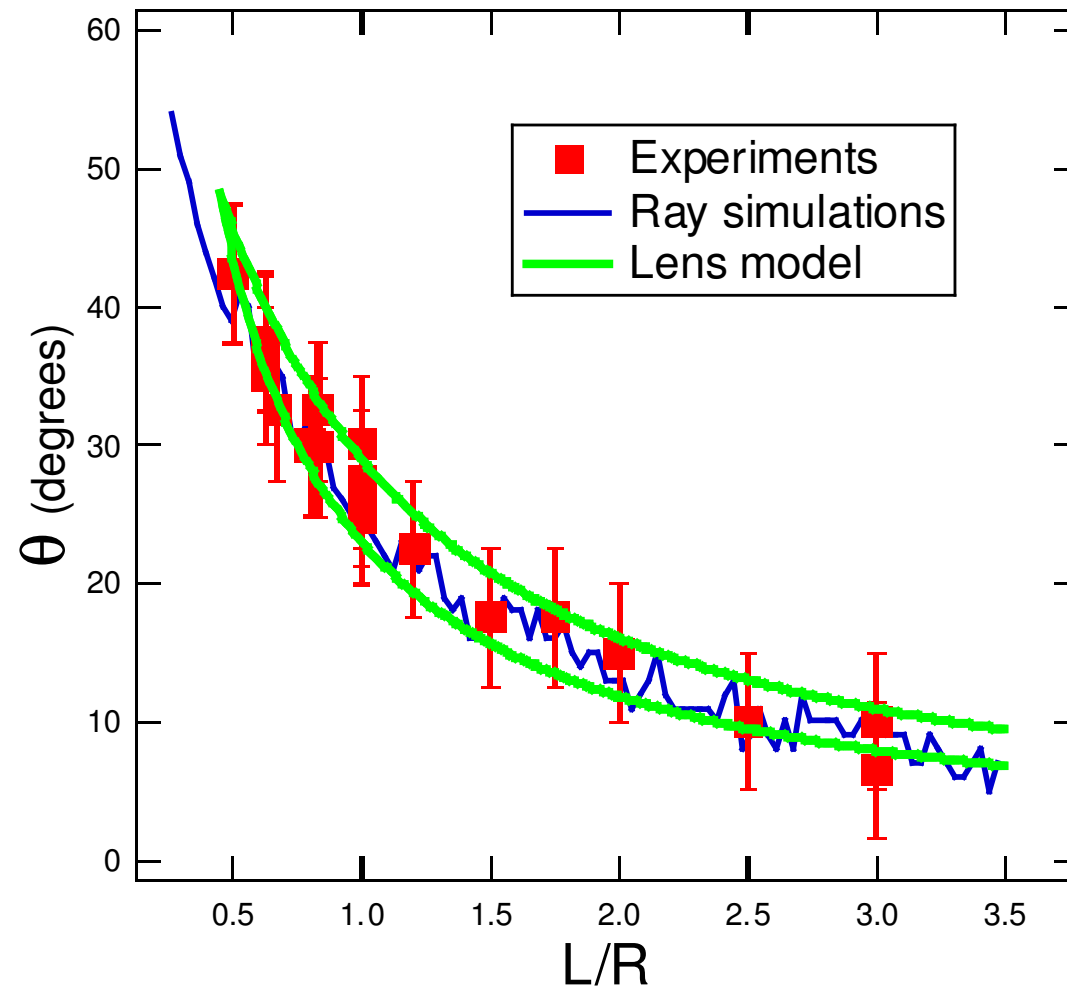
II Stadium: directional emission

Emission in the far-field pattern



II Chaotic cavity: lens model

ANALYTIC



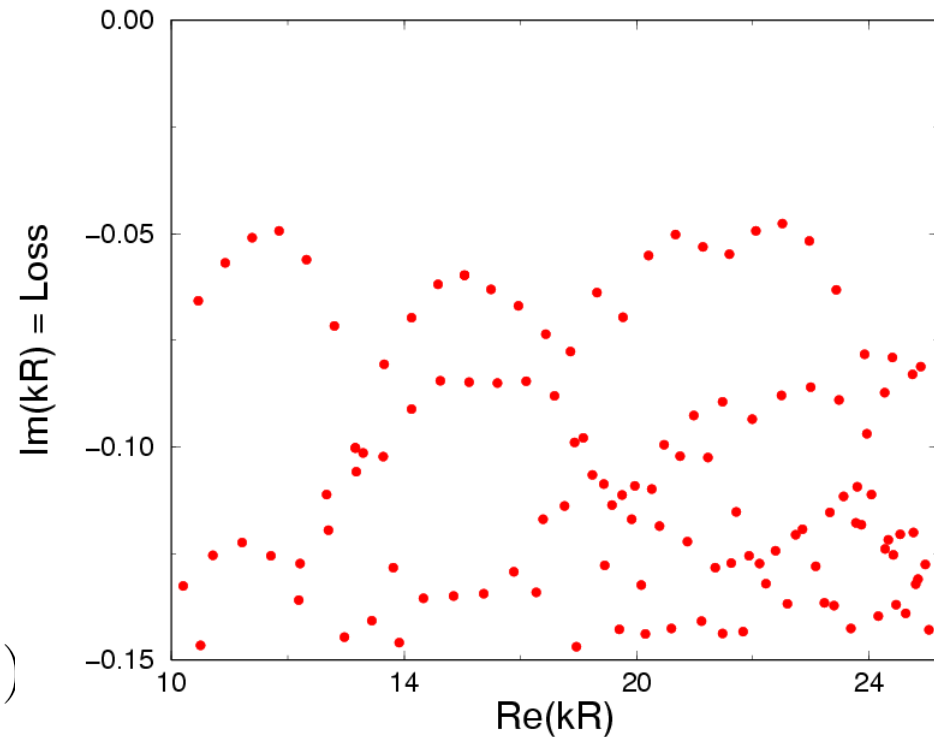
II Wave numerical simulations

Helmholtz equations inside and outside +
dielectric boundary conditions

Boundary element method

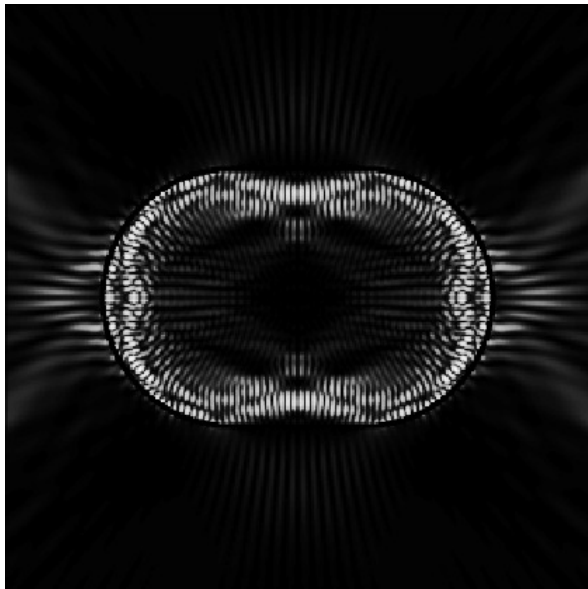
$$\psi_{int}(\vec{r}) = \oint ds \mu_{int}(s) G_{int}(\vec{r} - \vec{r}_s)$$

$$\psi_{ext}(\vec{r}) = \oint ds \mu_{ext}(s) G_{ext}(\vec{r} - \vec{r}_s)$$

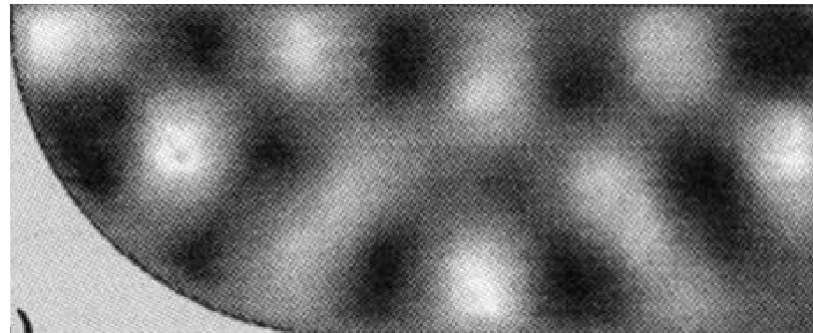


II Wave simulations

Open cavity

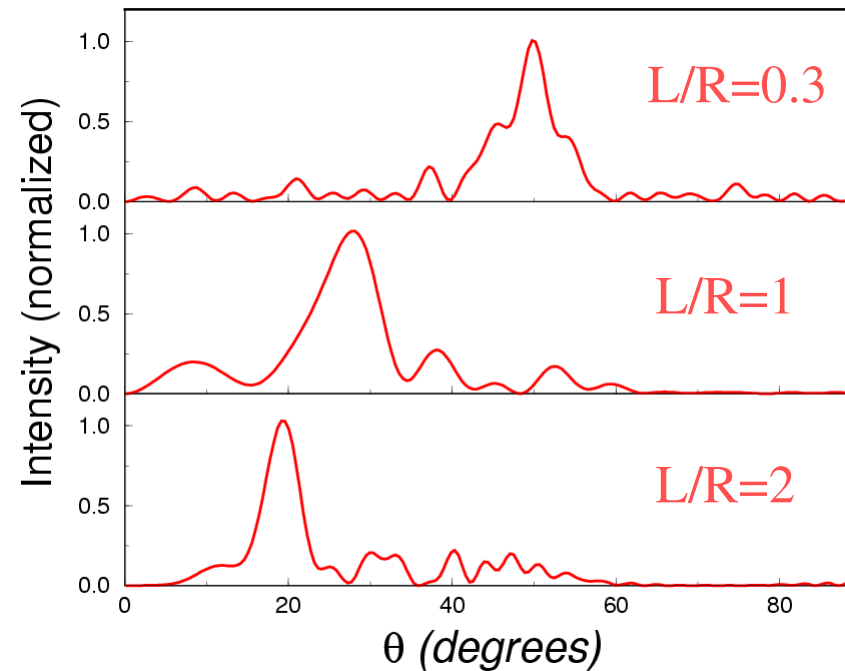
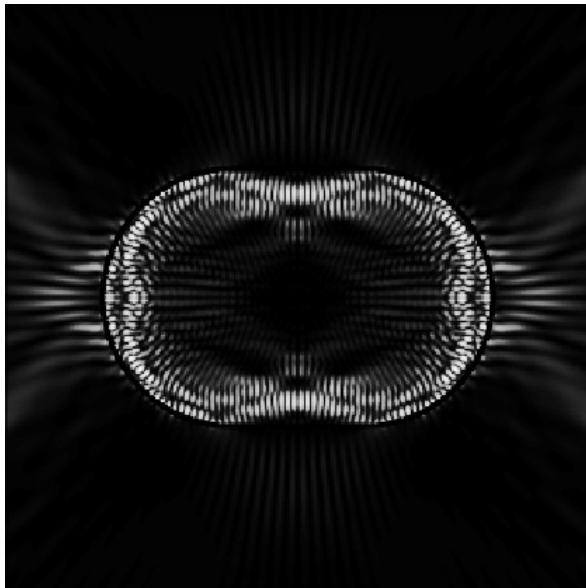


