

# Two vanishing theorems for holomorphic vector bundles of mixed sign

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**Summary.** We give (tiny) generalizations of vanishing theorems of Kodaira and Kobayashi for vector bundles over compact complex manifolds. Both yield cohomology vanishing for some vector bundles of mixed sign curvature. Their proof relies on heat kernel estimates.

**Keywords.** Vanishing theorems – heat kernel – holomorphic vector bundle – semi-positive curvature – mixed sign curvature

## Introduction

Let  $X$  be a compact complex analytic manifold of dimension  $n$  endowed with a hermitian metric  $\omega$ , and  $L$  be a holomorphic hermitian line bundle over  $X$ . We denote by  $ic(L)$  the curvature form of  $L$ , and by  $\alpha_1(x) \leq \dots \leq \alpha_n(x)$  its (ordered) eigenvalues with respect to  $\omega$  at a given point  $x$  of  $X$ . The  $\alpha_j$ 's are continuous functions on  $X$ , but they need not to be  $C^\infty$ . The first aim of this note is to prove the following theorem generalizing the Kodaira (coarse) vanishing theorem :

**Theorem 1** *For some  $q = 1, \dots, n$ , we suppose that  $L$  has at least  $n - q + 1$  nonnegative eigenvalues and moreover that the function  $\alpha_q^{-6n}$  is integrable over  $X$ . Then, for any holomorphic vector bundle  $E$  over  $X$ , and for any  $i \geq q$  the following vanishing*

$$H^i(X, E \otimes L^k) = 0$$

*holds as soon as  $k$  is sufficiently large.*

An easy consequence is the following

**Corollary 1** *On a Kähler manifold, a semi-positive line bundle such that  $\int_X \alpha_1^{-6n} < +\infty$  is ample.*

Theorem 1 is some kind of a vanishing theorem for vector bundles whose curvature can have arbitrarily large negative eigenvalues on a very small subset of  $X$ . An effective, or *precise* version of it should be much more powerful. Such a precise version would have to be quite different from the vanishing theorem of Ancona and Gaveau [1]. For instance, if  $X$  is assumed to be Kähler, and  $L$  satisfies the conditions of theorem 1 for some  $q$ , choose  $E$  in order that  $\text{ic}(E)_x < -(\lambda_1/16)\omega_x \otimes \text{Id}_{T_x^*X}$  at a point  $x$  where  $c(L)_x = 0$  (here  $\lambda_1$  is the first eigenvalue of the ordinary laplacian on functions on  $X$ , see [1]); then Theorem 1 yields the asymptotic vanishing of the cohomology in degree at least  $q$  although it is not likely that  $E \otimes L^k$  satisfies the assumptions of the main theorem of [1] for any  $k \geq 0$ .

Corollary 1 is, up to the author's knowledge, the first attempt in characterizing ampleness with a weaker condition than definite positivity (except for the case of isolated degeneracy points). Of course, the integral condition is very restrictive, and it is clear that such a case cannot appear in low dimension, as  $\alpha_1$  is a root of a polynomial of degree  $n$  with  $C^\infty$  coefficients. It is also clear that one cannot go much further, as the degree of the restriction of an ample line bundle must be positive on any curve. If one could drop the Kähler assumption on  $X$ , Corollary 1 could be viewed as a generalization of the projectivity criterion of Kodaira. In our case, this aspect of Corollary 1 is much weaker than [8].

Our method for proving Theorem 1 is simply a rationalization of the one we explained in our recent paper [4] under a strict positivity hypothesis. In [4] the vanishing of the cohomology was obtained by bounding its dimension by some quantity of the form  $Ck^n e^{-\alpha_0 k^\varepsilon}$ , thus smaller than one for a sufficiently large  $k$ . This bound was derived from the heat kernel estimate (1a) of [4]. Here we avoid the strict positivity assumption by replacing the uniform convergence to zero of the heat kernel by the Lebesgue bounded convergence theorem. The exponent  $6n$  is then shown to be the smallest integer we can afford in order to apply this Lebesgue convergence argument.