Closed cavity



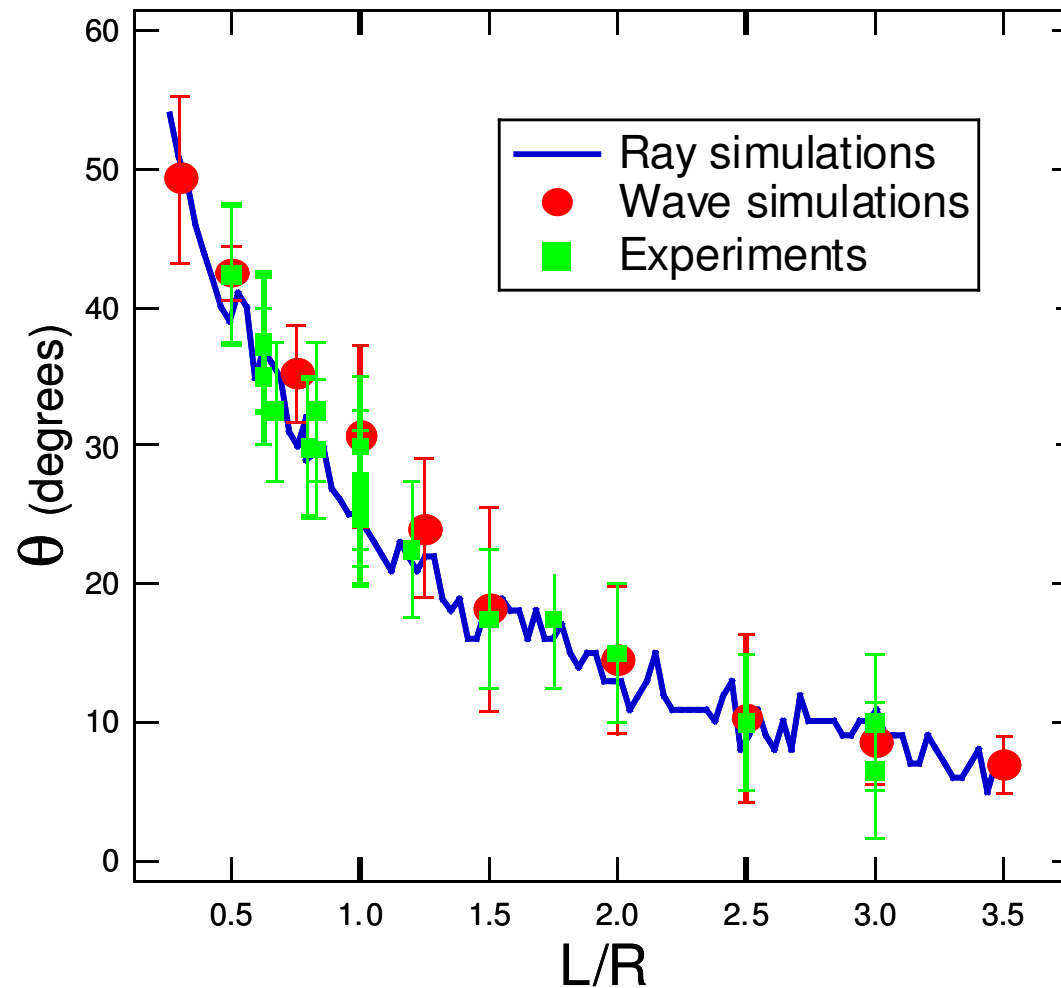
II Wave simulations

Far-field pattern



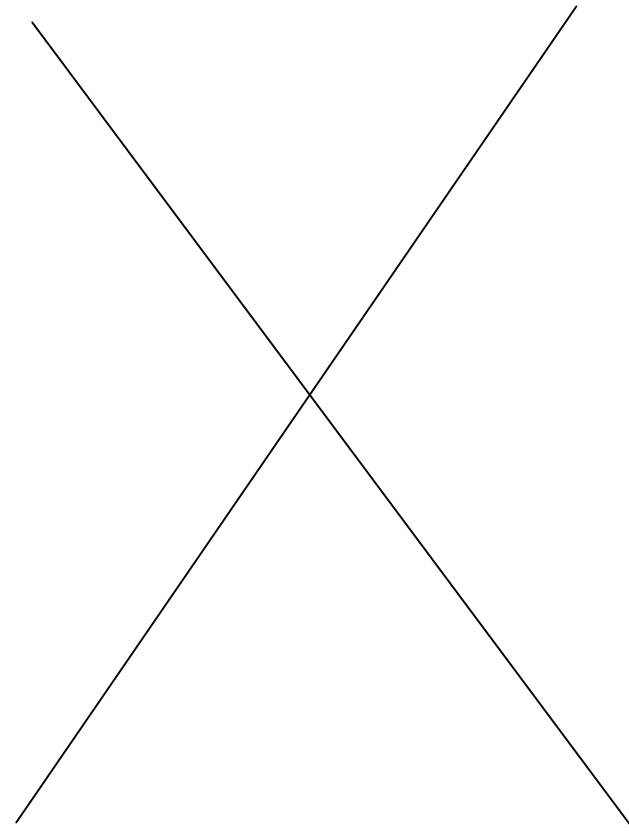
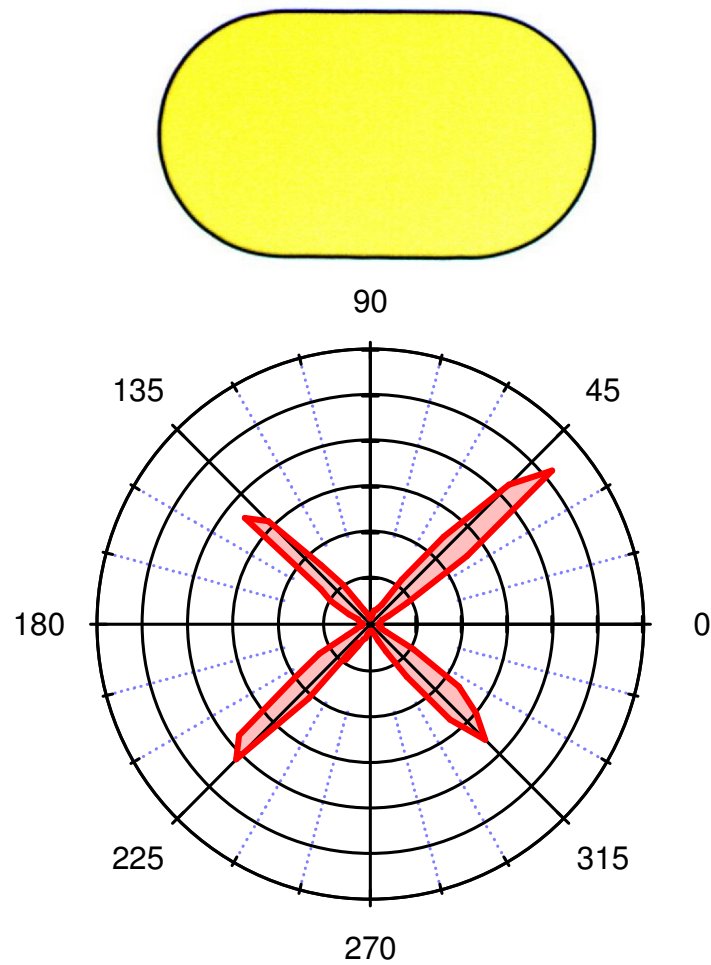
Individual well confined wavefunctions

II Refractive escape: Rays or waves ?