The second section is concerned with the proof of a precise vanishing theorem in the spirit of S. Kobayashi [7]. We say that  $X$  is special if it admits a metric  $\omega$  such that  $\partial\bar{\partial}\omega = 0$ . Given a holomorphic hermitian vector bundle  $E$ , we define in section 2 a tensor  $A_q$  which depends on the curvatures of  $E$ ,  $K_X$  and on the torsion of the metric  $\omega$ . It has the particularity to test simultaneously the positivity of  $E$  on  $(0, q)$ - and  $(n, q)$ -forms, and therefore allows mixed sign curvature on  $E$  when determining its sign. For instance,  $A_0$  only depends on  $E$  through the trace (with respect to  $\omega$ ) of its curvature, and does not depend on  $K_X$ . However, we must point out that, when  $q$  increases, estimates on  $A_q$  are much more difficult to obtain. In case  $X$  is Kähler and  $E$  is Hermite-Einstein, it is easier to compute. Our second main result is

**Theorem 2** *Let  $X$  be a compact complex special manifold, and  $E$  a holomorphic vector bundle on  $X$ . We suppose that  $A_q$  is semi-positive on  $X$ , then*

- (i)  $\dim H^q(X, E) \leq \binom{n}{q} \text{rk } E$  and the harmonic representatives of the cohomology are parallel ; moreover
- (ii)  $H^q(X, E) = 0$  if  $A_q$  is positive definite at some point.

This theorem follows directly from the standard Bochner technique and constructions of Demailly [6] and Bismut [3] which allow us to generalize the results

of [7] to  $q$ -forms over a non Kähler manifold. We also briefly discuss this theorem : we give an example, and an asymptotic version. As the Lebesgue convergence argument of the first section generalizes other results in [4], we conclude this paper with one of them.

## 1. The proof of Theorem 1

Let us first introduce some notations :  $X$  is a compact complex analytic manifold of dimension  $n$ , endowed with a hermitian metric  $\omega$  and associated volume element  $dV = \omega^n/n!$ .  $E$  (resp.  $L$ ) is a hermitian holomorphic vector bundle of rank  $r$  (resp. 1). We shall write  $E(k) = E \otimes L^k$ . We call  $\bar{\square}_k^q = (\bar{\partial}^* + \bar{\partial})^2$  the  $\bar{\partial}$ -laplacian (with respect to the given metrics) acting on  $(0, q)$ -forms with values in  $E(k)$ . The associated *heat kernel*  $e_k^q(t, x, y)$  is the smooth kernel of the operator  $e^{-\frac{2t}{k} \bar{\square}_k^q}$ . It enjoys the following expansion : for  $j = 0, 1, \dots$ , let  $\mu_j^k$  be the eigenvalues of  $\bar{\square}_k^q$  (counted with multiplicities), and  $(\psi_j)_j$  be an orthonormal  $L^2$  basis of eigenforms associated to the  $\mu_j^k$ 's, then

$$e_k^q(t, x, y) = \sum_{j \geq 0} e^{-\frac{2t}{k} \mu_j^k} \psi_j(x) \otimes \psi_j^*(y).$$

We refer to [4] for details. For a multiindex  $J$ , we put  $\bar{\alpha}_J = \sum_{j \notin J} \alpha_j - \sum_{j \in J} \alpha_j$ . We set also the function  $\frac{\alpha}{\sinh \alpha t}$  to be  $\frac{1}{t}$  when  $\alpha = 0$  and we define the function  $e_0^q(t, x)$  on  $\mathbb{R}_+^* \times X$  to be  $r(4\pi)^{-n} (\sum_{|J|=q} e^{t\bar{\alpha}_J}) \prod_{j=1}^n \frac{\alpha_j(x)}{\sinh \alpha_j(x)t}$ . Now, our main tool is

**Proposition 1** *When  $k \rightarrow +\infty$ , the following equivalent holds :*

$$k^{-n} e_k^q(k^\varepsilon, x, x) \sim e_0^q(k^\varepsilon, x) \quad (1)$$

*uniformly with respect to  $x \in X$  for any given  $\varepsilon \in ]0, 1/6[$ .*

*Proof.* A careful reading of [5] shows that in order to get the larger possible  $\varepsilon$  in the statement of Theorem 2 (which implies our Proposition 1) the only requirement is that the localization principle (Proposition 1 of [5]) is valid for  $t = k^\varepsilon$  on a ball sufficiently small for the validity of the approximations required. For any small  $\eta > 0$  the radius  $r_k = k^{-\frac{1}{3}-\eta}$  satisfies this condition. It is then easy to check that Theorem 2 of [5] holds for  $\varepsilon = \frac{1}{6} - 2\eta$ .