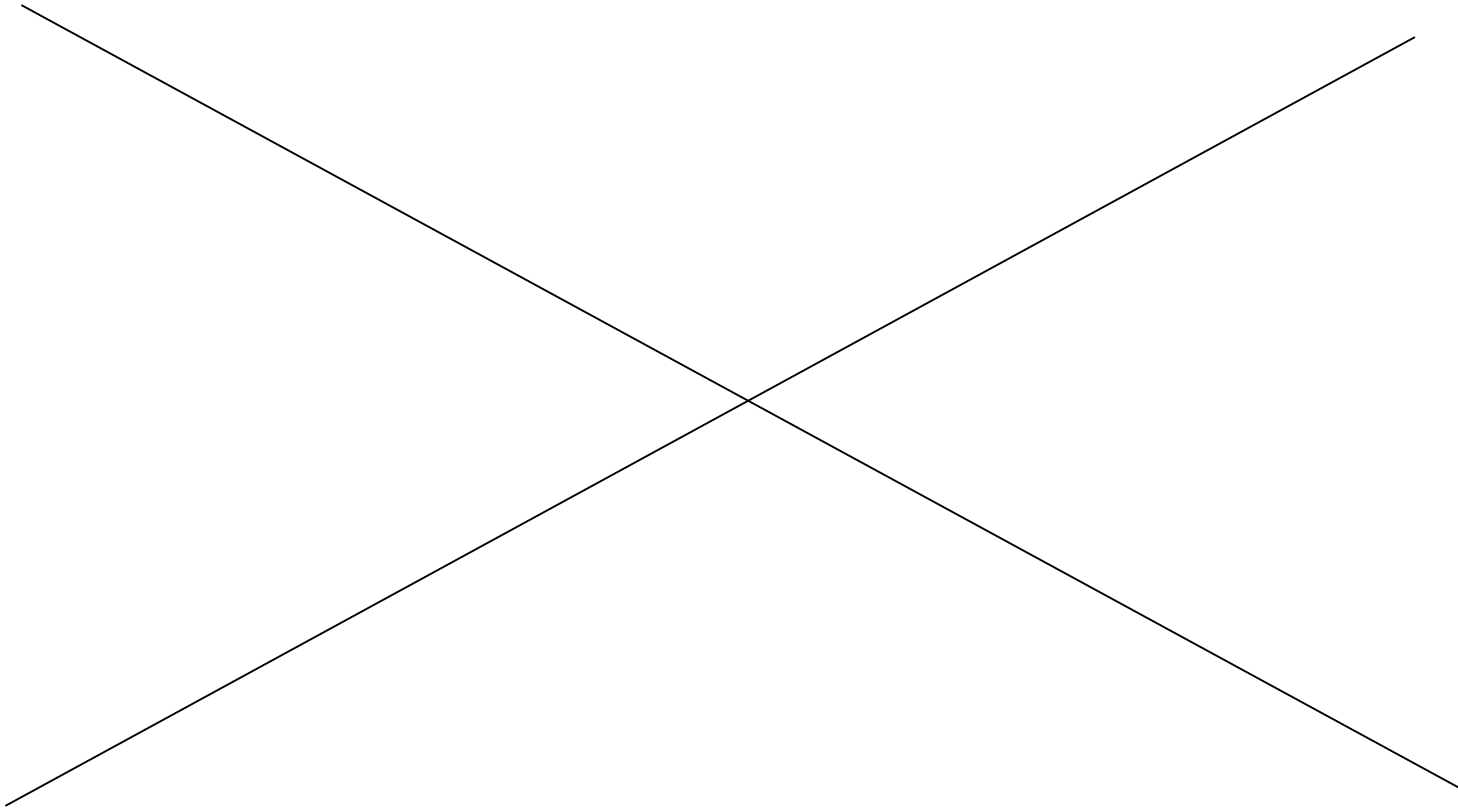


*PRA 75
033806
(2007)*

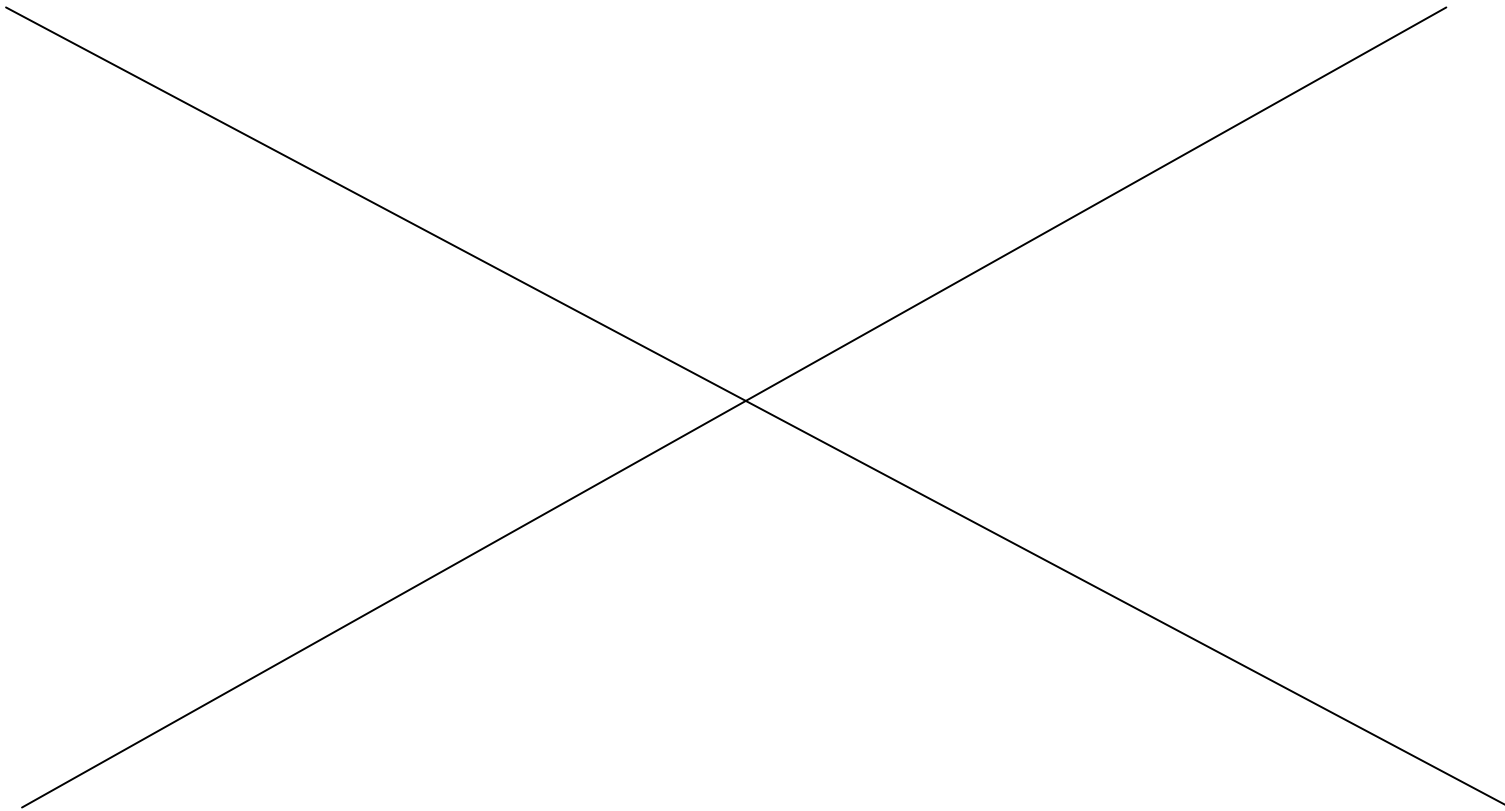
II Consequences of the lens model



II Polygons



II Polygons



Outline

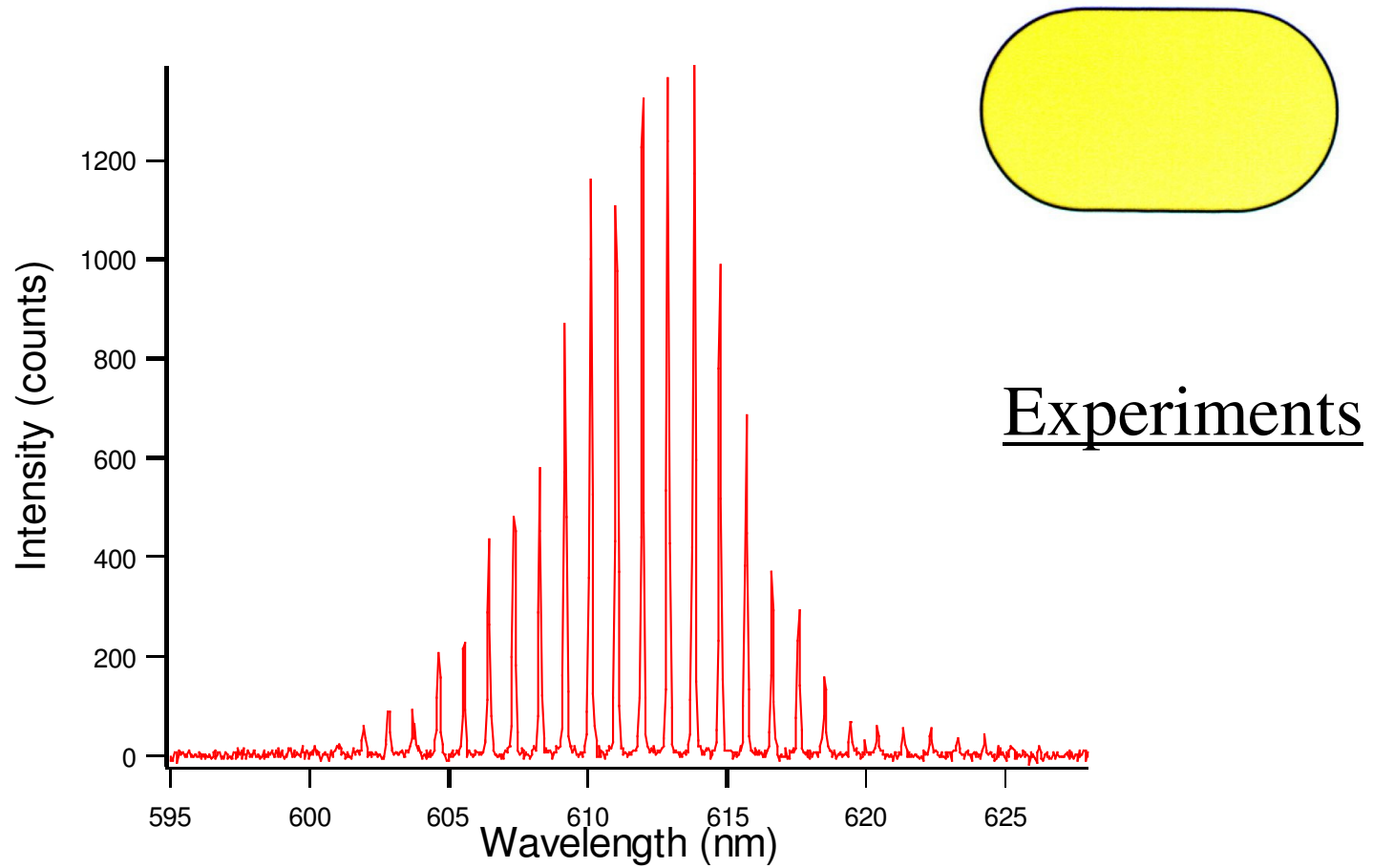
I Micro-lasers and wave chaos

II Existing tools (*what we can do*)

- Wavefunctions
- Directions of emission
- [Spectra](#)

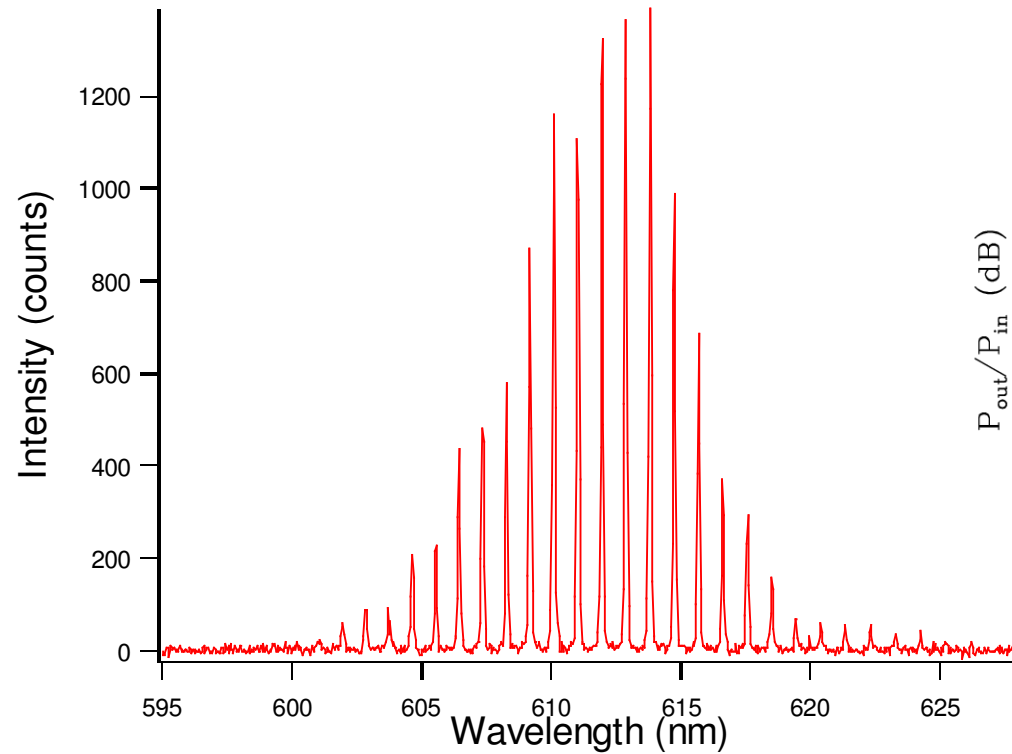
III Open questions

II Spectra

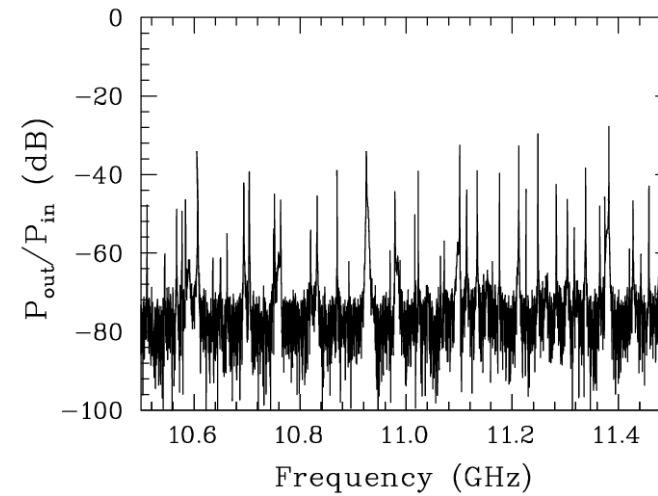


II Spectra

Open cavity



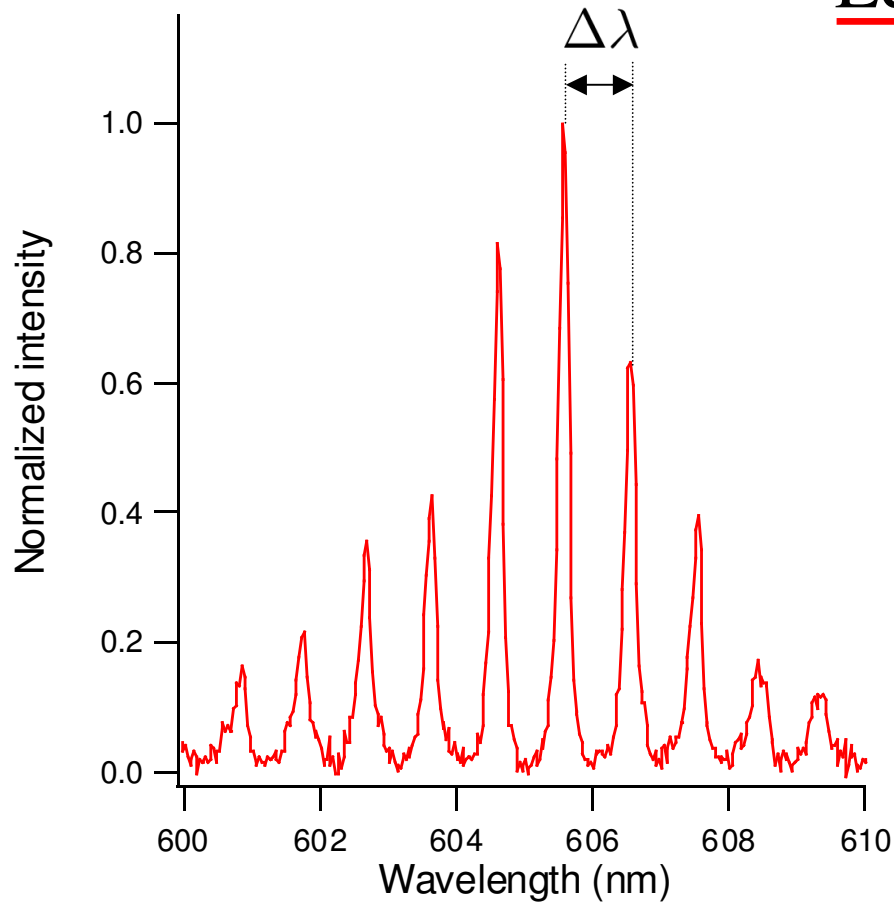
Closed cavity



Alt et al., PRE 60, 2851-2857 (1999)

II Spectra: periodic orbit

Length L of the periodic orbit ?



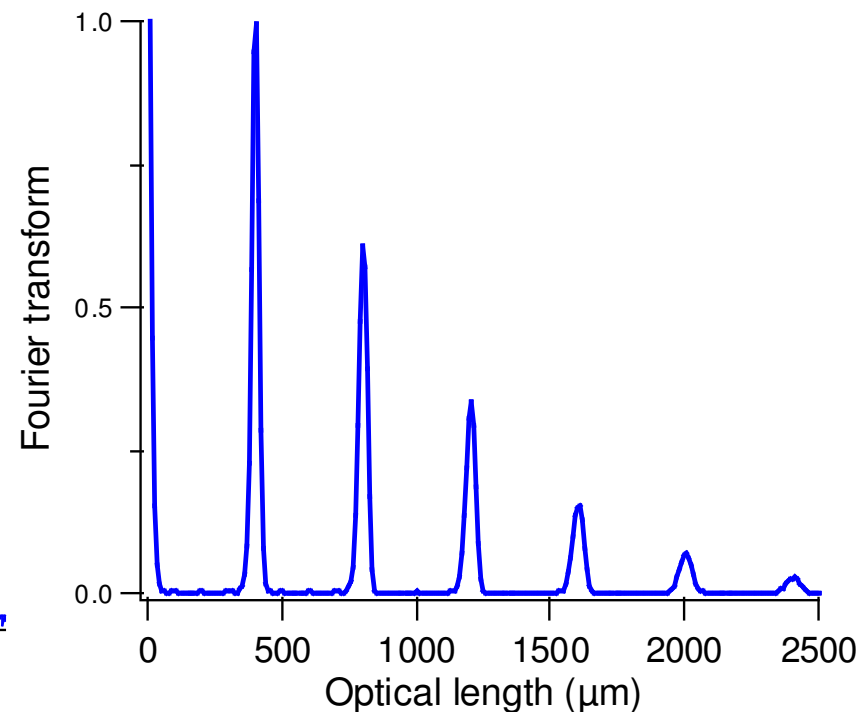
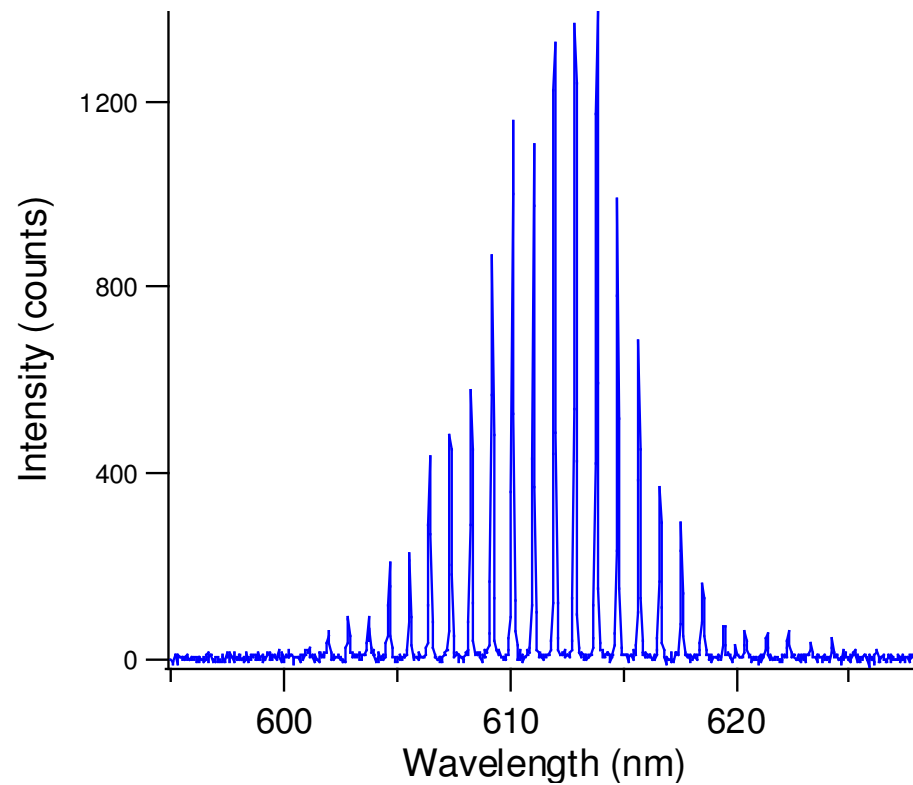
$$\frac{2\pi}{\lambda} n L = 2\pi m$$