We now fix  $q$  and suppose that  $L$  satisfies the assumptions of Theorem 1. The first step of the proof of Theorem 1 is the same as for Theorem 1.3 of [4]. As this paper is more technical, we shall be a little more precise. For some real number  $\alpha$  we put  $\alpha = \alpha^+ - \alpha^-$ . Now, it is easy to check that

$$\frac{\alpha e^{-\alpha t}}{\sinh \alpha t} \leq e^{-2\alpha^+ t} \left( |\alpha| + \frac{1}{t} \right)$$

for any  $\alpha \in \mathbb{R}$  and  $t > 0$ . Because the  $\alpha_j$ 's are ordered, one has  $\bar{\alpha}_J \leq -\alpha_1 - \dots - \alpha_i + \alpha_{i+1} + \dots + \alpha_n$  for any set of indices  $J$  of cardinal  $i$ . Thus Proposition 1 together with the nonnegativeness of  $\alpha_q$  yield

$$e_k^i(k^\varepsilon, x, x) \leq Ck^n(\alpha_q + k^{-\varepsilon})e^{-2\alpha_q k^\varepsilon} \quad (2)$$

for some constant  $C$  only depending on  $\varepsilon$ . As  $\alpha_q > 0$  outside a closed set  $Z$  of measure 0, the function  $e_k^i(k^\varepsilon, x, x)$  converges pointwisely to 0 on  $X \setminus Z$ . Now, it is clear from the infinite sum expansion of the heat kernel (and the Hodge identification between cohomology and harmonic forms) that

$$\dim H^i(X, E(k)) \leq \int_X e_k^i(k^\varepsilon, x, x) dV. \quad (3)$$

This last inequality reduces Theorem 1 to the following lemma thanks to the Lebesgue bounded convergence theorem :

**Lemma 1** *For  $\varepsilon = \frac{2n}{12n+1}$  there exists a constant  $C$  such that*

$$e_k^i(k^\varepsilon, x, x) \leq C(\alpha_q(x)^{-6n} + 1)$$

for  $k \geq 1$  and  $i \geq q$ .

*Proof* . We shall denote by  $C$  any constant independant of  $x$  and  $k$ . We simply remark that the function  $u^{\frac{n}{\varepsilon} - \frac{1}{2}}e^{-u}$  is bounded on  $\mathbb{R}^+$ , thus

$$k^{n - \frac{\varepsilon}{2}}e^{-\alpha_q k^\varepsilon} \leq C\alpha_q^{-6n} \quad (4)$$

because  $\frac{n}{\varepsilon} - \frac{1}{2} = 6n$ . Now, on the subset of  $X$  where  $\alpha_q \leq k^{-\frac{\varepsilon}{2}}$  the inequalities (2) and (4) yield

$$\begin{aligned} e_k^i(k^\varepsilon, x, x) &\leq Ck^{n - \frac{\varepsilon}{2}}e^{-2\alpha_q k^\varepsilon} \\ &\leq C\alpha_q^{-6n}. \end{aligned} \quad (5)$$

But we have obvious inequalities outside this set :

$$\begin{aligned} e_k^i(k^\varepsilon, x, x) &\leq k^n e^{-k^{\frac{\varepsilon}{2}}} \\ &\leq C \end{aligned} \quad (6)$$

The bounds (5) and (6) yield the lemma, thus Theorem 1.

*Remark* . Theorem 1 already derives from (3) when the upper limit of the integral is strictly less than one, which allows a weaker growth condition on  $\alpha_q^{-1}$ . However, this doesn't seem to have a simple geometric interpretation.

*Proof of Corollary 1*. We suppose that  $L$  is endowed with a metric with semi-positive curvature satisfying the condition  $\int_X \alpha_1^{-6n} < +\infty$ . Due to the solution of the conjecture of Grauert-Riemenschneider ([6], [9])  $X$  must be Moishezon (this is in our case a direct consequence of Theorem 1 and Riemann-Roch). By a famous theorem of Moishezon, a Kähler manifold is Moishezon if and only if it is projective. Thus  $X$  is projective, and Theorem 1 applies with  $E$  any coherent sheaf. This is due to the existence of global syzygies for coherent sheaves on a projective manifold. This means that  $L$  is ample.

*Remark* . In order to drop the Kähler assumption on  $X$  in corollary 1, it would be sufficient to show the following : *If  $\mathcal{I}$  is the ideal sheaf of a point in  $X$ ,  $\mathcal{I}$  and*

$\mathcal{I}^2$  admit a global syzygy. For instance, we do not know if a point in a Moishezon manifold is a complete intersection.

## 2. A precise vanishing theorem for vector bundles

In this section,  $X$  is a special compact complex manifold. According to the definition of the introduction, this means that the metric  $\omega$  is  $\partial\bar{\partial}$ -closed.  $E$  is a hermitian holomorphic vector bundle. We define the commutator of two operators  $A, B$  on the bigraduated algebra of  $(p, q)$ -forms with values in  $E$  of (total) degree  $a, b$  to be  $[A, B] = AB - (-1)^{ab}BA$ . We also identify a form and the operator it defines by wedge product, and call  $A$  the adjoint of  $\omega$ . Then, if we call  $\tau = [A, \partial\omega]$  the torsion, and  $T_\omega = -[\partial\omega, (\partial\omega)^*]$  the ‘‘torsion operator’’ associated to the metric  $\omega$ , and  $D = D' + D''$  the Chern connection on  $E$ , we have the following Bochner formula for the associated antiholomorphic laplacian (cf. Demailly [6], p. 217) :

$$\bar{\square} = \square_\tau + [ic(E), A] + T_\omega \quad (7)$$

where  $\bar{\square} = [D'', (D'')^*]$  and  $\square_\tau = [D' + \tau, (D' + \tau)^*]$ . As a  $\mathcal{C}^\infty(0, q)$ -form  $u$  with values in  $E$  defines a  $(n, q)$ -form  $\tilde{u}$  with values in  $K_X^* \otimes E$  (this correspondance is a holomorphic isometry) we can sum up the two versions of Eq. (7) to get, if we put  $A_q u = -Aic(E)u + ic(E)A\tilde{u} - ic(K_X)A \otimes id_E \tilde{u} + T_\omega u + T_\omega \tilde{u}$  :

$$2\bar{\square}u = \square_\tau u + \square_\tau \tilde{u} + A_q u. \quad (8)$$

Now, Bismut [3] has proved that, if  $\omega$  is special,  $D' + \tau$  is the part of type  $(1, 0)$  of a Chern connection on  $E$  for a different holomorphic structure on  $X$ . Thus, the construction of Demailly ([6], Proposition 3.6) which shows that the operator  $\square u + \square \tilde{u}$  can be interpreted as a rough laplacian associated to some riemannian connection  $\nabla$  on the  $\mathcal{C}^\infty$  vector bundle  $\wedge^{0,q} T^* X \otimes E$  can be applied to  $\square_\tau$  (namely, the connection  $\nabla$  has its  $(0, 1)$  part identified with  $D' + \tau$  while its  $(1, 0)$  part acts on  $(0, q)$ -forms similarly to  $(D' + \tau)^*$  on  $(n, q)$ -forms, the construction of Bismut allows us to find a holomorphic structure on  $X$  such that this  $(1, 0)$  part is the  $\bar{\partial}$  operator). We get an identity

$$2\bar{\square}u = \nabla^* \nabla u + A_q u. \quad (9)$$

Theorem 2 is a straightforward consequence of Eq. (9). Following the lines of [4], the heat kernel proof goes like this : if we call  $e(t, x, y)$  (resp.  $e_g(t, x, y)$ ) the heat kernel associated to  $2\bar{\square}$  (resp. to the de Rham laplacian on functions on  $X$ ) the Kato inequality yields

$$e(t, x, x) \leq \binom{n}{q} \text{rk } E e_g(t, x, x) \quad (10)$$

on the whole of  $X$ . Moreover, if  $A_q$  is bounded below by  $\alpha g \otimes h$  ( $\alpha > 0$ ) on an open set  $\Omega$  ( $g$  and  $h$  are the induced metrics on  $TX$  and  $E$  respectively), we get on  $\Omega$

$$e(t, x, x) \leq \binom{n}{q} \text{rk } E e^{-\alpha t} e_g(t, x, x). \quad (11)$$