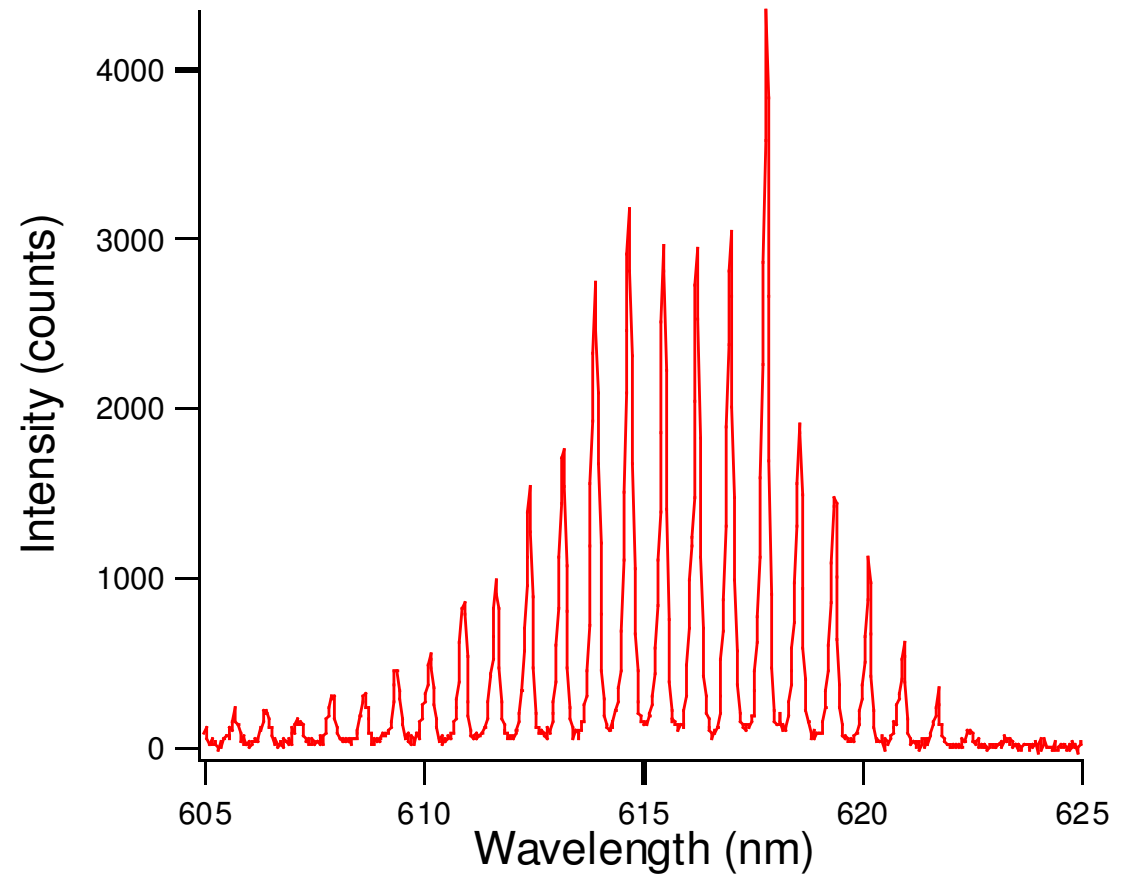
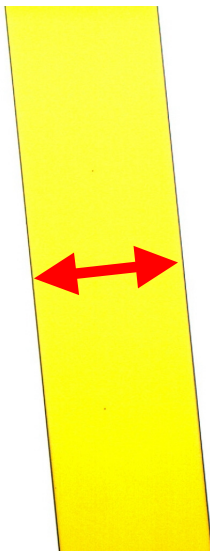
$$\frac{\Delta\lambda}{\lambda^2} n L = 1$$

II Spectra: periodic orbit

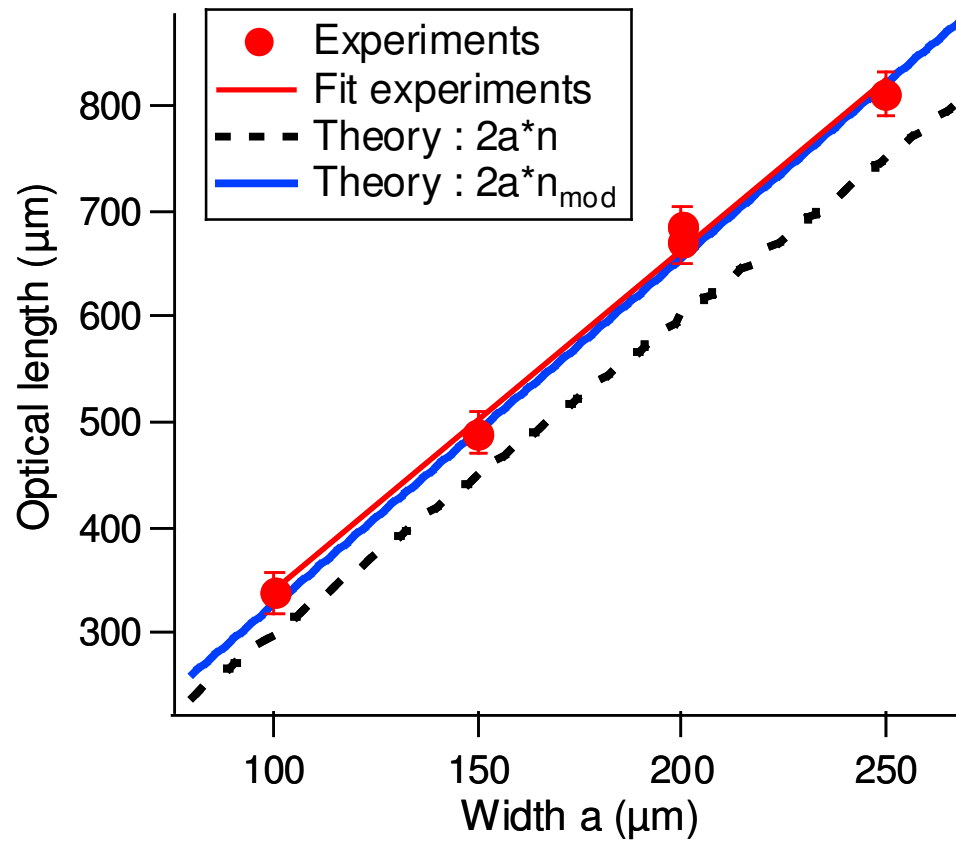
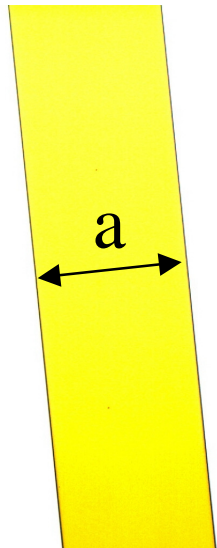
Fourier transform



II Test: Fabry-Perot resonator



II Spectra: periodic orbit

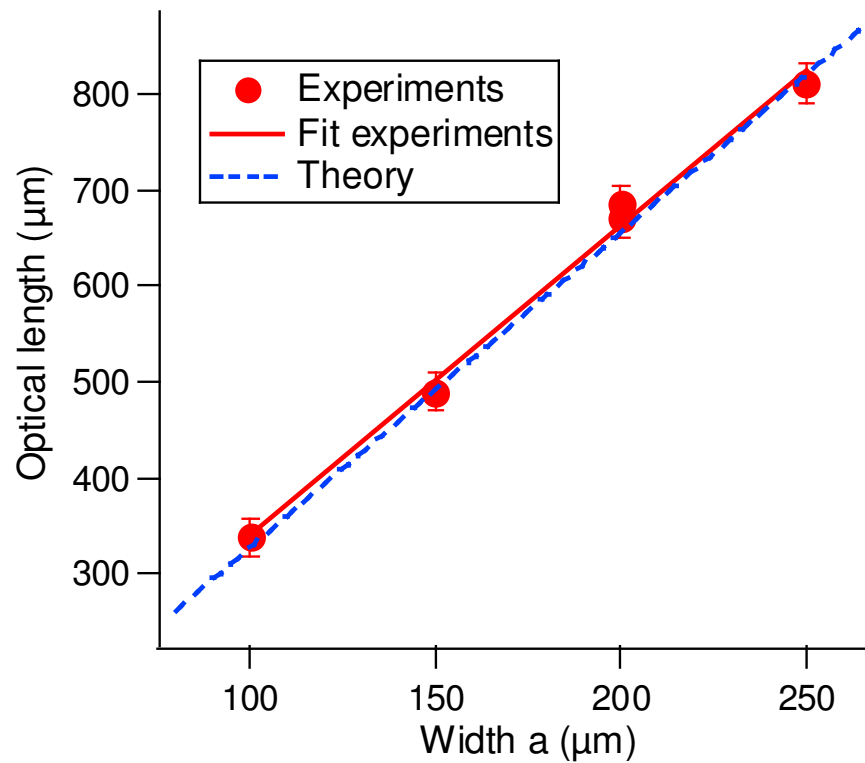
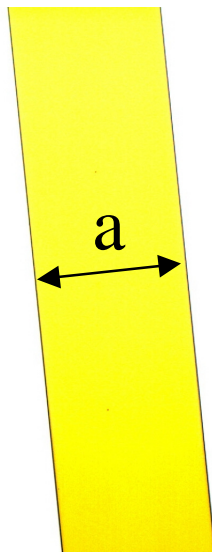


n (with group
velocity correction)
= 1.64
↑
Inferred
independently

Without any adjusted parameter

II Spectra: periodic orbit

Direct measure of the geometrical length

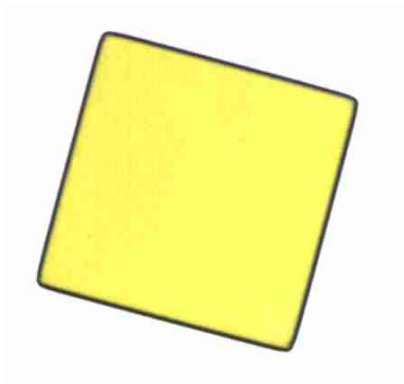


$$n = \underline{1.64}$$

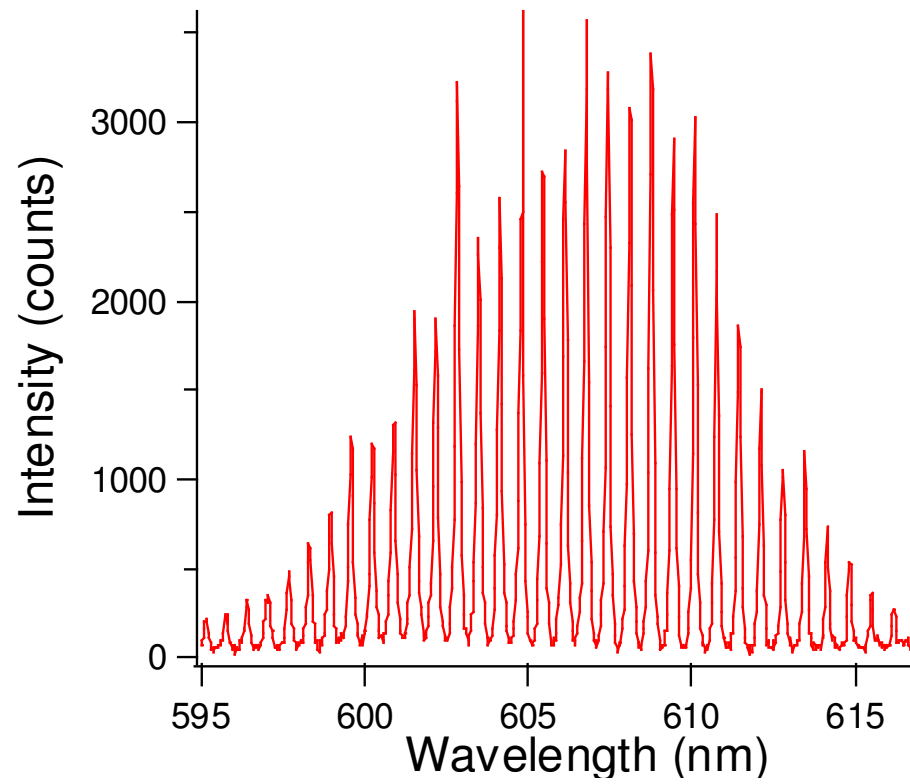
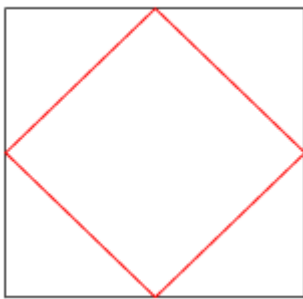
Without any adjusted parameter