Integrating Eq. (10) over  $X$ , and letting  $t \rightarrow +\infty$  yields the first assertion of (i) in Theorem 2, while considering both bounds (10) and (11) yields (ii). The fact that  $\bar{\square}$ -harmonic sections are parallel with respect to  $\nabla$  follows directly from (9). The reason why Eq. (9) is of a quite different nature from Eq. (7) is that a rough laplacian controls much more the global behaviour of sections than any positive operator such as  $\square_\tau$ . However, both equations coincide when  $q = 0$  (you can see it by simply adding  $\bar{\square}$  on both sides of (7) and remarking that the de Rham laplacian  $\Delta = \bar{\square} + \square_\tau$  is a rough laplacian when  $q = 0$ ). Before we come to some special cases, we note that, when  $X$  is Kähler, and  $q = 0$ , we have simply  $A_0 = -\text{tr}_\omega \text{ic}(E) \otimes \text{id}_{T^*X}$ . Thus theorem 2 asserts that some mean negativity is sufficient for the vanishing of global sections. When  $E$  is Hermite-Einstein with respect to  $\omega$  for a constant  $\lambda$ ,  $A_0 = -\lambda \text{id}_E \otimes \text{id}_{T^*X}$ . This shows that theorems 1, A and D in [7] are contained in Theorem 2. Now, our hope is that we can control the low degrees cohomology groups of  $E$  with the help of Theorem 2 when  $E$  has some mean negativity. If we note  $\Theta_E$  the endomorphism of  $\wedge^{0,q}T^*X \otimes E$  defined by  $\text{ic}(E)$ , a straightforward computation gives :

$$A_q = 2\Theta_E - \text{id}_{\wedge^{0,q}T^*X} \otimes \text{tr}_\omega \Theta_E - \text{ic}(K_X)A \otimes \text{id}_E \tilde{.}$$

It is already difficult to determine the sign of this tensor when  $q = 1$ . For example, when  $E = TX$ , it equals  $2\Theta_{TX}$ . As no tangent bundle is Nakano positive, Theorem 2 (ii) never applies to this situation but, as  $T\mathbb{P}^n$  is Nakano semi-positive, (i) bounds the dimension of  $H^{0,1}(\mathbb{P}^n, T\mathbb{P}^n)$  by the maximal rank of a trivial real analytic subbundle of  $T\mathbb{P}^n \otimes \bar{T}^*\mathbb{P}^n$  (which is much smaller than  $n^2$ ). In the case where  $E$  is a cotangent bundle, we have  $A_1 = 2(\Theta_{T^*X} - \Theta_{K_X} \otimes \text{id}_{T^*X})$ . It is known that the Griffiths negativity of  $T^*X$  yields the Griffiths positivity of the whole tensor. Unfortunately, Theorem 2 requires Nakano positivity. This shows that Theorem 2 is not a very powerful tool when  $q \neq 0$ . We conclude this section with the asymptotic version, which could be of more use in special situations.

**Corollary 2** *Let  $X$  be a special compact complex manifold,  $E$  (resp.  $L$ ) a holomorphic vector (resp. line) bundle over  $X$ . If the eigenvalues  $\alpha_1 \leq \dots \leq \alpha_n$  of  $\text{ic}(L)$  with respect to  $\omega$  verify  $\alpha_1 + \dots + \alpha_q > \alpha_{q+1} + \dots + \alpha_n$  for some  $q$ , the following vanishing*

$$H^q(X, E \otimes L^k) = 0$$

*holds as soon as  $k$  is sufficiently large.*

*Proof.* This is an immediate consequence of Theorem 2 as the part of the tensor  $A_q$  corresponding to  $L$  is diagonal with entries  $-\bar{\alpha}_J \geq \alpha_1 + \dots + \alpha_q - \alpha_{q+1} - \dots - \alpha_n$  (with the notations of section 1). Note that, when  $q > n/2$ , Corollary 2 is a consequence of Theorem 1 as our eigenvalues condition implies  $\alpha_q > 0$ . One can also prove it directly with the equivalent of Proposition 1, this proof shows that  $\omega$  needs not to be special because the extra torsion terms will be dominated by the curvature of  $L$ . Furthermore, one can verify that  $\bar{\alpha}_J \leq 0$  already yields the vanishing if the curvature of  $L$  vanishes nowhere on  $X$ , or even if it vanishes and satisfies the condition that  $\int_X (\sum_{j=1}^n |\alpha_j|)^{-6n} < +\infty$ .