*PRA 76 023830
(2007)*

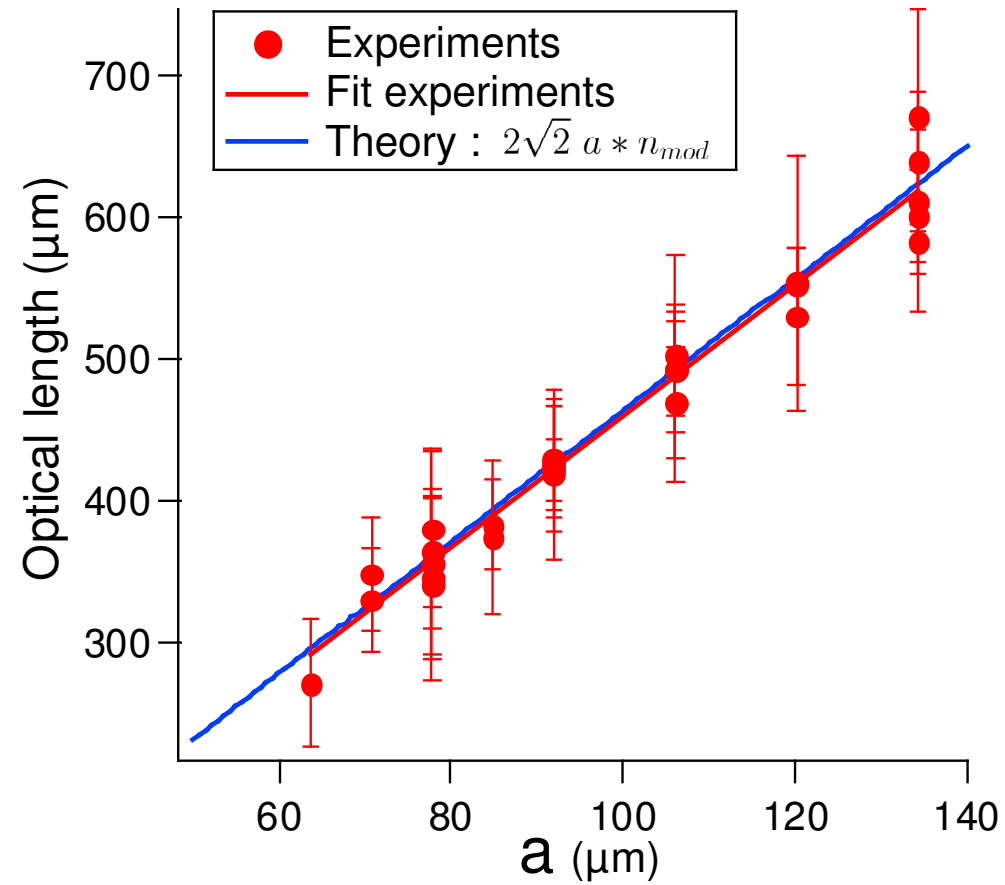
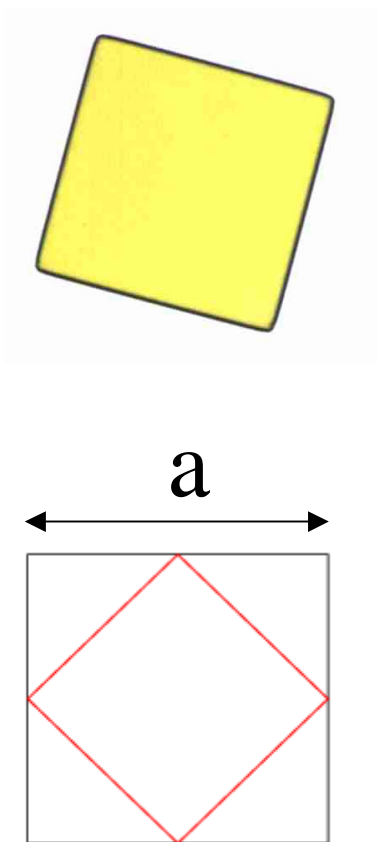
II Spectra: square



Diamond



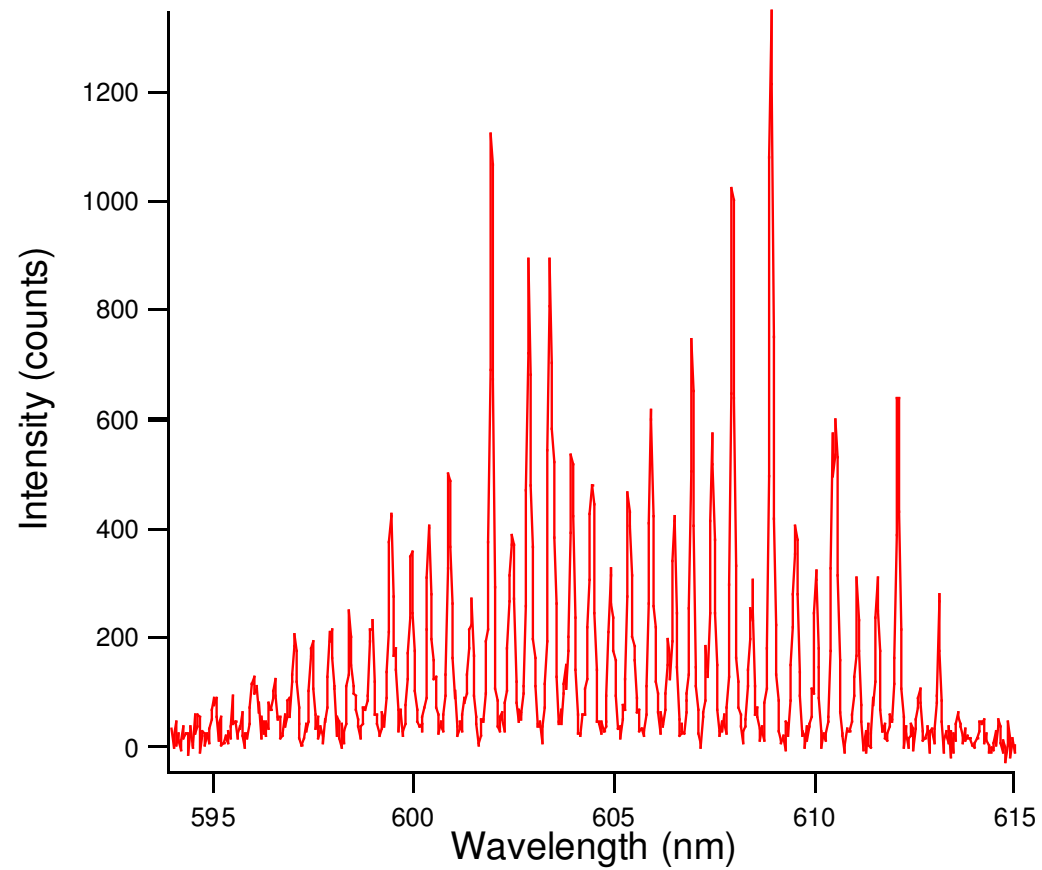
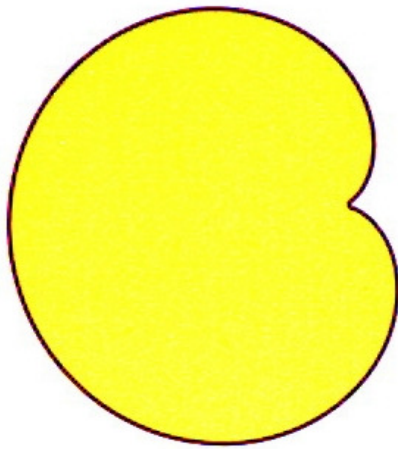
II Spectra: square



Without any adjusted parameter

II Spectra: periodic orbits

Cardioid



Outline

I Micro-lasers and wave chaos

II Existing tools (*what we can do*)

III Open questions

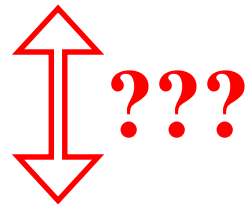
- Prediction of the dominant periodic orbit
- Resonances & lasing modes
- Diffraction on a dielectric corner

III Spectra: trace formula

Density of states

$$d(k) \propto \sum_m \frac{\text{Im } k_m}{(k - \text{Re } k_m)^2 + (\text{Im } k_m)^2} \quad \underline{\text{Wave physics}}$$

Integrable
Chaotic



Semi-classical limit $k \rightarrow \infty$

$$d(k) \propto \sum_p \mathbf{r}_p C_p \cos(\mathbf{n}kL_p + \varphi_p) \quad \underline{\text{Classical physics}}$$

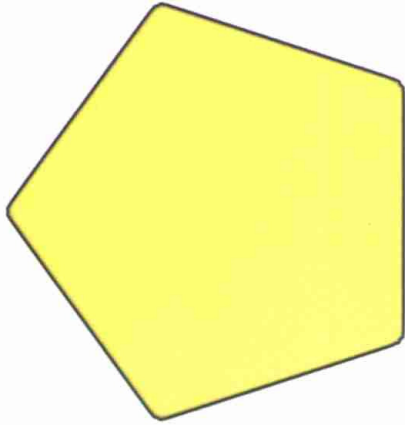


Proved for Fabry-Perot and disk

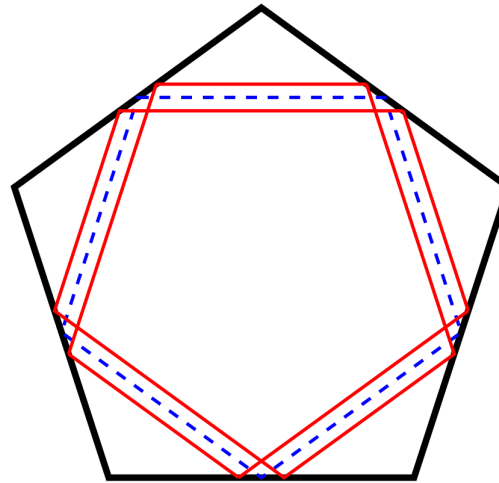
III Spectra and trace formula (a)

Periodic orbit: isolated or in family ?

Odd number of sides



Photography

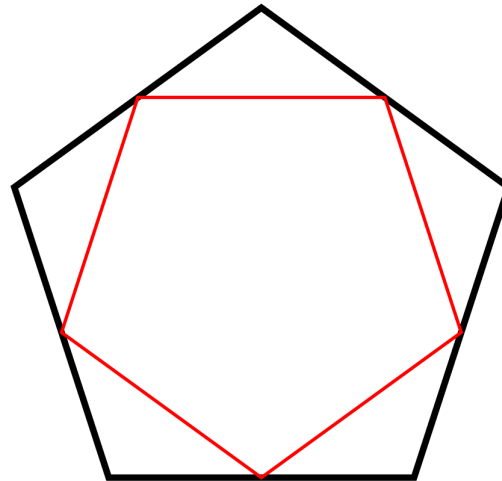
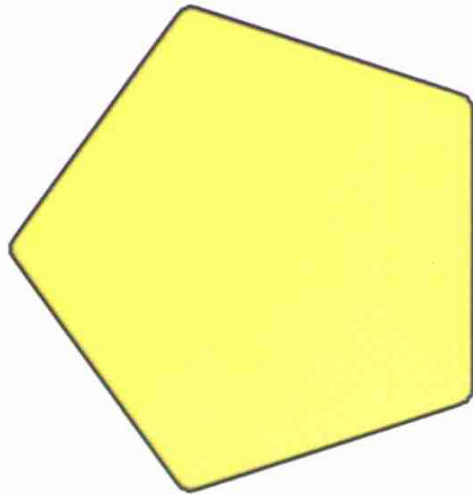


..... Isolated
 ————— In family

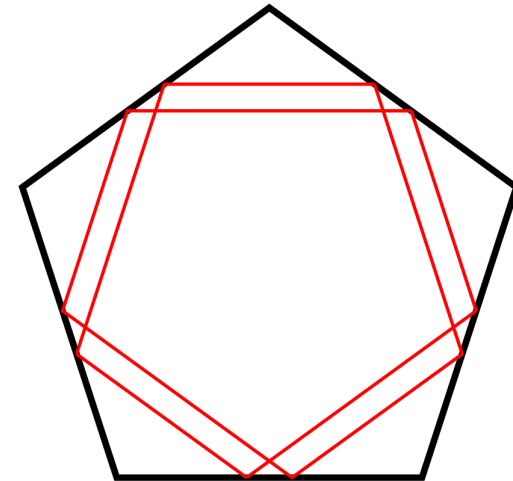
III Spectra and trace formula (a)