### 3. Eigenvalue estimates

As a conclusion to this note, we state the result that was its original motivation. We come back to the situation of section 1 and we denote by  $\mu_1^k$  the first nonzero eigenvalue of  $\bar{\square}_k^0$  (acting on  $E(k)$ ) on  $X$ .

**Proposition 2** *If  $\alpha_1 \geq 0$  satisfies  $\int_X \alpha_1^{-6n} < +\infty$  we have when  $k \rightarrow +\infty$  :*

$$k^{-\frac{10n+1}{12n+1}} \mu_1^k \rightarrow +\infty.$$

*Proof* . This proposition follows directly from the method of the proof of Lemma 3.2 in [4] because it is shown there that  $e^{-2\mu_1^k k^{\varepsilon-1}} \leq \int_X e_k^1(k^\varepsilon, x, x) dV$  with the same  $\varepsilon$  as in Lemma 1. Taking the Log of this inequality yields the result. Of course, the same estimate holds for the first nonzero eigenvalue of  $\bar{\square}_k^q$  under the same condition for  $\alpha_q$ .

*Remark* . This gives some intermediate between Lemma 3.2 in [4] (or Theorem 1 in Bismut and Vasserot [2]) and the result of the last section in [2] on  $\mu_1^k$  when  $L$  is ample but endowed with a nonnecessarily positively curved metric. We can also obtain different growth conditions on  $\mu_1^k$  from other vanishing orders on  $\alpha_1$ , but always on a set of measure zero. However, this is still very far from the control on  $\mu_1^k$  when  $L$  is only assumed to be semi-positive everywhere and positive at one point conjectured by Siu (see [4] for references). This is yet the best possible result in this direction that we can derive from an equivalent for the heat kernel  $e_k^q$  because this equivalent does not depend on the geometry of the bundle  $E$ . Thus, in Proposition 2, we are not only bounding the first eigenvalue of  $\bar{\square}_k^0$  for  $L^k$  but for all the class of bundles  $E(k)$ , and one cannot expect to control this except if  $L$  itself is ample. Thus, any heat kernel insight in semi-positivity will require a higher order expansion, involving the geometry of the bundles  $L$ ,  $E$  and probably  $TX$ .

In order to show one geometric consequence of Proposition 2, let us recall from [4,5] that the *distortion function* associated to an ample hermitian line bundle  $L$  is defined by  $b_k(x) = |s_1(x)|^2 + \dots + |s_N(x)|^2$  where the family  $(s_j)$  is an orthonormal basis of  $H^0(X, L^k)$  for the  $L^2$  inner product. For  $k$  large, it is viewed as the distortion between the metric of  $L^k$  induced by the one given on  $L$  and the restriction to  $X$  of the Fubini-Study metric of  $\mathcal{O}(1) \simeq L^k$ . S. Zhang [10] has called *metric semiample* an ample line bundle such that  $(b_k)^{\frac{1}{k}}$  converges to 1 uniformly on  $X$ . It is known (independently due to G. Tian and the author [5]) that a positive line bundle is metric semiample. Using the proposition above, we can show that an ample line bundle endowed with a nonnecessarily positively curved metric satisfying the condition  $\int_X \alpha_1^{-6n} < +\infty$  is metric semiample, which gives a partial answer (in the case of smooth  $X$ ) to a question raised in [10]. In fact, we have the more precise statement (whose proof follows directly from the method of [5] together with the estimates of section 1 and Proposition 2) :

**Proposition 3** *There exist positive constants  $C_1, C_2$  such that*

$$C_1 k^{n(1-\varepsilon)} \leq b_k(x) \leq C_2 k^n$$

*uniformly over  $X$ .*

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