Isolated

In family

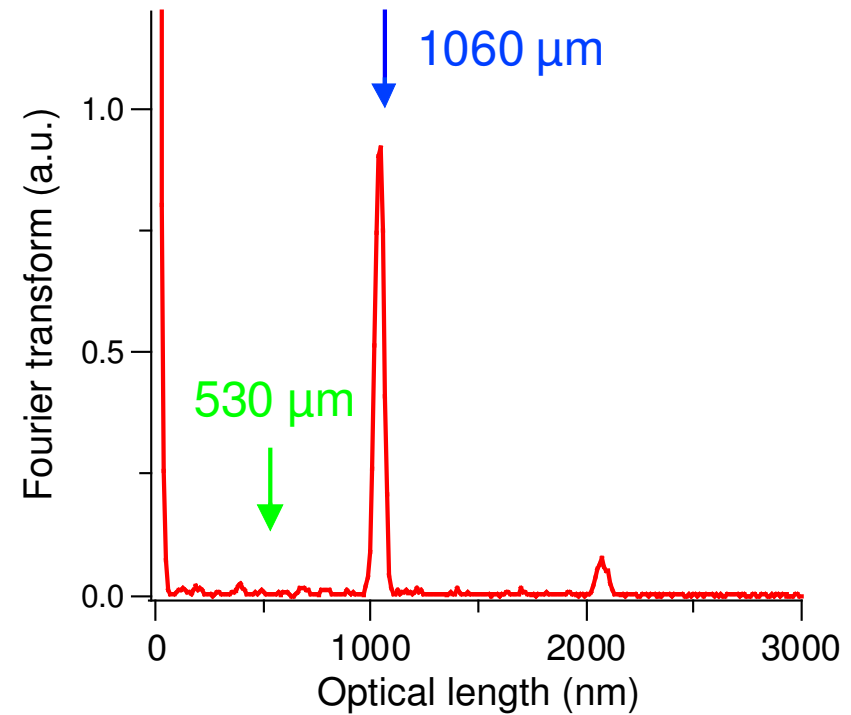
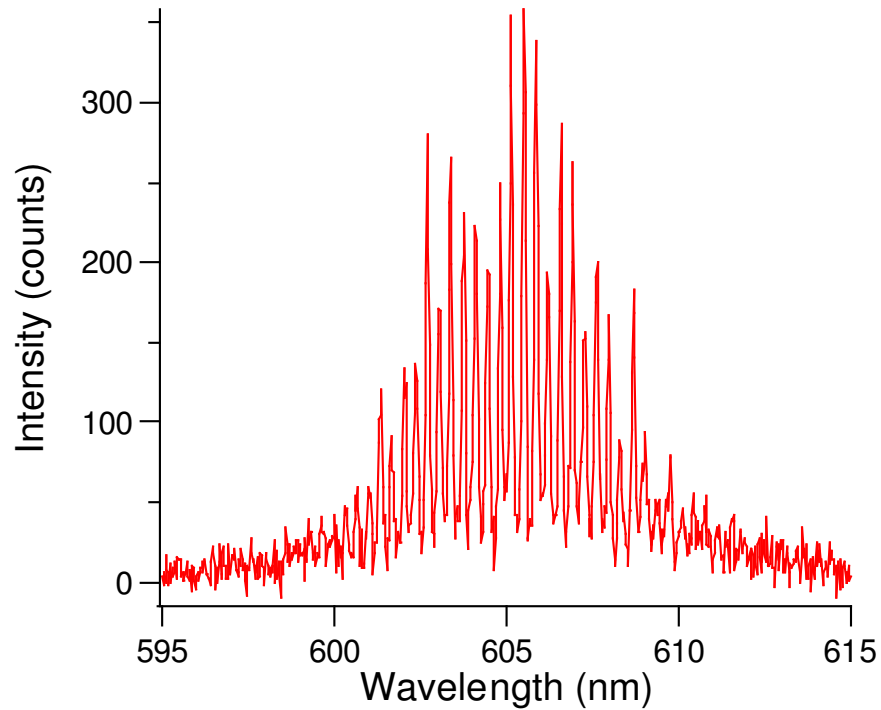


Single
pentagon



Double
pentagon

III Spectra and trace formula (a)

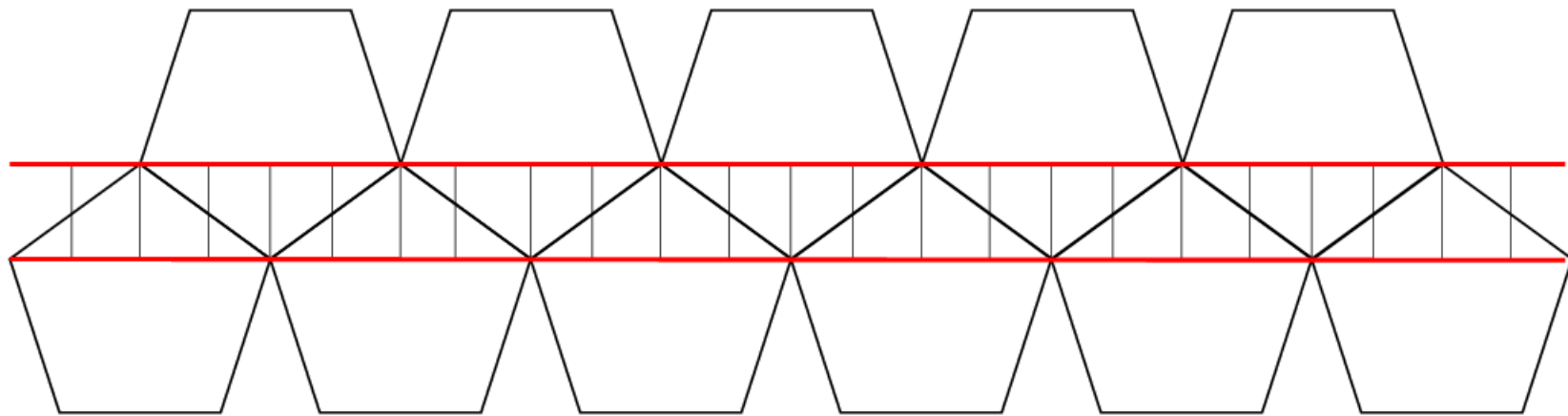


$$a = 80 \mu\text{m}$$

Single pentagon $\Rightarrow 5 a \cos\left(\frac{\pi}{5}\right) n_{full} = 530 \mu\text{m}$

Double pentagon $\Rightarrow 10 a \cos\left(\frac{\pi}{5}\right) n_{full} = 1060 \mu\text{m}$

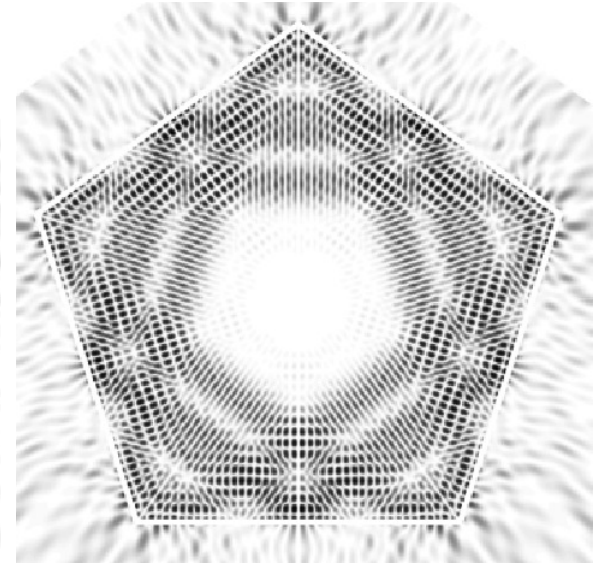
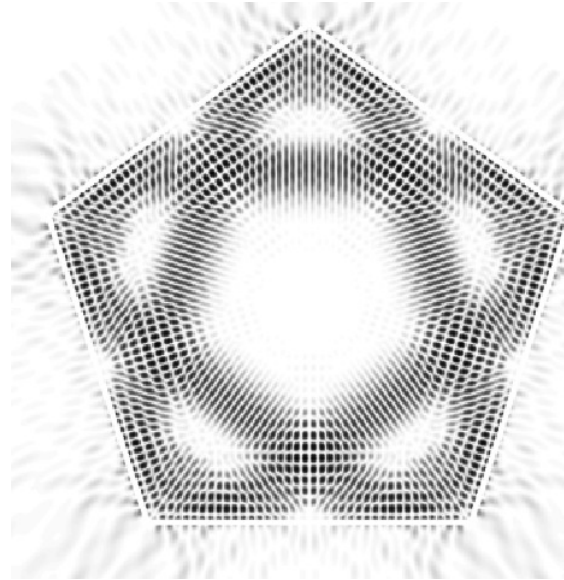
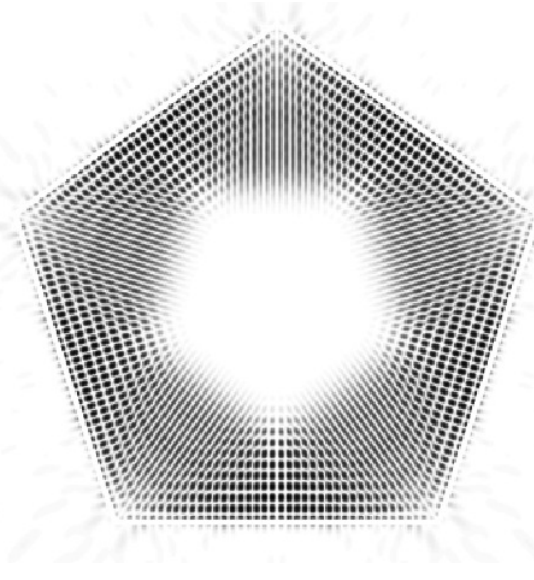
III [Superscars]



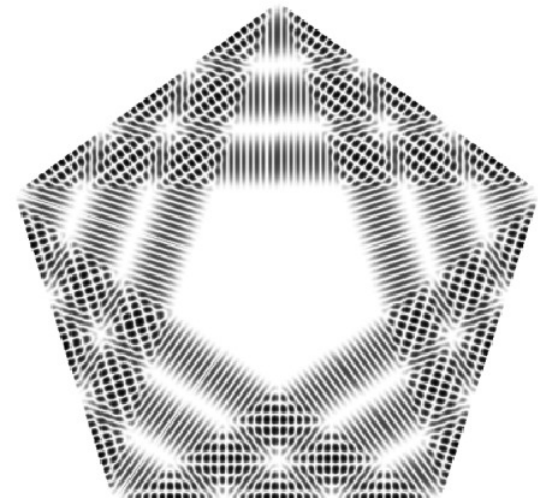
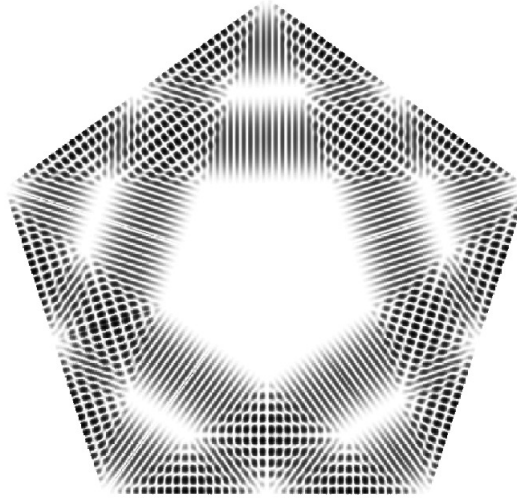
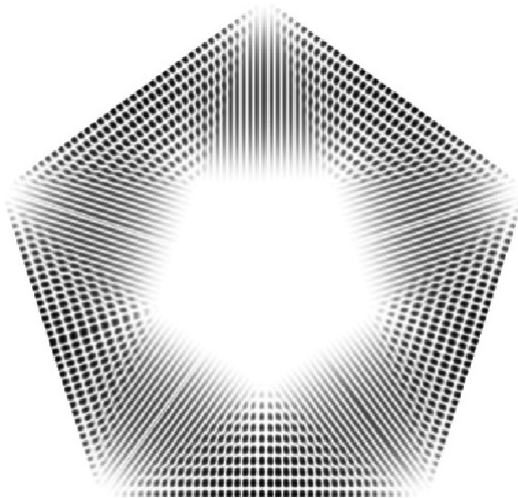
Superscar model \Rightarrow no light in the center

Numerical simulations

43

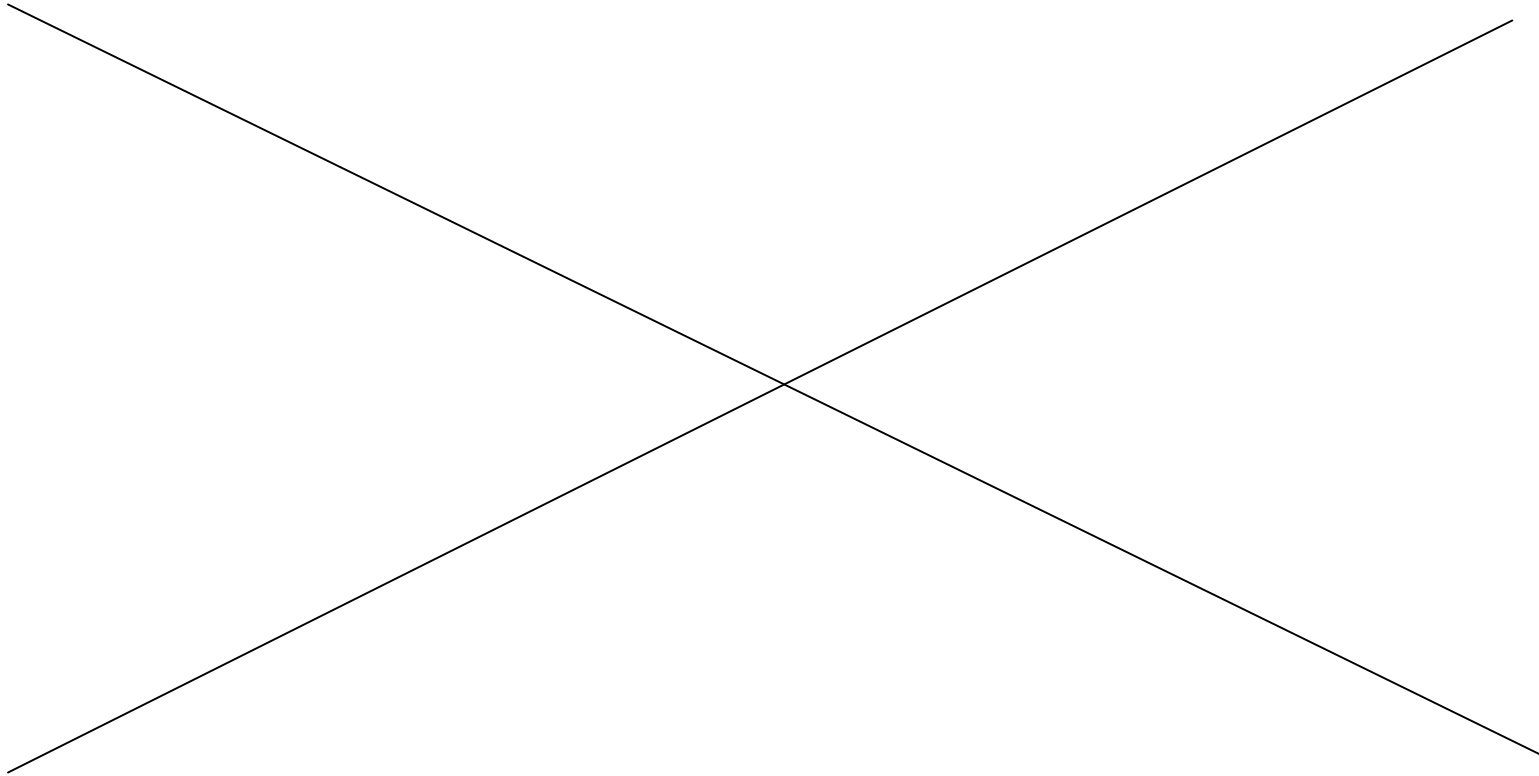


Superscar model

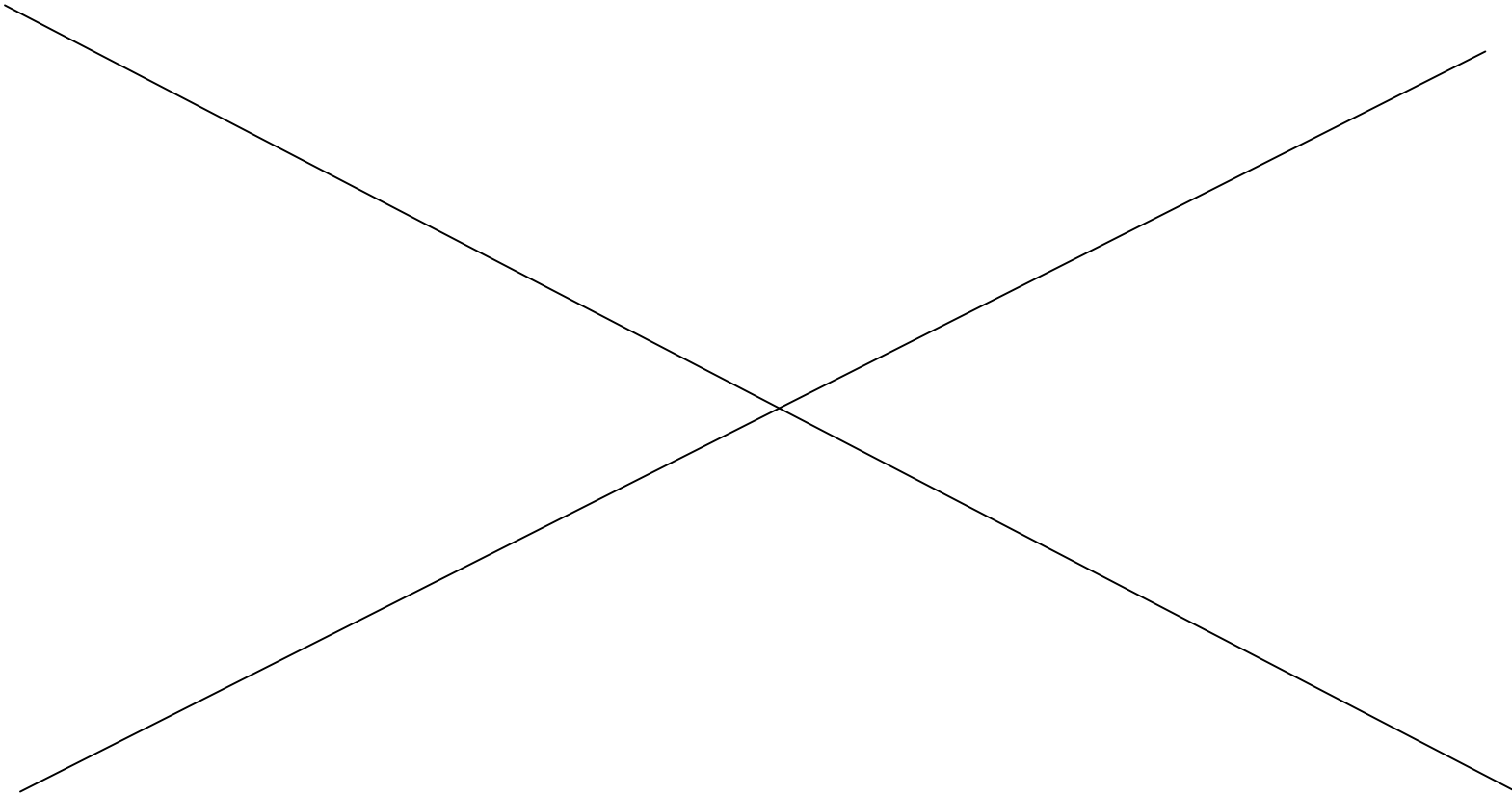


III Spectra and trace formula (b)

Change of dominant periodic orbit with a parameter



III Spectra (b): experiments



III Trace formula: summary

$$d(k) \propto \sum_p \mathbf{r}_p C_p \cos(\mathbf{n}kL_p + \varphi_p)$$

Semi-classical limit $k \rightarrow \infty$

- Proved for Fabry-Perot and disk
- Evidenced by experiments and numerical simulations

How to see the sub-dominant periodic orbits ?

Outline

I Micro-lasers and wave chaos

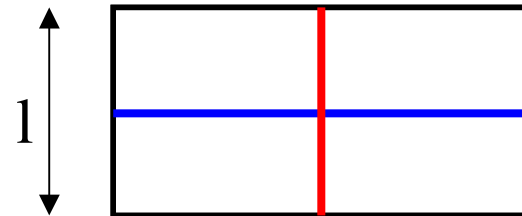
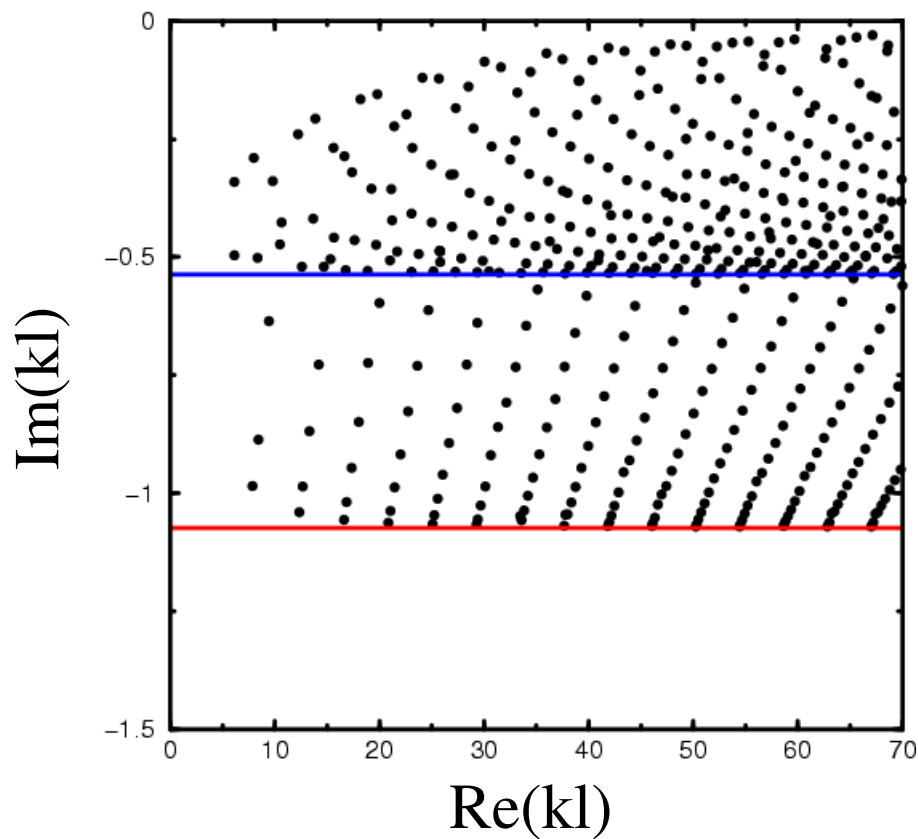
II Existing tools (*what we can do*)

III Open questions

- Prediction of the dominant periodic orbit
- Resonances & lasing modes
- Diffraction on a dielectric corner

III Lasing and losses

Rectangle

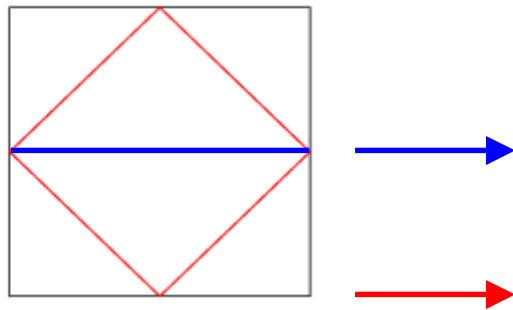


$$r^2 e^{iknL} = 1$$



$$|r|^2 e^{-k_i n L} = 1$$

III Lasing and losses



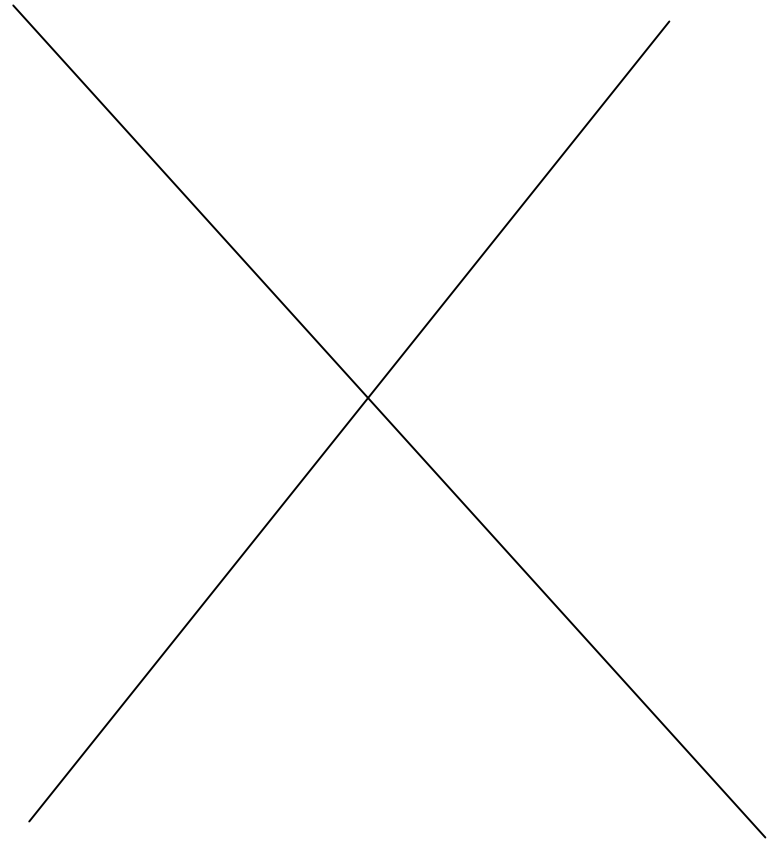
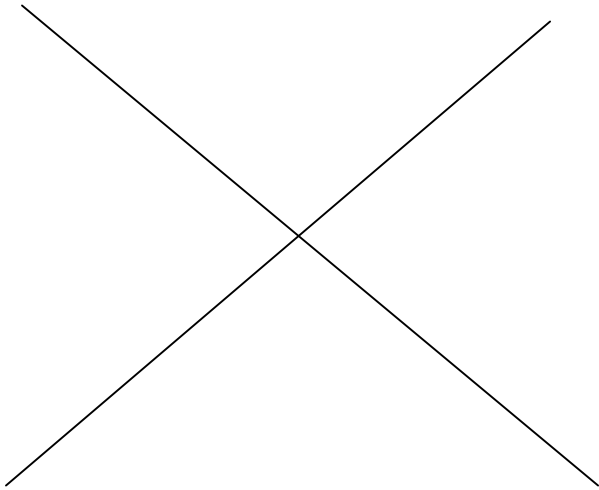
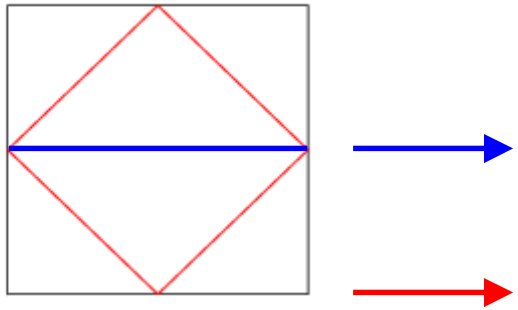
Question

- Diamond disappears when Fabry-Perot appears.

or

- Diamond and FP coexist, but the out-coupling of FP is too big to see diamond.

III Lasing and losses



III Lasing and losses

For a periodic orbit

Laser threshold \longleftrightarrow Losses *Experimental evidences*

Questions

- Which connexion with trace formula (c_p) ?
- Which meaning for resonances ?

III Models for micro-lasers

Laser effect → To fill the resonances with photons
(*second quantization*)

Resonances are not orthogonal

III Models for micro-lasers

- Helmholtz equations with n complex

$$(\Delta + n^2 k^2) \psi = 0 \quad n = n_0 - i\gamma$$

Nosich et al. JOSAA **25** 2884 (2008)

- Maxwell-Bloch equations

$$\ddot{E}^+ = \frac{1}{\epsilon(\mathbf{x})} \nabla^2 E^+ - \frac{4\pi}{\epsilon(\mathbf{x})} \ddot{P}^+ \quad \text{Türeci, Stone et al. Nonlinearity } \mathbf{22} \text{ C1 (2009)}$$

$$\dot{P}^+ = - (i\omega_a + \gamma_{\perp}) P^+ + \frac{g^2}{i\hbar} E^+ D \quad \text{Harayama et al. PRL } \mathbf{82} \text{ 3803 (1999)}$$

$$\dot{D} = \gamma_{\parallel} (D_0 - D) - \frac{2}{i\hbar} \left(E^+ (P^+)^* - P^+ (E^+)^* \right)$$

Outline

I Micro-lasers and wave chaos

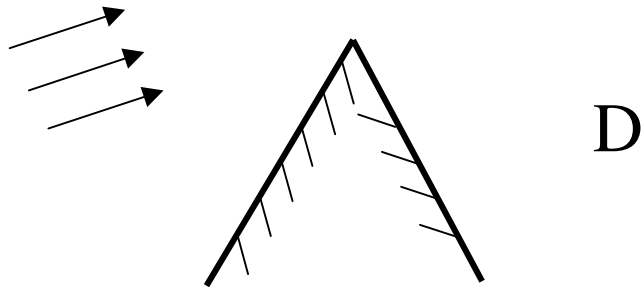
II Existing tools (*what we can do*)

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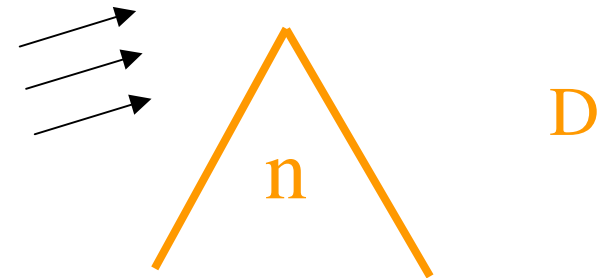
III Diffraction

Metallic



Sommerfeld (1896)

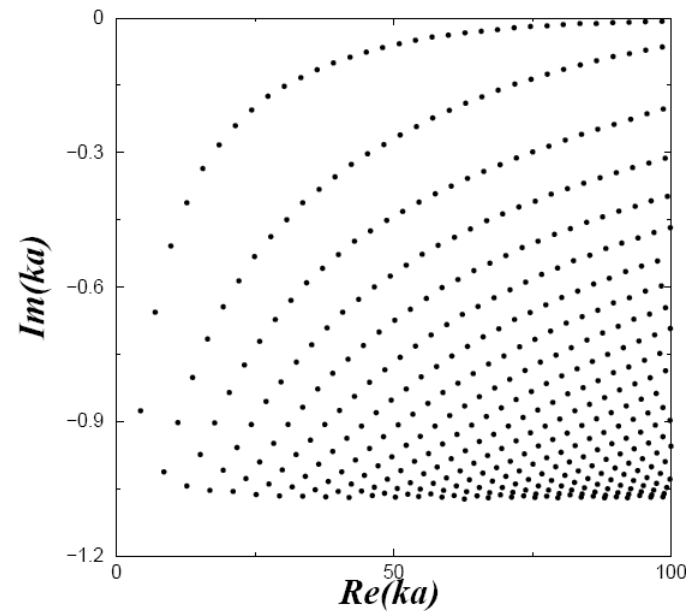
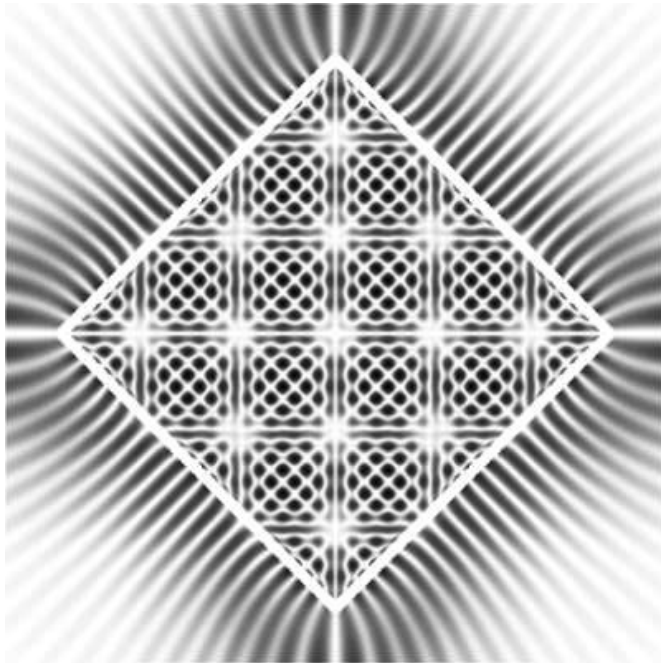
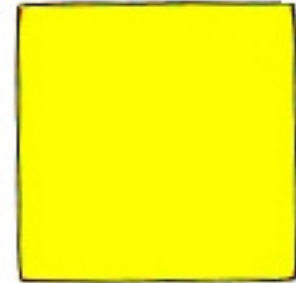
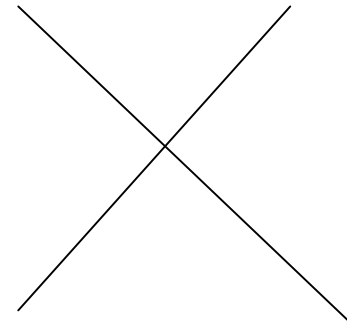
Dielectric



???

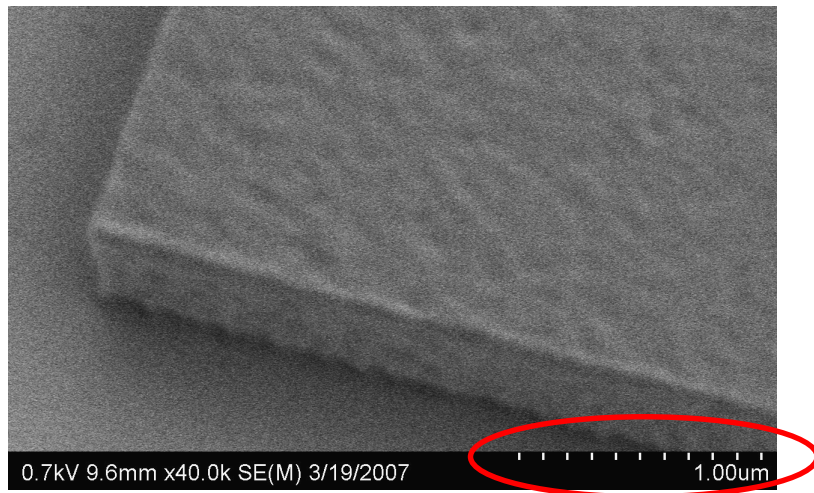
III Diffraction

Square: [not](#) (yet) integrable

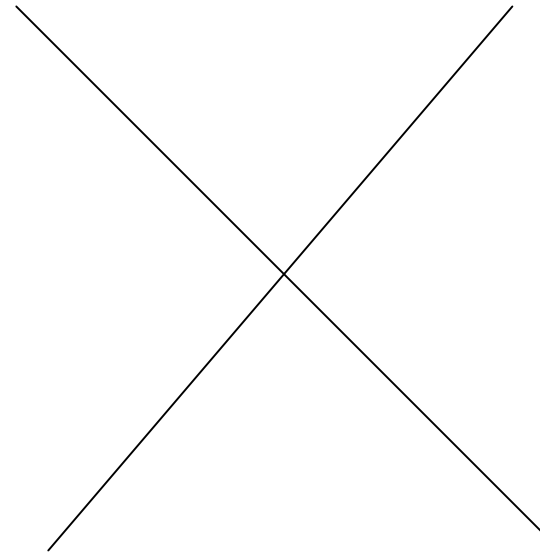


*Rémy
Dubertrand,
Thesis.*

III Diffraction



Collaboration with C. Ulysse (LPN)



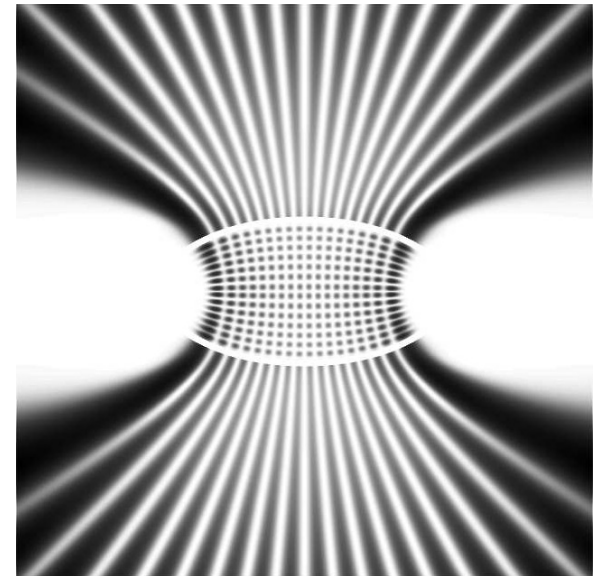
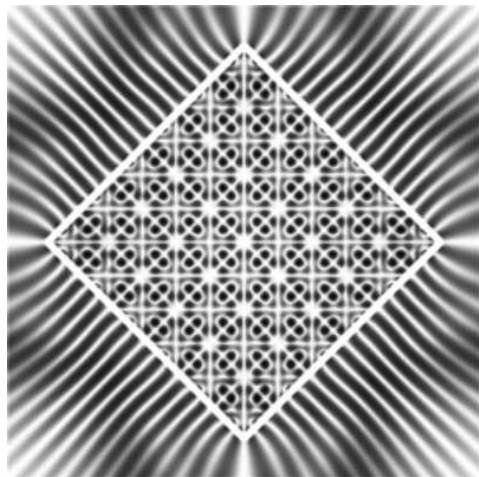
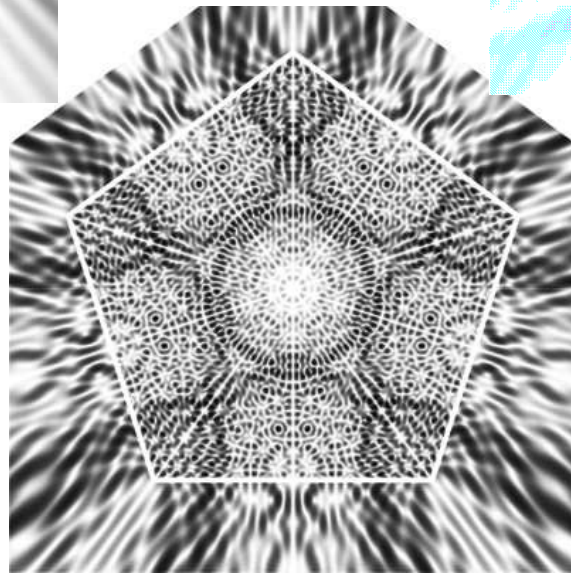
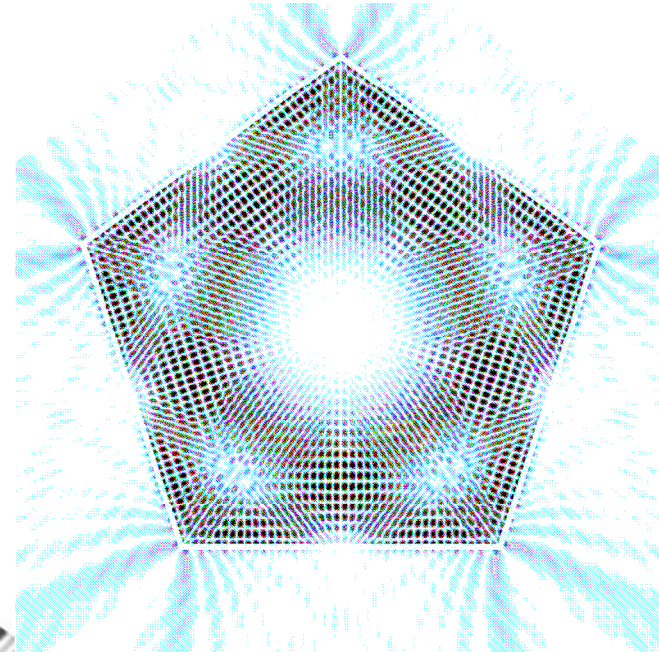
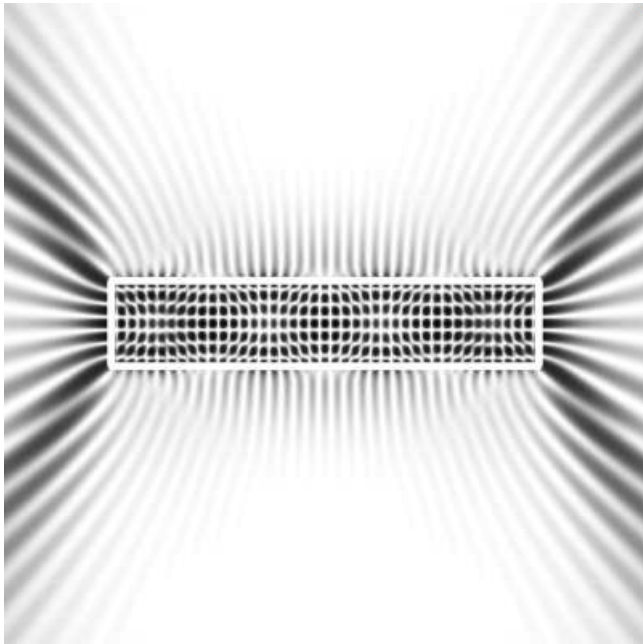
Summary & Perspectives

Organic micro-lasers

- Well-behaved microlasers → | Practical applications (*sensors*)
| Open billiards
- Existing tools for wave chaos → | Far-field patterns
| Periodic orbits
| *Losses*

Open questions

- Trace formula for dielectric billiards
- Resonances & lasing modes
- Diffraction on a dielectric corner



C. Schmit &
R. Dubertrand